EDGE-ODD GRACEFULNESS OF $C_5 \Theta P_n$ AND $C_5 \Theta 2P_n$

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ABSTRACT

A $(p, q)$ connected graph is edge-odd graceful graph if there exists an injective map $E(G) \rightarrow \{1, 3, ..., 2q-1\}$ so that induced map $f^+: V(G) \rightarrow \{0, 1, 2, 3, ..., (2k -1)\}$ defined by $f^+(x) \equiv \sum f(x, y) \pmod{2k}$, where the vertex $x$ is incident with other vertex $y$ and $k = \max \{p, q\}$ makes distinct labeling. In this article, the edge-odd graceful labelings of $C_5 \Theta P_n$ and $C_5 \Theta 2P_n$ are obtained.

Keywords: Graceful graph, edge-odd graceful labeling, edge-odd graceful.

INTRODUCTION


Solairaju and Chitra [2009] obtained edge-odd graceful labeling of some graphs related to paths. They proved that the graph $C_3 \Theta P_n$ and $C_3 \Theta 2P_n$ are edge-odd graceful. Solairaju, and Sasikala [2008] got gracefulness of a spanning tree of the graph of product of $P_n$ and $C_m$.

Solairaju, and Vimala [2008] gracefulness of a spanning tree of the graph of Cartesian product of $S_m$ and $S_n$. Solairaju and Muruganantham [2009] proved that ladder $P_2 \times P_n$ is even-edge graceful (even vertex graceful). They found [2009] the connected graphs $P_n \circ nC_3$ and $P_n \circ nC_7$ are both even vertex graceful, where $n$ is any positive integer. They also obtained that the connected graph $P_n \Delta nC_4$ is even vertex graceful, where $n$ is any even positive integer.

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SECTION-2: EDGE-ODD GRACEFUL LABELING OF ARMED CROWN GRAPH C₅ ⊕ Pₙ

The following definitions are given now.

**Definition 2.1:** **Graceful Graph:** A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set \{0, 1, 2, ..., m\} such that when each edge uv is assigned the label \( |f(u) - f(v)| \) and the resulting edge labels are distinct. Then the graph G is graceful.

**Definition 2.2:** **Edge-odd graceful graph:** A (p, q) connected graph has edge-odd graceful labeling if there exists an injective map \( f: E(G) \rightarrow \{1, 3, ..., 2q-1\} \) so that induced map \( f+: V(G) \rightarrow \{0, 1, 2, ..., (2k-1)\} \) defined by \( f+(x) \equiv \sum f(x, y) \pmod {2k} \), where the vertex x is incident with other vertex y and \( k = \max\{p, q\} \) makes distinct labelings. Then the graph G is edge-odd graceful.

**Definition 2.3:** **Armed crown** \( C₅ ⊕ Pₙ \) is a connected graph such that each vertex of a circuit \( C₅ \) is identified with any pendant vertex of the paths \( Pₙ \). It has \( 5n \) vertices and \( 5n \) edges. Its vertex set is \{\( V₁, V₂, ..., V₅n \)\} and edge is \{\( VᵢVi₊₁: i = 1 \) to \( n; i = (n+1) \) to \( 2n; i = (2n+1) \) to \( 3n; i = (3n+1) \) to \( 4n \); \( i = (4n+1) \) to \( 5n \)\} \cup \{\( VₙVₙ₊₁, Vₙ₊₁V₂ₙ₊₁, V₂ₙ₊₁V₃ₙ₊₁, V₃ₙ₊₁V₄ₙ₊₁, V₄ₙ₊₁V₅ₙ \}. This graph is given in figure 1.

![Figure-1: Armed crown graph C₅ ⊕ Pₙ](image)

**Theorem 2.4:** The connected graph \( C₅ ⊕ Pₙ \) is edge-odd graceful for \( n \geq 2 \).

**Proof:** The figure 2 is the armed crown \( C₅ ⊕ Pₙ \) with \( 5n \) vertices and \( 5n \) edges, with some labelings to its edges.

