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EDGE-ODD GRACEFULNESS OF $C_5 \Theta P_n$ AND $C_5 \Theta 2P_n$

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ABSTRACT

A (p, q) connected graph is edge-odd graceful graph if there exists an injective map E(G) → {1, 3, …, 2q-1} so that induced map f_+: V(G)→ {0,1,2, 3, …, (2k-1)} defined by $f_+(x) \equiv \Sigma f(x, y)(\text{mod } 2k)$, where the vertex x is incident with other vertex y and k = max {p, q} makes distinct labeling. In this article, the edge-odd graceful labelings of $C_5 \,\Theta\, P_n$ and $C_5 \,\Theta\, 2P_n$ are obtained.

Keywords: *Graceful graph, edge-odd graceful labeling, edge-odd graceful.*

INTRODUCTION

Barrientos [2001] got that cyclic snakes are graceful. Barrientos [2002] investigated graceful labelings for chain and corona graph. Abhyankar and Bhat-Nayak [2000] got easiest graceful labeling of olive trees. Barrientos [2005a] obtained the graceful labeling for the union of cycle and complete bipartite graph. Barrientos [2007] analyzed graceful labeling for arbitrary super-subdivision of graph. Barrientos [2005b] found graceful labeling for graph with pendant edges.

Lee and Seah [1990] verified that kth power cycle is edge-graceful labeling. Wilson and Riskin [1998] obtained edge-graceful labeling for all odd cycles and their products. Shiu and Lam [2005] got super-edge-graceful labelings for multi-level wheel graphs, paths, circuits, and star graphs. Seoud Abdel Maqsoud, and Sheehan [2000] found graceful labeling for the union of cycles and paths. Panigrahi and Mishra [2008] investigated graceful labeling for all lobsters obtained from diameter four trees.

Lee, Pan, and Tsai [2005 showed that (p, p + 1) - graph is vertex-graceful for any positive integer p. Lee, Wang and Year [2005] identified super edge-graceful labeling for Eulerian graphs. Lee, Leung, and Ng [2009] proved that unicylic graphs are super vertex-graceful.

Solairaju and Chitra [2009] obtained edge-odd graceful labeling of some graphs related to paths. They proved that the graph $C_3 \,\Theta\, P_n$ and $C_3 \,\Theta\, 2P_n$ are edge -odd graceful. Solairaju, and Sasikala [2008] got gracefulness of a spanning tree of the graph of product of P_m and C_n,

Solairaju, and Vimala [2008] gracefulness of a spanning tree of the graph of Cartesian product of S_m and S_n. Solairaju and Muruganantham [2009] proved that ladder $P_2 \times P_n$ is even-edge graceful (even vertex graceful). They found [2009] the connected graphs P_n o nC_3 and P_n o nC_7 are both even vertex graceful, where n is any positive integer. They also obtained that the connected graph $P_n \Delta nC_4$ is even vertex graceful, where n is any even positive integer.

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SECTION-2: EDGE-ODD GRACEFUL LABELING OF ARMED CROWN GRAPH $C_5 \Theta P_n$

The following definitions are given now.

Definition 2.1: Graceful Graph: A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set {0, 1, 2, …, m} such that when each edge uv is assigned the label $|f(u) – f(v)|$ and the resulting edge labels are distinct. Then the graph G is graceful.

Definition 2.2: Edge-odd graceful graph: A (p, q) connected graph has edge-odd graceful labeling if there exists an injective map f: $E(G) \rightarrow \{1, 3, …, 2q-1\}$ so that induced map $f_+: V(G) \rightarrow \{0, 1, 2, …,(2k -1)\}$ defined by $f_+(x) \equiv \Sigma f(x, y) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes distinct labelings. Then the graph G is edge- odd graceful.

Definition 2.3: Armed crown $\mathbf{C_5} \Theta P_n$ is a connected graph such that each vertex of a circuit C_5 is identified with any pendant vertex of the paths P_n. It has 5n vertices and 5n edges. Its vertex set is $\{V_1, V_2,…, V_{5n}\}$ and edge is $\{V_iV_{i+1}$: i = 1 to n; i = (n+1) to (2n); i = (2n+1) to (3n); i = (3n+1) to (4n) i = (4n+1) to (5n)$\} \cup \{V_nV_{n+1}, V_{n+1}V_{2n+1}, V_{2n+1}V_{3n+1}, V_{3n+1}V_{4n+1}, V_{5n+1}V_n \}$. This graph is given in figure 1.

