DOMINATION PARAMETERS OF SOME GRAPHS AND ITS REALIZATION

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ABSTRACT

The domination parameters of a graph G of order n has been already introduced in [9]. It is defined as \( D \subseteq V(G) \) is a dominating set of G, if every vertex \( v \in V-D \) is adjacent to atleast one vertex in D. In this paper we established various domination parameters of some graphs such as path, cycle, wheel, star, r-corona and complete bipartite graph with m, n vertices. Also established the relation between this parameters and illustrated an example for some graphs which is deviated from its general formula.

1. INTRODUCTION

A graph \( G = (V, E) \), were V is a finite set of elements called vertices and E is a set of unordered pairs of distinct vertices of G called edges. The degree of a vertex \( v \) in G is the number of edges incident on it. A graph G is said to be k-regular if all its vertices are of degree k. Every pair of its vertices are adjacent in G, is said to be complete, the complete graph on 'n' vertices is denoted by \( K_n \).

A graph G is said to be bipartite or bigraph if the vertex set of V(G) can be partitioned in to two subsets X and Y such that every edge of G has one in X and the other end in Y. A bipartite graph G with \( |X| = m \) and \( |Y| = n \) is said to be complete if every element in one partition is adjacent with all elements of the other partition and is denoted by \( K_{m,n} \). The graph \( K_{1,n} \) is called a Star graph.

Let \( u, v \) be the vertices of a graph G, a u-v walk of G is an alternating sequences \( u = u_0, e_1, u_1, u_2, ..., u_{n-1}, e_n, u_n = v \) of vertices and edges beginning with vertex u and ending with vertex v such that \( e_i = u_{i-1}u_i \) for all \( i = 1, 2, ..., n \). The number of edges in a walk is called its length. A walk in which all the vertices are distinct is called a path. A path on 'n' vertices is denoted by \( P_n \). A closed path is called a cycle, a cycle on 'n' vertices are denoted by \( C_n \). Let G = (V, E) be a simple connected graph, for any vertex \( v \in V \), the open neighborhood is the set \( N(v) = \{u \in V/ u v \in E\} \) and the closed neighborhood of v is the set \( N(v) \cup \{v\} \). For a set \( S \subseteq V \), the open neighborhood of S is \( N(S) = \bigcup_{v \in S} N(v) \) and the closed neighborhood of S is \( N(S) = N(S) \cup S \).

Definition 1.1: A set \( D \subseteq V \) is a dominating set of G if every vertex \( v \in V-D \) is adjacent to atleast one vertex of D. We call a dominating set D is a minimal if there is no dominating set \( D' \subseteq V(G) \) with \( D' \subset D \) and \( D' \neq D \). Further we call a dominating set D is minimum if these is no dominating set \( D' \subseteq V(G) \) with \( |D'| < |D| \). The cardinality of a minimum dominating set is called the domination number denoted by \( \gamma(G) \) and the minimum dominating set D of G is also called a \( \gamma \)- set.

Definition 1.2: A dominating set D is said to be a total dominating set if every vertex in V is adjacent to some vertex in D. The total domination number of G. denoted by \( \gamma_t(G) \) is the minimum cardinality of a dominating set.

Definition 1.3: A dominating set D of a graph G is an independent dominating set, if the induced subgraph \( \langle D \rangle \) has no edges. The independent domination number \( \gamma_i(G) \) is the minimum cardinality of an independent dominating set.

Definition 1.4: A dominating set D is said to be connected dominating set, if the induced subgraph \( \langle D \rangle \) is connected. The connected domination number \( \gamma_c(G) \) is the minimum cardinality of a connected dominating set.

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Definition 1.5: A dominating Set D of a graph G is said to be a paired dominating set if the induced subgraph <D> contains at least one perfect matching, paired domination number \( \gamma_p(G) \) is the minimum cardinality of a paired dominating set.

Definition 1.6: A dominating Set D of G is a split dominating set if the induced subgraph <V – D> disconnected Split domination number \( \gamma_s(G) \) is the minimum cardinality of a split dominating set.

Definition 1.7: A dominating Set D of G is a non split dominating set, if the induced subgraph <V – D> is connected. Non split domination number \( \gamma_{ns}(G) \) is the minimum cardinality of a non split dominating set.

Definition 1.8: Let D be a \( \gamma^- \) set of G. A dominating set \( D_1 \) contained in V – D is called an inverse dominating set of G with respect to D. The inverse domination number \( \gamma'(G) \) is the minimum cardinality of all inverse dominating set of G, the vertices of \( \gamma'(G) \) is called \( \gamma^- \) set.

Definition 1.9: A dominating set D of a graph G is called a global dominating set, if D is also a dominating set of G. The global domination number \( \gamma_g(G) \) in the minimum cardinality of a global dominating set.

Definition 1.10: A dominating set D is called a perfect dominating set, if every vertex in V – D in adjacent to exactly one vertex in D. The perfect domination number \( \gamma_{pt}(G) \) is the minimum cardinality of a perfect dominating set.

Definition 1.11: If D = \{x\} is a dominating set of G, then x is called a dominating vertex of G. A vertex \( v \in V(G) \) is said to be a \( \gamma^- \) required vertex of G, if v lies in every \( \gamma^- \) set of G.

Definition 1.12: Let x be any real value, then its upper sealing of x is denoted as \( \varphi_x \) and is defined

\[
\varphi_x = \begin{cases} 
  x & \text{if } x \text{ is an integer} \\
  k, \text{ where } k \text{ is an integer lies in the interval } x < k < x + 1 & \text{otherwise}
\end{cases}
\]

the lower sealing of x is denoted as \( \lambda_x \) and is defined by

\[
\lambda_x = \begin{cases} 
  x & \text{if } x \text{ is an integer} \\
  k, \text{ where } k \text{ is an integer lies in the interval } x - 1 < k < x & \text{otherwise}
\end{cases}
\]

Lemma 2.1: Let G be a connected graph with \( \delta(G) \geq 2 \), then \( \gamma(G) + \gamma'(G) = n \) if and only if G = P4 or C4.

Lemma 2.2: Let G be a connected graph with \( \delta = 1 \) and \( \Delta = n \) then \( \gamma(G) + \gamma'(G) = n + 1 \) if and only if G = k1, n.