![Figure-2: One of arbitrary labelings for edges of the graph C₅ ⊕ Pₙ](image)
Define \( f: E(G) \to \{1, 3, \ldots, 2q-1\} \) by
\[
f(e_i) = 2i - 1, \quad i = 1, 2, 3, \ldots, 5n
\] (1)

Define \( f+: V(G) \to \{0, 1, 2, \ldots, (2k-1)\} \) by
\[
f+(v) = \sum f(uv) \mod (2k), \text{ where this sum run over all edges through } v
\] (2)

Hence the map \( f \) and the induced map \( f_+ \) provide labels as distinct odd numbers for edges and also the labelings for vertex set has distinct values in \( \{1, 2, \ldots, (2k-1)\} \). Hence the graph \( C_5 \Theta P_n \) is edge-odd graceful.

**Example 2.5:** The connected graph \( C_5 \Theta P_6 \) is edge – odd graceful.

The edge – odd graceful labeling of the graph of \( C_5 \Theta P_6 \) with 30 vertices and 30 edges is as follows:

![Figure-3: Edge-odd labelings of the graph C5 \( \Theta \) P6](image)

**Example 2.6:** The connected graph \( C_5 \Theta P_5 \) is edge – odd graceful.

The figure 4 is the graph of \( C_5 \Theta P_5 \) with (25) vertices and (25) edges, with some edge-odd graceful labeling in vertices and edges as follows:

![Figure-4: Edge-odd labelings of the graph C5 \( \Theta \) P5](image)

**SECTION 3: BI-ARMED CROWN \( C_5 \Theta 2P_n \) IS EDGE-ODD GRACEFUL**

**Definition 3.1:** Bi-armed crown \( C_5 \Theta 2P_n \) is a connected graph such that each vertex of a circuit \( C_5 \) is identified with any pendant vertex of two paths \( P_n \). It has \( (10n - 5) \) vertices and \( (10n - 5) \) edges. Its vertex set is \( \{V_1, V_2, \ldots, V_{(10n-5)}\} \) and edge set is \( \{V_iV_{i+1} : i = 1 \text{ to } (2n-1); i = (2n) \text{ to } (4n-2); i = (4n - 1) \text{ to } (6n -3); i = (6n - 2) \text{ to } (8n -4) ; i = (8n - 3) \text{ to } (10n - 5)\} \cup \{V_nV_{2n}, V_{2n}V_{4n-2}, V_{4n-2}V_{6n-3}, V_{6n-3}V_{8n-4}, V_{8n-4}V_n\}. \)
Theorem 3.2: The connected graph (bi-armed crown graph) $C_5 \Theta 2P_n$ is edge – odd graceful.

Proof: The figure 6 is the armed crown $C_5 \Theta 2P_n$ with $(10n - 5)$ vertices and $(10n - 5)$ edges, with some arbitrary labeling to edges as follows.

Define $f: E(G) \rightarrow \{1, 3, \ldots, 2q-1\}$ by
$$f(e_i) = 2i - 1, \quad i = 1, 2, 3, \ldots, (10n-5) \quad (1)$$

Define $f+: V(G) \rightarrow \{0, 1, 2, \ldots, (2k-1)\}$ by
$$f+(v) = \Sigma f(uv) \mod (2k), \text{ where this sum run over all edges through } v \quad (2)$$

Hence the map $f$ and the induced map $f_+$ provide labels as odd numbers for edges with all distinct and also the labelings for vertex set has distinct values in $\{1, 2, \ldots, (2k-1)\}$. Hence the graph $C_5 \Theta 2P_n$ is edge-odd graceful.
Example 3.3: The connected graph $C_5 \Theta 2P_4$ is edge – odd graceful.

The figure 7 is the armed crown $C_5 \Theta 2P_4$ with 35 vertices and 35 edges, with edge-odd graceful labeling to vertices and edges.

Example 3.4: The connected graph $C_5 \Theta 2P_5$ is edge – odd graceful.

The figure 8 is the armed crown $C_5 \Theta 2P_5$ with 45 vertices and 45 edges, with edge-odd graceful labeling to vertices and edges.

REFERENCES


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