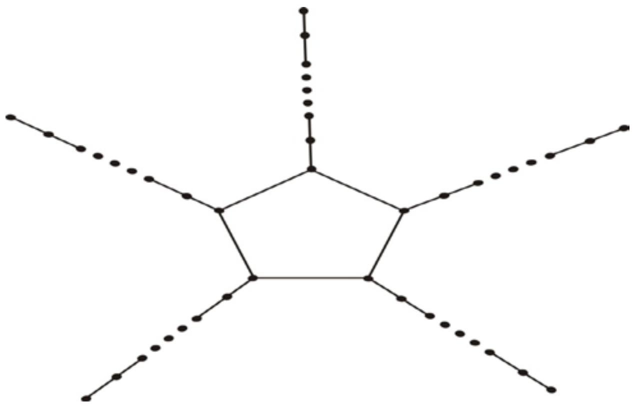

Figure-1: Armed crown graph $C_5 \Theta P_n$

Theorem 2.4: The connected graph $C_5 \Theta P_n$ is edge – odd graceful for n ≥ 2.

Proof: The figure 2 is the armed crown $C_5 \Theta P_n$ with 5n vertices and 5n edges, with some labelings to its edges.

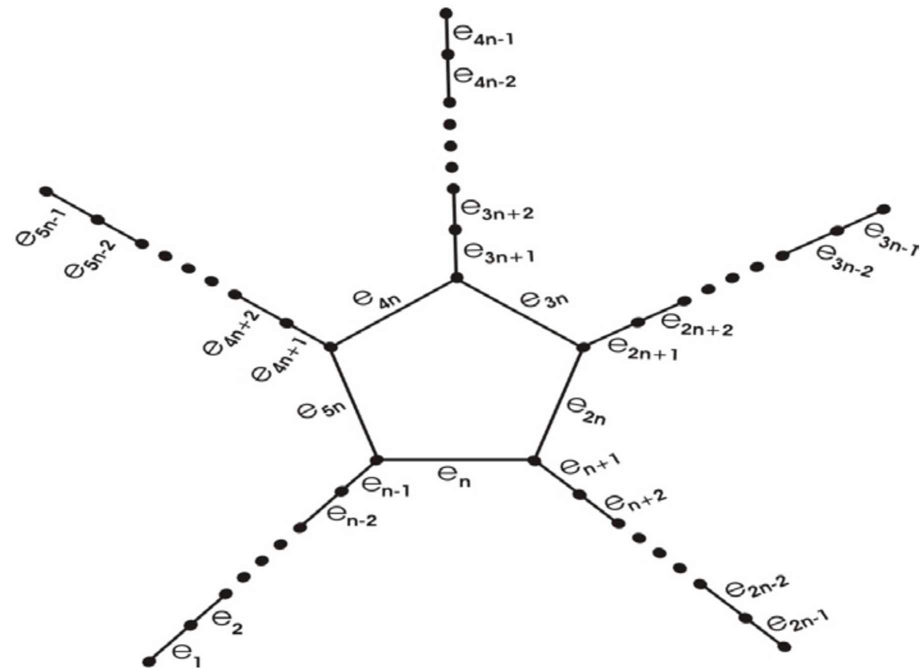

Figure-2: One of arbitrary labelings for edges of the graph $C_5 \Theta P_n$.

Define f: E(G) → {1, 3, …, 2q-1} by

$$f(ei) = 2i - 1, i = 1, 2, 3, \ldots, 5n \tag{1}$$

Define f_+: V(G) → {0,1,2,…, (2k-1)} by

$$f_+(v) \equiv \Sigma f(uv) \bmod (2k), \text{ where this sum run over all edges through } v \tag{2}$$

Hence the map f and the induced map f_+ provide labels as distinct odd numbers for edges and also the labelings for vertex set has distinct values in {1, 2,…., (2k-1)}. Hence the graph $C_5 \Theta P_n$ is edge-odd graceful.