Lemma 2.3: For any tree with \( n \geq 2 \) with more than two pendant vertices then there exists a vertex v \( v \in V \) such that \( \gamma(T - v) = \gamma(T) \).

Lemma 2.4: For any path \( P_n \), \( \gamma(P_n) \leq \gamma'(P_n) \) \( \forall n \geq 3 \).

Proof: Since \( P_n \) is a path with n vertices then

\[
\gamma(P_n) = \begin{cases} 
  \gamma'(P_n) - 1 & \text{if } n = 3k, \forall k = 1, 2 \ldots \\
  \gamma'(P_n) & \text{otherwise}
\end{cases}
\]

therefore, \( \gamma(P_n) \leq \gamma'(P_n) \) \( \forall n \geq 2 \)

Note: Let G be a path of length n then

\[
\gamma(P_n) = \left\lfloor \frac{n}{3} \right\rfloor \quad \forall \ n > 3
\]

\[
\gamma'(P_n) = \left\lfloor \frac{n}{3} \right\rfloor + 1
\]

Lemma 2.5: Let G be a cycle of length four then \( \gamma(G) = \gamma'(G) = \gamma_d(G) = \gamma_s(G) = \gamma_p(G) = \gamma_t(G) = \gamma_{ns}(G) = \gamma_{ns}(G) = \gamma_t(G) = 2 \).

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Proof: Let \( v_1, v_2, v_3 \) and \( v_4 \) are the vertices of \( C_4 \), each vertex \( v_i \) connected with \( v_{i+1} \), \( i = 1, 2, 3 \) and \( v_4 \) is connected with \( v_1 \) in \( G \). Hence \( v_1, v_3 \) and \( v_2, v_4 \) are the edges in \( \overline{G} \). Let \( D = \{v_1, v_2\} \) be the vertices of \( G \). Clearly \( D \) satisfies the conditions for total domination, connected dominating, paired domination, global domination and non split domination therefore, \( \gamma(G) = \gamma_t(G) = \gamma_c(G) = \gamma_p(G) = \gamma_g(G) = \gamma_{ns}(G) = 2 \).

Let \( D' = \{v_3, v_4\} \) satisfies the condition for the inverse domination, therefore, \( \gamma'(G) = 2 \). Let \( D_1 = \{v_1, v_3\} \) satisfies the condition for independent domination and split domination, therefore, \( \gamma_i(G) = \gamma_s(G) = 2 \).

Hence, \( \gamma(G) = \gamma_t(G) = \gamma_p(G) = \gamma_g(G) = \gamma_{ns}(G) = \gamma'(G) = \gamma_i(G) = \gamma_s(G) = 2 \).

Note: \( C_4 \) is the smallest simple connected graph which satisfies the conditions of all dominations parameters with its cardinality is two.

Lemma 2.6: For any complete graph \( K_n \),
\[
\gamma(G) = \gamma'(G) = \gamma_i(G) = \gamma_s(G) = 3 \quad \text{and} \quad \gamma_t(G) = \gamma_p(G) = 2
\]

Theorem 2.7: For any path \( P_n \), \( n > 2 \) the domination parameters satisfies the following.

i) \( \gamma(G) = \gamma_t(G) = \gamma_p(G) = \gamma_g(G) = \left\lceil \frac{n}{3} \right\rceil \)

ii) \( \gamma'(G) = \left\lfloor \frac{n}{3} \right\rfloor + 1 \) if \( n = 4k + 1 \) \( k = 1, 2, \ldots \)

iii) \( \gamma_c(G) = \begin{cases} 2k + 1 & \text{if } n = 4k + 1 \ k = 1, 2, \ldots \\ 2 \left\lceil \frac{n}{4} \right\rceil & \text{otherwise} \end{cases} \)

iv) \( \gamma_p(G) = 2 \left\lceil \frac{n}{4} \right\rceil \)

v) \( \gamma_s(G) = \gamma_{ns}(G) = n - 2 \)

Proof:

i) Let \( P_n \) be the path and its vertices are denoted by \( v_1, v_2, v_3, \ldots, v_n \) for all \( n > 2 \). Now subdivide the path into sub graphs \( G_1, G_2, G_3, \ldots, G_k \) such that each sub graphs \( G_i, i = 1, 2, \ldots, k \) containing three consecutive vertices from the beginning.

That is \( G_1 = \{v_1, v_2, v_3\} \); \( G_2 = \{v_4, v_5, v_6\} \); \( G_3 = \{v_7, v_8, v_9\} \), \ldots, \( G_k = \{v_{n-2}, v_{n-1}, v_n\} \) if \( n = 3k \), collect all the vertices \( \{v_{2k+i} \in G_k, k = 1, 2, 3, \ldots, \text{and } i = k - 1 \text{ is the required minimum dominating set of } G \).

That is, \( D = \{v_2, v_5, v_8, v_{11}, \ldots, v_{n-1}\} \), we have collected exactly one element form each \( G_i, i = 1, 2, \ldots, k \).

Hence, \( |D| = k \)
\[
= \left\lfloor \frac{n}{3} \right\rfloor \quad \text{[Since } n = 3k] \\
= \left\lfloor \frac{n}{3} \right\rfloor \quad \text{[} \frac{n}{3} \text{ is an integer } \left\lfloor \frac{n}{3} \right\rfloor = \frac{n}{3} \] \]

Suppose \( n = 3k - 1 \), then the last partition \( G_k \) contains only last two vertices. That is, \( G_k = \{v_{n-1}, v_n\} \) then \( S = \{v_{2k+i} \text{ }/ \text{ } k = 1, 2, \ldots, i = k - 1 \text{ and } 2k + i \leq n - 2 \} \) now \( S = \{v_2, v_5, v_8, \ldots, v_{n-3}\} \) then \( D = S \cup \{v_{n-1}\} \) and \( D = S \cup \{v_n\} \) is the required minimum dominating sets of \( G \) with cardinality \( k \).

\[
n = (k - 1) 3 + 2 \Rightarrow \frac{n}{3} = (k - 1) + \frac{2}{3}
\]
\[
\Rightarrow \left\lfloor \frac{n}{3} \right\rfloor = [k - 1] + \left\lfloor \frac{2}{3} \right\rfloor = (k - 1) + 1 \Rightarrow |D| = \left\lfloor \frac{n}{3} \right\rfloor .
\]

If \( n = 3k - 2 \), as in the above case all sub graphs \( G_i, i = 1, 2, \ldots, k - 1 \). Containing three vertices and the last partition \( G_k \) containing the only vertex \( v_n \).
Now \( S = \{v_{2k+i}/k = 1, 2, 3, \ldots; 2k+i \leq n-1\text{ and } i = k-1\} \) then \( D = S \cup G_k \) is the minimum dominating set with cardinality \( k \)

\[(i)\]

We have \( n = 3k - 2 \Rightarrow n = 3(k - 1) + 1 \)

\[
\begin{align*}
\left\lfloor \frac{n}{3} \right\rfloor & = k - 1 + \frac{1}{3} \\
\left\lfloor \frac{n}{3} \right\rfloor & = k - 1 + \left\lfloor \frac{1}{3} \right\rfloor \Rightarrow \left\lceil \frac{n}{3} \right\rceil = |D|
\end{align*}
\]

Therefore, in each case \( \gamma(G) = \left\lceil \frac{n}{3} \right\rceil \)

In all cases the induced subgraph \( D \) are independent in \( G \). therefore, \( \gamma_i(G) = \gamma(G) = \left\lceil \frac{n}{3} \right\rceil \)

Since, \( P_n \) is a tree the induced subgraph \( <V-D> \) is disconnected \( \Rightarrow \gamma_i(G) = \gamma(G) = \left\lceil \frac{n}{3} \right\rceil \)

Clearly \( D \) is dominating set of \( G \).

Hence, \( \gamma(G) = \gamma_i(G) = \gamma_d(G) = \gamma_e(G) = \left\lceil \frac{n}{3} \right\rceil \)

(ii) Case (i): if \( n = 3k \) by case (i) \( D = \{v_{2k+i}/k = 1, 2, 3, \ldots; i = k-1 \text{ and } 2k+i \leq n\} \)

That is, \( D = \{v_2, v_5, v_8, \ldots, v_{n-2}, v_n\} \) is the minimum dominating set of \( P_n \).

Now choose the elements of \( v \in V - D \) such that

\[S = \{v_{3i+1}/i = 0, 1, 2, \ldots \text{ and } 3i+1 \leq n\} \text{ then} \]

\[S' = \{v_1, v_4, v_7, v_{10}, \ldots, v_{n-2}\}, \text{ now } v_n \text{ is not adjacent to any vertex } v_i \in S'\]

Let \( D' = S' \cup \{v_n\}, D' = \{v_1, v_4, v_7, \ldots, v_{n-3}, v_{n-1}\} \) now we have selected one vertex from each subgraph \( G_k, k = 1, 2, 3, \ldots, k - 1 \) such that \( v_{3i+1} \in G_{i+1} \) and two elements \( v_{n-2} \) and \( v_n \) from \( G_k \).

Therefore, \( |D'| = k - 1 + 2 \)

\[= n + 1 \]

\[= \left\lceil \frac{n}{3} \right\rceil + 1 \]

Case (ii): \( n = 3k-2 \) by case (i) the sub graphs \( G_i, i = 1, \ldots, k - 1 \) containing exactly three vertices and \( G_k \) contains only one vertex \( \{v_n\} \) and \( D = \{v_2, v_5, \ldots, v_{n-2}, v_n\} \) by case (i) \( D' = \{v_{3i+1}/i = 0, 1, 2, \ldots \text{ and } 3i+1 \leq n\} \)

\[D' = \{v_1, v_4, v_7, \ldots, v_{n-3}, v_{n-1}\} \] is the required inverse dominating set of \( G \).

\[|D'| = (k - 2) + 2 = k \]

We have, \( n = 3k - 2 \)

\[
\begin{align*}
\left\lfloor \frac{n}{3} \right\rfloor & = k - 1 + \frac{1}{3} \\
\left\lfloor \frac{n}{3} \right\rfloor & = k - 1 + \left\lfloor \frac{1}{3} \right\rfloor \\
\left\lfloor \frac{n}{3} \right\rfloor & = k - 1 \quad \{\because \left\lfloor \frac{1}{3} \right\rfloor = 0\} \\
\Rightarrow \quad k & = \left\lceil \frac{n}{3} \right\rceil + 1 \\
\Rightarrow \quad |D| & = \left\lceil \frac{n}{3} \right\rceil + 1
\end{align*}
\]
Case (iii): \( n = 3k – 1 \) by the previous argument each subgraph \( G_i, i = 1, 2, \ldots , k – 1 \) containing exactly three vertices and \( G_k \) contains two vertices \( \{v_{n – 1}, v_n\} \) then

\[
D = \left\{ \frac{2k + i}{k = 1, 2, \ldots ; 2k + i \leq n \text{ and } i = k – 1} \right\}
\]

is the dominating set of \( G \) and

\[
D' = \left\{ \frac{3i + 1}{i = 0, 1, 2, \ldots \text{ and } 3i + 1 \leq n} \right\}
\]

\( D' = \{v_1, v_4, v_7, \ldots , v_{n – 1}\} \) is the inverse dominating set with respect to \( D \) in \( G \) and \( |D'| = k \). that is,

\[
n = 3(k – 1) + 2 \Rightarrow \frac{n}{3} = k + \frac{1}{3} + 0
\]

\[
\Rightarrow k = \left\lfloor \frac{n}{3} \right\rfloor + 1
\]

\[
\Rightarrow |D'| = \left\lfloor \frac{n}{3} \right\rfloor + 1
\]

Hence, \( \gamma'(P_n) = \left\lfloor \frac{n}{3} \right\rfloor + 1 \) for all \( n \).

(iii) if \( n = 4k + 1 \). Divide the vertices of \( G \) into \( k \) partition such that each partition \( G_i, i = 1, \ldots , k – 1 \) containing four vertices and the last partition \( G_k \) contains exactly five vertices

\[
G_1 = \{v_1, v_2, v_3, v_4\}; \ G_2 = \{v_5, v_4, v_7, v_8\}; \ldots \; . \ G_k = \{v_{n – 4}, v_{n – 3}, v_{n – 2}, v_{n – 1}, v_n\}
\]

Choose middle two vertices from \( G_i, i = 1, \ldots , k \) and three vertices from \( G_k \).