Example 2.5: The connected graph $C_5 \Theta P_6$ is edge – odd graceful.

The edge – odd graceful labeling of the graph of $C_5 \Theta P_6$ with 30 vertices and 30 edges is as follows:

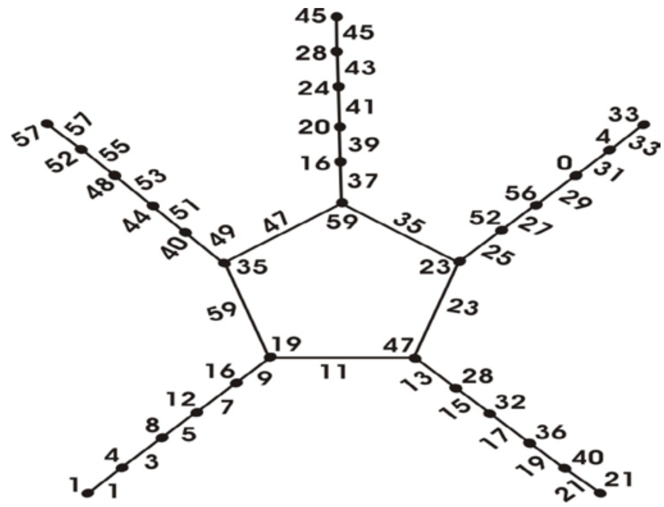

Figure-3: Edge-odd labelings of the graph $C_5 \Theta P_6$

Example 2.6: The connected graph $C_5 \Theta P_5$ is edge – odd graceful.

The figure 4 is the graph of $C_5 \Theta P_5$ with (25) vertices and (25) edges, with some edge-odd graceful labeling in vertices and edges as follows:

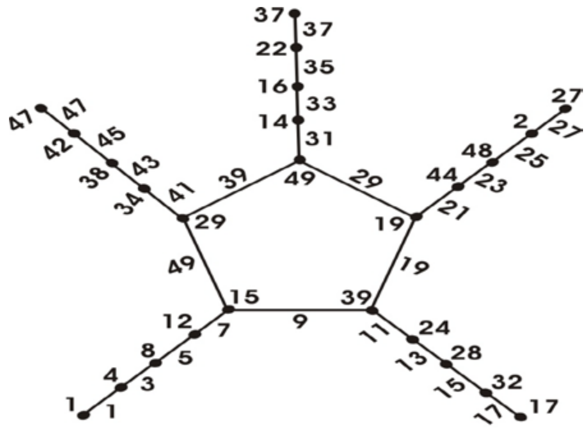

Figure-4: Edge-odd labelings of the graph $C_5 \Theta P_5$

SECTION 3: BI-ARMED CROWN $C_5 \Theta 2P_n$ IS EDGE-ODD GRACEFUL

Definition 3.1: Bi-armed crown $\mathbf{C}_5 \Theta 2P_n$ is a connected graph such that each vertex of a circuit C_5 is identified with any pendant vertex of two paths P_n. It has (10n - 5) vertices and (10n - 5) edges. Its vertex set is $\{V_1, V_2, \ldots, V_{(10n-5)}\}$ and edge set is $\{V_iV_{i+1}$: i = 1 to (2n-1); i = (2n) to (4n-2); i = (4n - 1) to (6n -3); i = (6n - 2) to (8n - 4) i = (8n - 3) to (10n - 5)$\} \cup \{V_nV_{2n}, V_{2n}V_{4n-2}, V_{4n-2}V_{6n-3}, V_{6n-3}V_{8n-4}, V_{8n-4}V_n\}$.

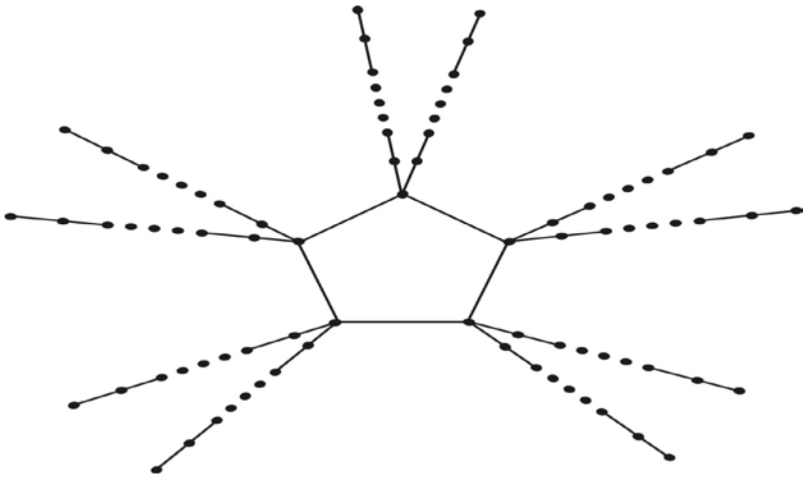

Figure-5: Bi-armed crown graph C_5 Θ $2P_n$

Theorem 3.2: The connected graph (bi-armed crown graph) C_5 Θ $2P_n$ is edge – odd graceful.

Proof: The figure 6 is the armed crown C_5 Θ $2P_n$ with (10n - 5) vertices and (10n - 5) edges, with some arbitrary labeling to edges as follows.

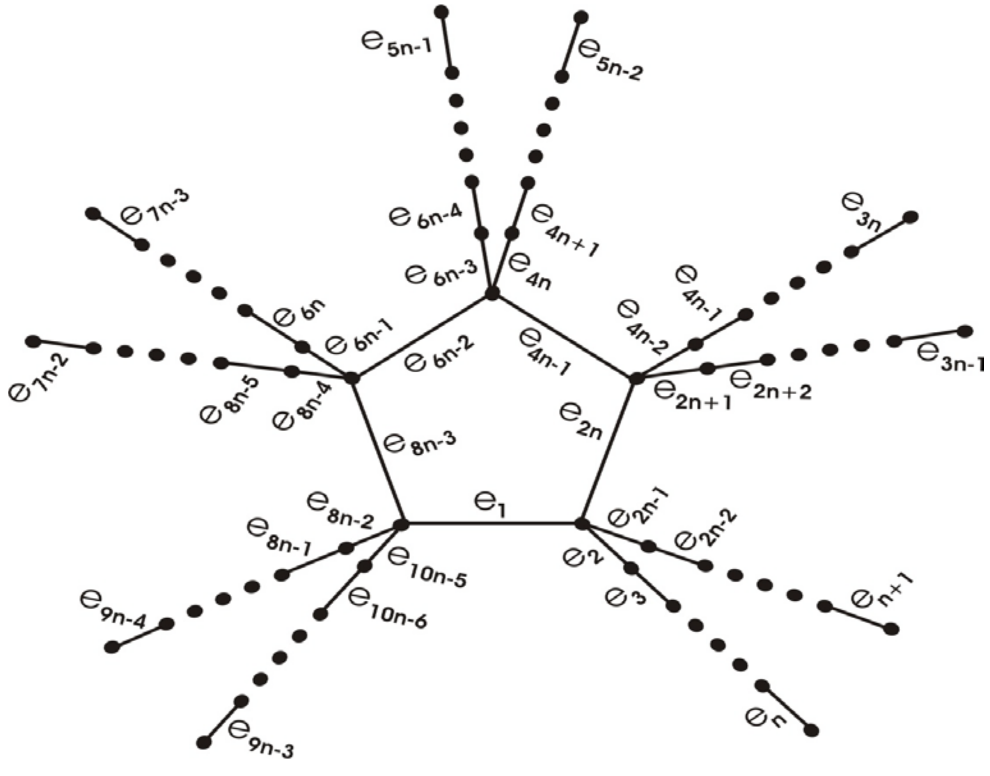

Figure-6: One of the arbitrary labelings of the graph C_5 Θ $2P_n$

Define f: E(G) → {1, 3, …, 2q-1} by

$$f(ei) = 2i - 1, \; i = 1, 2, 3, \ldots, (10n-5) \qquad (1)$$

Define f_+: V(G) → {0, 1, 2, …, (2k-1)} by

$$f_+(v) \equiv \Sigma f(uv) \bmod (2k), \text{ where this sum run over all edges through v} \qquad (2)$$

Hence the map f and the induced map f_+ provide labels as odd numbers for edges with all distinct and also the labelings for vertex set has distinct values in {1, 2,…., (2k-1)}. Hence the graph C_5 Θ $2P_n$ is edge-odd graceful.