\( D_t(G) = \left\{ v_{2k + i} / k = 1, 2, \ldots ; i = k – 1 \text{ and } 3k + i \leq n \right\} \cup \{v_{n – 1}\} \) is the required minimum total dominating set of \( G \) and \( |D_t(G)| = 2(k – 1) + 3 \Rightarrow |D_t(G)| = 2k + 1 \)

That is, \( \gamma_t(P_n) = 2k + 1 \) if \( n = 4k + 1 \)

if \( n \neq 4k + 1 \), then the vertices is of the form \( \{v_1, v_2, v_3, v_4\}; \ G_2 = \{v_5, v_6, v_7, v_8\}; \ldots \) the last partition \( G_k \) is either \( \{v_{n – 5}, v_{n – 4}, v_{n – 3}, v_{n – 2}, v_{n – 1}, v_n\} \) or \( G_k = \{v_{n – 6}, v_{n – 5}, v_{n – 4}, v_{n – 3}, v_{n – 2}, v_{n – 1}, v_n\} \)

In both \( G_k \) we have to select two pair of vertices for the total domination of \( G_k \) and a pair of vertices \( \{(G_{2k + i}, \ v_{3k + i}) / k = 1, 2, \ldots ; i = k – 1 \text{ and } 3k + i \leq n -6\} \).

Therefore, \( |D_t(G)| = (k – 1)2 + 4 \)

\( = 2k + 2 \) (ii)

\( \begin{align*}
\text{If } n = 4k + 2 & \Rightarrow \frac{n}{4} = k + \frac{2}{4} \\
\left\lfloor \frac{n}{4} \right\rfloor & = k + \left\lfloor \frac{2}{4} \right\rfloor \\
& = k + 1
\end{align*} \)

\( \Rightarrow 2(k + 1) = 2\left\lfloor \frac{n}{4} \right\rfloor \)

\( \Rightarrow |D_t(G)| = 2\left\lfloor \frac{n}{4} \right\rfloor \left\lfloor |D_t(G)| = k + 1 \right\rfloor \)

If \( n = 4k + 3 \)

\( \begin{align*}
\frac{n}{4} & = k + \frac{3}{4} \Rightarrow \left\lfloor \frac{n}{4} \right\rfloor = k + \left\lfloor \frac{3}{4} \right\rfloor \\
& = k + 1
\end{align*} \)

\( \Rightarrow 2(k + 1) = 2\left\lfloor \frac{n}{4} \right\rfloor \)

\( \Rightarrow |D_t(G)| = 2\left\lfloor \frac{n}{4} \right\rfloor \)

\( \Rightarrow \gamma_t(P_n) = \begin{cases} 2k + 1 & \text{if } n = 4k + 1 \\ 2\left\lfloor \frac{n}{4} \right\rfloor & \text{otherwise} \end{cases} \)
(iv) by (iii) divide the vertices of $P_n$ into $k$ subsets such that each subset containing four vertices is of the form $G_1 = \{v_1, v_2, v_3, v_4\} \; ; \; G_2 = \{v_5, v_6, v_7, v_8\} \ldots$ then the $k^{th}$ partition $G_k$ is any one of the following.

\[
G_k = \begin{cases} \
\{v_{n-3}, v_{n-2}, v_{n-1}, v_n\} & \text{if } n = 4k \\
\{v_{n-4}, v_{n-3}, v_{n-2}, v_{n-1}, v_n\} & \text{if } n = 4k + 1 \\
\{v_{n-5}, v_{n-4}, v_{n-3}, v_{n-2}, v_{n-1}, v_n\} & \text{if } n = 4k + 2 \\
\{v_{n-6}, v_{n-5}, v_{n-4}, v_{n-3}, v_{n-2}, v_{n-1}, v_n\} & \text{if } n = 4k + 3 
\end{cases}
\]

If $n = 4k$, the pair of middle two vertices in each partition of $G_i$, $i = 1 \ldots k$ is the required minimum paired dominating set of $G$.

That is $D_p(G) = \{v_{2i}, v_{2i+1} \; / \; i = 1, 3, 5, \ldots, \; \text{and} \; 2i + 1 \leq n\}$ then $D_p(G) = \{v_2, v_3, v_5, v_7, v_{10}, v_{11}, \ldots, v_{n-2}, v_{n-1}\}$

\[
\Rightarrow |D_p(G)| = 2k = 2 \left[ \frac{n}{4} \right] \\
\Rightarrow 2 = \left[ \frac{n}{4} \right] \quad [\square n = 4k]
\]

If $n = 4k + 1$, select middle two vertices from each partition $G_i$, $i = 1 \ldots, k-1$, and any two pair of vertices which forms a paired dominating set of $G_i$.

\[
|D_p(G)| = 2(k-1) + 4 = 2k + 2 = 2(k + 1)
\]

\[
= 2 \left( \frac{n - 1}{4} + 1 \right) \quad [\square n = 4k + 1]
\]

\[
\Rightarrow 2 = \left[ \frac{n}{4} \right] \quad \text{[} n - 1 \text{ is a multiple of 4}
\]

\[
\Rightarrow \gamma_p(G) = 2 \left[ \frac{n}{4} \right]
\]

In similar, if $n = 4k + 2$

\[
|D_p(G)| = 2(k-1) + 4 = 2k + 2 = 2(k + 1)
\]

\[
= 2 \left( \frac{n - 2}{4} + 1 \right) \quad [\square n = 4k + 2]
\]

\[
\Rightarrow 2 = \left[ \frac{n}{4} \right] \quad \text{[} n - 2 \text{ is a multiple of 4}
\]

\[
\Rightarrow \gamma_p(G) = 2 \left[ \frac{n}{4} \right]
\]

In similar $|D_p(G)| = 2 \left[ \frac{n}{4} \right]$, if $n = 4k + 3$.

Hence, \(\gamma_p(G) = 2 \left[ \frac{n}{4} \right] \quad \forall \; n \geq 2\)

(v) Let $G = \{v_1, v_2, \ldots, v_n\}$ be the vertex of $P_n$ then by definition

(vi) $D_c(G) = G - \{v_1, v_n\}$ and $D_{ns}(G) = G - \{v_1, v_n\}$

Therefore, $\gamma_c(G) = \gamma_{ns}(G) = n - 2 \; \text{for all } n > 3$.
Result 2.8: If G is a path with n vertices then
\[ \gamma \leq \gamma'_i \leq \gamma_s \leq \gamma' \leq \gamma_m \leq \gamma_c \]

The following table represents the values of the various domination parameters of \( P_n, n \leq 10 \).

<table>
<thead>
<tr>
<th>( P_n )</th>
<th>( \gamma )</th>
<th>( \gamma'_i )</th>
<th>( \gamma_s )</th>
<th>( \gamma' )</th>
<th>( \gamma_m )</th>
<th>( \gamma_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_3 )</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( P_4 )</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( P_5 )</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>( P_6 )</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>( P_7 )</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>( P_8 )</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>( P_9 )</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>( P_{10} )</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

Corollary 2.9: For any integer \( n \geq 4 \), the only graph which satisfy the condition
\[ \gamma(P_n) = \gamma_i(P_n) = \gamma_s(P_n) = \gamma_g(P_n) = \gamma'_i(P_n) = \gamma_t(P_n) = \gamma_p(P_n) = \gamma_m(P_n) = \gamma_c(P_n) = 2 \] is \( P_4 \)

Proposition 2.10: For any integer \( n \geq 4 \)
\[ \gamma(P_n) = \gamma_i(P_n) = \gamma_s(P_n) = \gamma_g(P_n) = 2 \iff n = 4, 5 \]

Proof: by (i) of Theorem 2.7
\[ \gamma(P_n) = \gamma_i(P_n) = \gamma_s(P_n) = \gamma_g(P_n) = \left\lfloor \frac{n}{3} \right\rfloor \]

Hence, \( \gamma(P_n) = \gamma_i(P_n) = \gamma_s(P_n) = \gamma_g(P_n) = 2 \iff n = 4, 5 \).

Theorem 2.11: For any path \( P_n, n > 3, G = \overline{P_n} \) then
\[ \gamma(G) = \gamma_i(G) = \gamma'(G) = \gamma_m(G) = \gamma_c(G) = 2 \]

Proof: Let \( v_1, v_2, v_3, \ldots, v_n \) be the vertices of the graph \( P_n \) and each vertex \( v_i \), \( i = 2, \ldots, n-1 \) is connected with \( v_{i-1} \) and \( v_{i+1} \), and \( v_1 \) and \( v_n \) are connected only with \( v_2 \) and \( v_{n-1} \) respectively, that is
\[ d(v_i) = \begin{cases} 2 & i = 1, n \ 
\end{cases} 
\]

Let \( G = \overline{P_n} \); then \( d(v_i) = d(v_n) = n-1 \), \( v_1, v_n \in G \)
\[ d(v_i) = n-2 \quad \forall \ v_i \in G; i = 2, \ldots, n-1 \]

In \( G \) the vertices \( v_1 \) and \( v_n \) are connected with all vertices of \( G \) other than \( v_2 \) and \( v_{n-1} \) respectively.

Now \( \{v_1, v_n\} \) and any vertex set \( \{v_i, v_j\} \) is the minimum dominating set of \( G = \overline{P_n} \).

Since \( v_1, v_{n-1} \in E(P_n) \)
which are independent in \( G \) and is the minimum independent dominating set of \( G \).

Since \( n > 3 \) for any set \( \{v_i, v_j\} \) is a dominating set of \( P_n \) then any pair of vertices \( \{v_i, v_n\} \subset V - D \) is the inverse dominating set of \( G \). Since \( v_1, v_n \notin E(\overline{P_n}) \), therefore,\( \{v_1, v_n\} \) is the total, connected and non split dominating set of \( G \).

Hence, \( \gamma(G) = \gamma_i(G) = \gamma'_i(G) = \gamma_m(G) = \gamma_c(G) = 2 \forall n \geq 3 \) where \( G = \overline{P_n} \).
Result 2.12: For any integer \( n \geq 4 \)
\[
\gamma(P_n) = \gamma'(P_n) = \gamma(\overline{P_n}) = \gamma'(\overline{P_n}) = 2 \text{ iff } n = 4, 5
\]

Corollary 2.13: If \( G \) is a connected simple graph with \( |V(G)| > 3 \), \( D = \{v\} \) is the only minimum dominating set of \( G \) and \( \gamma'(G) = |V| - 1 \) then \( G \) is a star graph.

Proof: Let \( G \) be any graph with \( |V(G)| = n \).

Since \( D=\{v\} \) is the only minimum dominating set of \( G \), all vertices of \( G \) are connected with \( v \). Also \( \gamma'(G) = |V| - 1 \) then the inverse dominating set of \( G \) consists all vertices of \( G \) other than \( V \), therefore, no vertices of \( G - V \) are adjacent to each other which implies every vertices of \( G \) other, than \( v \) are pendent vertices \( \Rightarrow d(v_i) = n - 1 \) and \( d(v) = 1 \), for all \( v_i \in G \) and \( v_i \neq v \).

Hence \( G \) is a star graph.

Theorem 2.14: For any integer \( n \geq 3 \).
(i) \( \gamma(C_n) = \gamma'(C_n) = \gamma_i(C_n) = \gamma_s(C_n) = \left\lceil \frac{n}{3} \right\rceil \)

(ii) \( \gamma_b(C_n) = \left\lfloor \frac{n}{3} \right\rfloor \) if \( n \geq 3 \) and \( n \neq 5 \).

(iii) \( \gamma(C_n) = \begin{cases} 2k+1 & \text{if } n = 4k + 1 \\ 2 \left\lfloor \frac{n}{4} \right\rfloor & \text{otherwise} \end{cases} \)

(iv) \( \gamma_p(C_n) = 2 \left\lfloor \frac{n}{4} \right\rfloor \)

(v) \( \gamma_c(C_n) = \gamma_{ns}(C_n) = n - 2 \)

Proof: Let \( G = C_n \) be a cycle of length \( n \) and its vertices are denoted by \( v_1, v_2, v_3, \ldots, v_n \), such that \( v_iv_{i+1} \in E(G) \), \( \forall i = 1 \ldots n - 1 \) and \( v_1v_n \in E(G) \), we are going to prove this theorem in three cases.