Example 3.3: The connected graph C_5 Θ $2P_4$ is edge – odd graceful.

The figure 7 is the armed crown C_5 Θ $2P_4$ with 35 vertices and 35 edges, with edge-odd graceful labeling to vertices and edges.

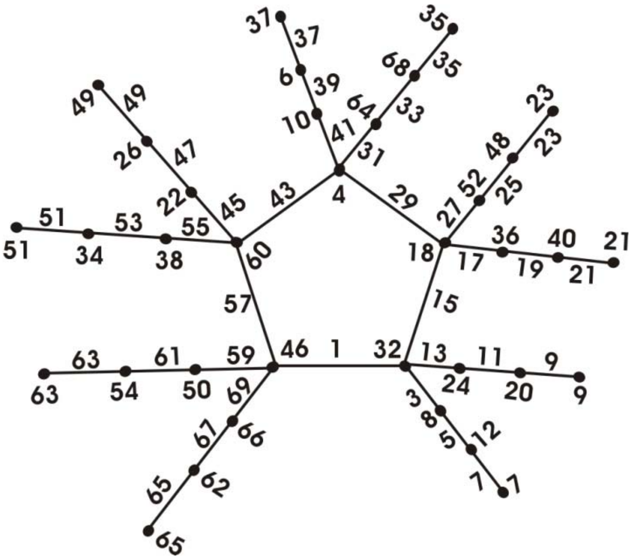

Figure-7: Edge-odd labelings for the graph C_5 Θ $2P_4$

Example 3.4: The connected graph C_5 Θ $2P_5$ is edge – odd graceful.

The figure 8 is the armed crown C_5 Θ $2P_5$ with 45 vertices and 45 edges, with edge-odd graceful labeling to vertices and edges.

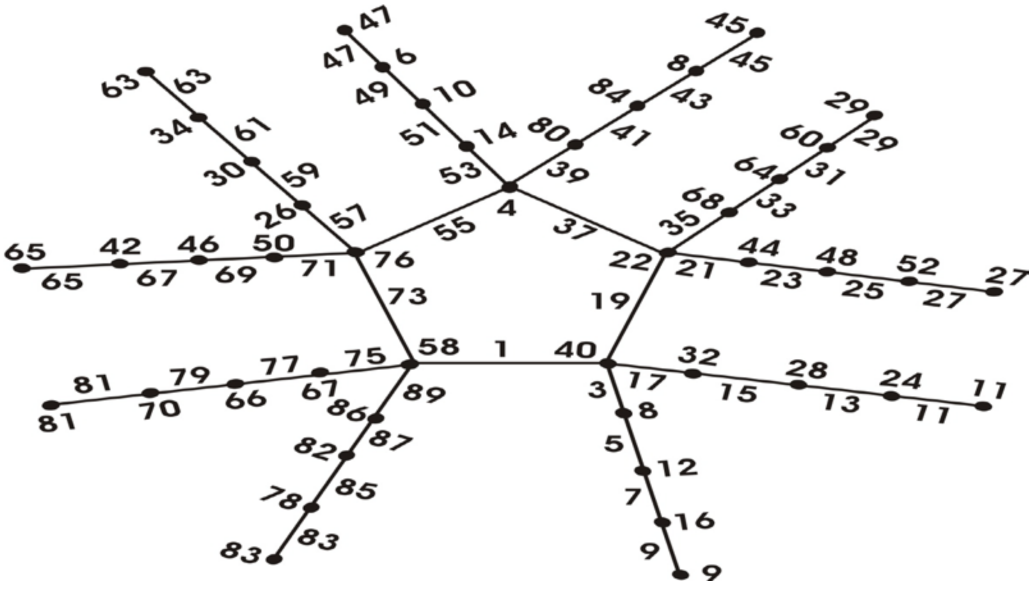

Figure-8: Edge – odd labelings of the graph C_5 Θ $2P_5$

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