Case (i): \( n = 3k, k = 1, 2, 3, \ldots \)
Choose, \( D_1 = \{v_{3i+1}, v_{3i+2}, v_{3i+3} \mid i = 0, 1, 2, \ldots \} \) and \( D_2 = \{v_{3i+2}, v_{3i+3} \mid i = 0, 1, 2, \ldots, 3i + 2 \leq n \} \) and \( D_3 = \{v_{3i+3} \mid i = 0, 1, 2, \ldots, 3i + 3 \leq n \} \)

Now \( D_1, D_2 \) and \( D_3 \) are the minimum dominating sets of \( C_n \) also the elements of \( D_i, i = 1, 2, 3, \ldots \) are independent and the induced subgraph \( \langle V - D \rangle \) is disconnected, clearly each set \( D_1, D_2 \) and \( D_3 \) are mutually disjoint. Therefore, \( D_2 \) and \( D_3 \) are the inverse dominating set of \( D_1 \) and vice versa.

The cardinality of \( D_1, D_2 \) and \( D_3 \) is \( \frac{n}{3} \), \( \left\lceil \frac{n}{3} \right\rceil \) \( [\square n \text{ is a multiple of } 3] \) \( \Rightarrow \gamma(C_n) = \gamma'(C_n) = \gamma_i(C_n) = \gamma_s(C_n) = \left\lceil \frac{n}{3} \right\rceil \) if \( n = 3k \).

Case (ii): \( n = 3k + 1 \).

Now divide the graph \( G \) into \( k + 1 \) induced sub graphs \( G_i, i = 1, 2, \ldots, k + 1 \) containing three vertices,
\( G_i = \{v_{3j+1}, v_{3j+2}, v_{3j+3} \mid i = 1, 2, \ldots, k; j = i - 1 \} \) and \( G_{k+1} = \{v_n\} \)

Let \( D = \{v_{3i+2} \mid i = 0, 1, \ldots, k \} \cup G_{k+1} \) is the required minimum dominating set of \( G \), all vertices of \( D \) are independent in \( G \) and the induced subgraph \( \langle V - D \rangle \) is disconnected.

Now \( |D| = k + 1 \)
\[
\frac{n}{3} = k + 1 \Rightarrow \left\lfloor \frac{n}{3} \right\rfloor = k + \left\lceil \frac{1}{3} \right\rceil
\]
\[
= k + 1
\]
\[
\Rightarrow |D| = \left\lfloor \frac{n}{3} \right\rfloor \quad \text{[by (i)]}
\]

Choose, \(G_1 = \{v_n, v_1, v_2\}; \ G_2 = \{v_3, v_4, v_5\}; \ldots; \ G_k = \{v_{n-4}, v_{n-3}, v_{n-2}\}\) and \(G_{k+1} = \{v_{n-1}\}\) select \(D' = \{v_{3i+1} / i = 0, 1, 2, \ldots, k-1\} \cup G_{k+1}\) consists the elements \(\{v_1, v_4, v_7, \ldots, v_{n-1}\}\) is the required inverse dominating set of \(C_n\) by the same proof given in case (ii)

Therefore, \(|D'| = \left\lfloor \frac{n}{3} \right\rfloor\]

**Case (iii):** If \(n = 3k + 2\),

Here also we divide the vertices of \(G\) as in case (ii)

\[
G_i = \left\{v_{3j + 1}, v_{3j + 2}, v_{3j + 3} / i = 1, 2, \ldots, k; \; j = i - 1\right\} \; \text{and} \; G_{k+1} = \{v_{n-1}, v_n\}
\]

Choose the elements of \(D\) as \(D = \{v_{3i+2} / i = 0, 1, 2, \ldots, k\}\) is the required minimum dominating set with cardinality \(k + 1\) and choose \(D' = \{v_{3i+1} / i = 0, 1, 2, \ldots, k\}\) then \(D' = \{v_1, v_4, v_7, \ldots, v_{n-1}\}\) is the required inverse dominating set with minimum cardinality \(k + 1\).

\[
n = 3k + 2 \Rightarrow \frac{n}{3} = k + \frac{2}{3}
\]

\[
\Rightarrow \left\lfloor \frac{n}{3} \right\rfloor = k + 1
\]

\[
\Rightarrow |D| = \left\lfloor \frac{n}{3} \right\rfloor \quad \text{[\(\Box\) \(D\) \(=\) \(k + 1\)]}
\]

Also each set \(D\) and \(D'\) are independent in \(C_n\). In all cases the induced subgraph \(<C_n - D>\) is disconnected.

Hence \(\gamma(C_n) = \gamma'(C_n) = \gamma_i(C_n) = \gamma_s(C_n) = \left\lfloor \frac{n}{3} \right\rfloor\)

If \(n = 4\), then the graph and its complement are given as below

![Figure 1](image1.png)

\[D = \{v_1, v_2\}; \ D' = \{v_1, v_2\}\]

\[\gamma(C_4) = \gamma(\overline{C_4}) = 2\]

Hence, \(\gamma_s(C_4) = 2\)

If \(n = 5\), the graph and its complement are represented as below and completed as above.

![Figure 2](image2.png)
In $G$, 
\[ D_1 = \{v_1, v_3\}; D_2 = \{v_1, v_4\}; D_3 = \{v_2, v_4\} \]
\[ D_4 = \{v_2, v_5\}; D_5 = \{v_3, v_5\} \] are the minimum dominating sets of $G$.

In $\overline{G}$,
\[ D_1 = \{v_1, v_2\}; D_2 = \{v_1, v_5\}; D_3 = \{v_2, v_3\} \]
\[ D_4 = \{v_3, v_4\}; D_5 = \{v_4, v_5\} \] are the minimum dominating sets.

Clearly, none of the dominating set of $G$ with cardinality two is a dominating set of $\overline{G}$.
\[ \Rightarrow \gamma_\overline{g}(C_5) \neq 2 \]
\[ \Rightarrow \gamma_\overline{g}(C_5) = 3 \]

In $C_n$, $n > 5$ all minimum dominating sets of $C_n$ in also a dominating set of $\overline{C_n}$

Hence, $\gamma_\overline{g}(C_n) = \left[ \frac{n}{3} \right] \forall n > 3$ and $n \neq 5$.

(iii) Case (i) if $n = 4k + 1$. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of the graph $G = C_n$ now divide $G$ into $k$ induced subgraph $G_i, i = 1, \ldots, k$ such that $G_{i+1} = \{v_{4i + 1}, v_{4i + 2}, v_{4i + 3}, v_{4i + 4}, i = 0, 1, 2, \ldots, k - 2\}$ and the last partition $G_k$ contains five vertices as $G_k = \{v_{n-4}, v_{n-3}, v_{n-2}, v_{n-1}, v_n\}$

Choose, $D_t = \{v_{4i + 2}, v_{4i + 3}, i = 0, 1, 2, \ldots, \frac{k - 2}{4}\} \cup \{v_{n-3}, v_{n-2}, v_{n-1}\}$ is the required minimum total dominating set with cardinality
\[ |D_t(G)| = (k - 1) + 3 = 2k + 1 \] (iv)

If $n \neq 4k + 1$. by case (i) of (iii)
\[ G_{i+1} = \{v_{4i + 1}, v_{4i + 2}, v_{4i + 3}, v_{4i + 4}, i = 0, 1, 2, \ldots, k - 1\} \] then the last partition
\[ G_{k+1} = \begin{cases} \{v_{n-1}, v_n\} & \text{if } n = 4k + 2 \\ \{v_{n-2}, v_{n-1}, v_n\} & \text{if } n = 4k + 3 \end{cases} \]
then $D_t(G) = \{v_{4i + 2}, v_{4i + 3}, i = 0, 1, 2, \ldots, \frac{k - 1}{4}\} \cup \{\text{any two elements of } G_{k+1}\}$

Now $D_t(G)$ is the minimum total dominating set with cardinality
\[ |D_t(G)| = 2 \left( k + 1 \right) \]
\[ n = 4k + 2 \text{ and } n = 4k + 3 \]
\[ \left[ \frac{n}{4} \right] = k + \frac{1}{2} \text{ and } \left[ \frac{n}{4} \right] = k + \frac{3}{4} \]
\[ \left[ \frac{n}{4} \right] = k + 1 \text{ and } \left[ \frac{n}{4} \right] = k + 1 \]

\[ \Rightarrow 2 \left[ \frac{n}{4} \right] = 2 \left( k + 1 \right) \text{ and } 2 \left[ \frac{n}{4} \right] = 2 \left( k + 1 \right) \]
\[ \Rightarrow |D_t(G)| = 2 \left[ \frac{n}{4} \right] \text{ if } n = \{4k + 2 \text{ and } 4k + 3\} \] [□ by (v)]

Hence, $\gamma_t(G) = \begin{cases} 2k + 1 & \text{if } n = 4k + 1 \\ 2 \left[ \frac{n}{4} \right] & \text{otherwise} \end{cases}$

iv) The same argument given in (iii)
Let $T = \{v_{4i+2}, v_{4i+3} \mid i = 0, 1, 2, \ldots, k - 1\}$ then $D_p(G) \cup \{v_{n-1}, v_n\}$ is the required minimum dominating set of $G$ and $|D_p(G)| = 2(k + 1)$.

If $n = 4k + 1$

$$\frac{n}{4} = k + \frac{1}{4} \Rightarrow \left\lfloor \frac{n}{4} \right\rfloor = k + \left\lfloor \frac{1}{4} \right\rfloor$$

$$\Rightarrow 2 \left\lfloor \frac{n}{4} \right\rfloor = 2(k + 1)$$

$$\Rightarrow |D_p(G)| = 2 \left\lfloor \frac{n}{4} \right\rfloor$$ [by 1]

if $n = 4k + 2$ and if $n = 4k + 3$

$$\frac{n}{4} = k + \frac{2}{4} \quad \text{and} \quad \frac{n}{4} = k + \frac{3}{4}$$

$$\left\lfloor \frac{n}{4} \right\rfloor = k + \left\lfloor \frac{2}{4} \right\rfloor \quad \text{and} \quad \left\lfloor \frac{n}{4} \right\rfloor = k + \left\lfloor \frac{3}{4} \right\rfloor$$

$$= k + 1 \quad \text{and} \quad = k + 1$$

$$\Rightarrow 2(k + 1) = 2 \left\lfloor \frac{n}{4} \right\rfloor$$

$$\Rightarrow |D_p(G)| = 2 \left\lfloor \frac{n}{4} \right\rfloor$$

Hence $\gamma_p(G) = 2 \left\lfloor \frac{n}{4} \right\rfloor \forall n > 3$.

v) Let $T = \{v_i, v_{i+1} \mid v_i \sim v_{i+1} \in E(G)\}$ the $V - T$ and $V - \{v_1, v_n\}$ is a connected and non split dominating sets of $G$.

Therefore, $\gamma_p(C_n) = \gamma_{ns}(C_n) = n - 2 \quad \forall n > 3$.

Hence the proof.

**Corollary 2.15:** If $G$ is one corona $(K_n \circ k_1)$ then

$$\gamma(G) = \gamma'(G) = \gamma_t(G) = \gamma_i(G) = \gamma_c(G) = \gamma_s(G) = \gamma_{ns}(G) = \gamma_{g}(G) = n$$

$$\gamma_p(G) = \begin{cases} n+1 & \text{if} \quad n = 2k + 1 \\ n & \text{if} \quad n = 2k \end{cases}$$

**Proof:** Let the vertex set of $G = K_n \circ K_1$ is represented in figure 1.

Let $S_1 = \{v_1, v_2, v_3, \ldots, v_n\}$ and $S_2 = \{u_1, u_2, u_3, \ldots, u_n\}$

Then $D = S_1$; $D' = S_2$

$D_T = S_1$; $D_i = S_2$

$D_c = S_1$; $D_{ns} = S_2$

$D_p = \{(v_{2i+1}, v_{2i+2}) \mid i = 0, 1, 2, \ldots, k - 1\}$

Therefore,

$$\gamma(G) = \gamma'(G) = \gamma_t(G) = \gamma_i(G) = \gamma_c(G) = \gamma_s(G) = \gamma_{ns}(G) = \gamma_{g}(G) = n$$

If $n = 2k + 1$

$D_p = \{v_{2i+1}, v_{2i+2}\} \cup \{v_{n-1}, v_n\} \mid i = 0, 1, \ldots, k - 1 \text{ and } 2i + 2 \leq 2k$ is the required minimum paired dominating set of $G$.
\[ |D_p(G)| = 2k + 2 \]
\[ = 2k + 1 + 1 \]
\[ = n + 1 \quad \square \quad 2k + 1 = n \]

Therefore, \( \gamma_p(G) = \begin{cases} n & \text{if } n \text{ is even} \\ n + 1 & \text{if } n \text{ is odd} \end{cases} \quad \forall \ n \geq 3 \)

**Corollary 2.16:** For any graph \( G \) is the \( r \) corona of \( K_n \), \( \forall \ n \geq 3 \)
\[ \gamma(G) = \gamma'(G) = \gamma_t(G) = \gamma_i(G) = \gamma_s(G) = \gamma_{ns}(G) = \gamma_i(G) = n \]
\[ \gamma_p(G) = \begin{cases} n & \text{if } n \text{ is even} \\ n + 1 & \text{if } n \text{ is odd} \end{cases} \]

**Theorem 2.17:** Let \( G \) be a barbeled graph then
\[ \gamma(G) = \gamma'(G) = \gamma_t(G) = \gamma_i(G) = \gamma_s(G) = \gamma_{ns}(G) = \gamma_i(G) = 2 \]
\[ \gamma_p(G) = \begin{cases} n & \text{if } n \text{ is even} \\ n + 1 & \text{if } n \text{ is odd} \end{cases} \]

**Proof:** The Barbelled graph \( G \) is given in figure as below.
The vertex \( u_i \) is connected with vertex \( v_i \in S_2 \) in \( G \), therefore \( u_i \) is connected with all the vertices of \( S_2 \) other than \( v_i \) in \( \overline{G} \). Similarly \( v_i \) is connected with \( u_i \in S_1 \) in \( G \). Therefore, \( v_i \) is connected with all the vertices of \( S_1 \) other than \( u_i \) in \( G \). Hence

\[
D_d(G) = \{u_i, v_i\} \quad \text{and} \quad D_p(G) = \{u_i, v_i\}
\]

\[
\gamma(G) = \gamma'(G) = \gamma_i(G) = \gamma_s(G) = \gamma_n(G) = \gamma_d(G) = \gamma_p(G) = 2.
\]

**Theorem 2.18:** Let \( G \) be any complete bipartite graph with \( m, n \) vertices then

\[
\gamma(G) = \gamma'(G) = \gamma_i(G) = \gamma_s(G) = \gamma_d(G) = \gamma_p(G) = 2 \quad \text{and} \quad \gamma_n(G) = \gamma_d(G) = \{m \text{ or } n \text{ whichever in less}\}
\]

**Proof:** The vertices of \( G = K_{m,n} \) are partitioned into two sets \( S_1 \) and \( S_2 \) such that

\[
S_1 = \{u_1, u_2, \ldots, u_m\} \quad \text{and} \quad S_2 = \{v_1, v_2, \ldots, v_n\}.
\]

Since \( G = K_{m,n} \),

\[
N(u_i) = S_2 \quad \forall \ i = 1, 2, \ldots, m \quad \text{and} \quad N(v_j) = S_1 \quad \forall \ j = 1, 2, \ldots, n.
\]

Therefore, each pair of vertices \( \{(u_i, v_j)/ u_i \in S_1 \quad \text{and} \quad v_j \in S_2\} \) is a dominating set of \( G \), \( u_i \), \( v_j \in E(G) \) which implies that

\[
D(G) = D_T(G) = D_d(G) = D_p(G) = D_g(G) = \left\{\left(u_i, v_j\right)/ u_i \in S_1 \quad \text{and} \quad v_j \in S_2\right\}
\]

Also, any induced subgraph \( <V - D> \) is connected.

Therefore,

\[
D_{ad}(G) = \left\{\left(u_i, v_j\right)/ u_i \in S_i; \quad v_j \in S_j, \quad i \neq j\right\}\].
\]

For any dominating set

\[
D = \left\{\left(u_i, v_j\right)/ u_i \in S_i; \quad v_j \in S_j\right\}
\]

\[
D' = \left\{\left(u_i, v_j\right)/ u_i \in S_i, \quad v_i \in S_j, \quad i \neq j\right\}
\]

is the inverse dominating set of \( D \).

Since, \( u_i \) is not adjacent with any element of \( S_1 \) in \( G \), \( u_i \), \( i = 1, \ldots, m \) is adjacent with all the vertices of \( S_1 \) in \( \overline{G} \),

Similarly \( v_j \), \( j = 1, \ldots, n \) is connected with all elements of \( S_2 \) in \( \overline{G} \) which implies \( (u_i, v_j) \) is a dominating set of \( \overline{G} \), that is, \( D_d(G) = \left\{\left(u_i, v_j\right)/ u_i \in S_1 \quad \text{and} \quad v_j \in S_2\right\}\)

Clearly \( +V(G) - S_1 \), and \( +V(G) - S_2 \), are disconnected. Also the elements of \( S_1 \) and \( S_2 \) are independent, hence independent and split dominating set of \( G \) is either \( S_1 \) or \( S_2 \) which having the minimum numbers of vertices.

That is, \( \gamma_d(G) = \gamma_s(G) = \left\{\begin{array}{ll}
m, & \text{if } m < n \\
n, & \text{otherwise}
\end{array}\right.\]

**Corollary 2.19:** If \( G = K_{m,n} \) then \( \overline{G} \) is a disconnected graph with two components and each components is a complete subgraph with \( m, n \) vertices.

Hence

\[
\gamma(\overline{G}) = \gamma'(\overline{G}) = \gamma_i(\overline{G}) = \gamma_s(\overline{G}) = \gamma_n(\overline{G}) = \gamma_d(\overline{G}) = \gamma_p(\overline{G}) = 2 \quad \text{and} \quad \gamma_d(\overline{G}) = \gamma_p(\overline{G}) = 4.
\]
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