



On πgb - Closed Sets in Topological Spaces

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Abstract

In this paper a new class of sets called πgb -closed set is introduced and its properties are studied. Further the notion of $\pi gb-T_{1/2}$ space and πgb -continuity are introduced.

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1. Introduction

Andrijevic [3] introduced a new class of generalized open sets in a topological space, the so-called b-open sets. This type of sets was discussed by Ekici and Caldas [11] under the name of γ -open sets. The class of b-open sets is contained in the class of semi-pre-open sets and contains all semi-open sets and preopen sets. The class of b-open sets generates the same topology as the class of preopen sets. Since the advent of these notions, several research paper with interesting results in different respects came to existence ([1, 3, 6, 11, 12, 21, 22, 23]). Levine [16] introduced the concept of generalized closed sets in topological space and a class of topological spaces called $T_{1/2}$ spaces. Extensive research on generalizing closedness was done in recent years as the notions of a generalized closed, generalized semi-closed, α -generalized closed, generalized semi-pre-open closed sets were investigated in [2,7,16,18,19]. The finite union of regular open sets is said to be π -open. The complement of a π -open set is said to be π -closed.

The aim of this paper is to study the notion of πgb -closed sets and its various characterizations are given in this paper. In Section 3, we study basic properties of πgb -closed sets. In Section 4, we characterize πgb -open sets. Finally in section 5, πgb -continuous and πgb -irresolute functions are discussed.

2. Preliminaries

Throughout this paper (X, τ) and (Y, τ) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ) $cl(A)$ and $int(A)$ denote the closure of A and the interior of A respectively. (X, τ) will be replaced by X if there is no chance of confusion.

Let us recall the following definitions which we shall require later.

Definition 2.1: A subset A of a space (X, τ) is called

- (1) a preopen set [17] if $A \subset int(cl(A))$ and a preclosed set if $cl(int(A)) \subset A$;
- (2) a semi-open set [15] if $A \subset cl(int(A))$ and a semi-closed set if $int(cl(A)) \subset A$;
- (3) a α -open set [20] if $A \subset int(cl(int(A)))$ and a α -closed set if $cl(int(cl(A))) \subset A$;
- (4) a semi-preopen set [1] if $A \subset cl(intcl(A))$ and a semi-pre-closed set if $int(cl(int(A))) \subset A$;
- (5) a regular open set if $A = int(cl(A))$ and a regular closed set if $A = cl(int(A))$;
- (6) b-open [3] or sp-open [8], γ -open [11] if $A \subset cl(int(A)) \cup int(cl(A))$.

The complement of a b-open set is said to be b-closed [3]. The intersection of all b-closed sets of X containing A is called the b-closure of A and is denoted by $bCl(A)$. The union of all b-open sets of X contained in A is called b-interior of A and is denoted by $bInt(A)$. The family of all b-open (resp. α -open, semi-open, preopen, β -open, b-closed, preclosed) subsets of a space X is denoted by $bO(X)$ (resp. $\alpha O(X)$, $SO(X)$, $PO(X)$, $\beta O(X)$, $bC(X)$, $PC(X)$) and the

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The sets $SO(X, x)$, $\alpha O(X, x)$, $PO(X, x)$, $\beta O(X, x)$ are defined analogously.

Lemma 2.2 [3]: Let A be a subset of a space X. Then

- (1) $bCl(A) = sCl(A) \cap pCl(A) = A \cup [Int(Cl(A)) \cap Cl(Int(A))];$
- (2) $bInt(A) = sInt(A) \cup pInt(A) = A \cap [Int(Cl(A)) \cup Cl(Int(A))];$

Definition 2.3: A subset A of a space (X, τ) is called

- (1) a generalized closed (briefly g-closed)[16] if $cl(A) \subset U$ whenever $A \subset U$ and U is open.
- (2) a generalized b-closed (briefly gb-closed)[13] if $bcl(A) \subset U$ whenever $A \subset U$ and U is open.
- (3) πg -closed [10] if $cl(A) \subset U$ whenever $A \subset U$ and U is π -open.
- (4) πgp -closed [24] if $pcl(A) \subset U$ whenever $A \subset U$ and U is π -open.
- (5) $\pi g\alpha$ -closed [14] if $\alpha cl(A) \subset U$ whenever $A \subset U$ and U is π -open.
- (6) πgsp -closed [25] if $spcl(A) \subset U$ whenever $A \subset U$ and U is π -open.
- (7) πgs -closed [4] if $scl(A) \subset U$ whenever $A \subset U$ and U is π -open.

Definition 2.4: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

- (1) π - irresolute [4] if $f^{-1}(V)$ is π - closed in (X, τ) for every π -closed of (Y, σ) ;
- (2) b-irresolute: [11] if for each b-open set V in Y, $f^{-1}(V)$ is b-open in X;
- (3) b-continuous: [11] if for each open set V in Y, $f^{-1}(V)$ is b-open in X.

3. πgb -closed sets

Definition 3.1: A subset A of (X, τ) is called πgb -closed if $bcl(A) \subset U$ whenever $A \subset U$ and U is π -open in (X, τ) . By $\pi GBC(\tau)$ we mean the family of all πgb - closed subsets of the space (X, τ) .

Theorem 3.2:

1. Every closed set is πgb -closed
2. Every g-closed is πgb -closed
3. Every α -closed set is πgb -closed
4. Every pre-closed set is πgb -closed
5. Every gb-closed set is πgb -closed
6. Every πg -closed set is πgb -closed.
7. Every πgp -closed set is πgb -closed
8. Every $\pi g\alpha$ -closed set is πgb -closed
9. Every πgs -closed set is πgb -closed.
10. Every πgb -closed set is πgsp -closed.

Proof: Straight forward Converse of the above need not be true as seen in the following examples.

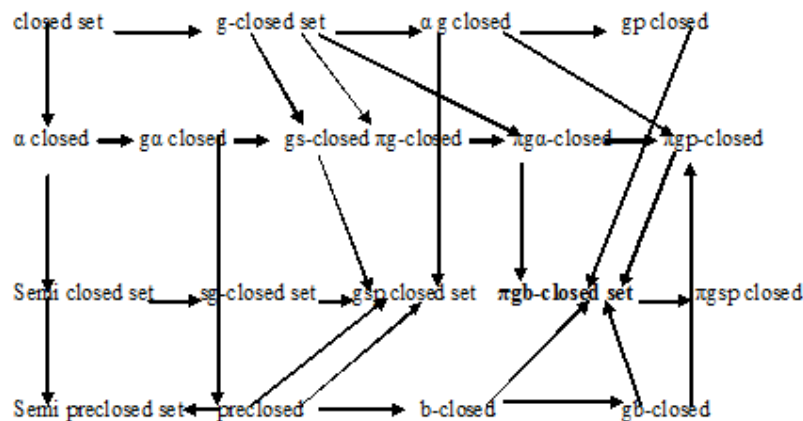
Example3.3: Consider $X = \{a, b, c, d\}$, $\tau = \{\Phi, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, X\}$. Let $A = \{c\}$. Then A is πgb -closed but not closed, g-closed, α -closed, pre-closed, gb-closed, πg -closed.

Example 3.4: Let $X = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Let $A = \{a\}$. Therefore A is πgb -closed but not $\pi g\alpha$ - closed.

Example3.5: Let $X = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$.

Let $A = \{a, b\}$. Therefore A is πgb -closed but not πgp - closed.

Remark 3.6: The above discussions are summarized in the following diagram.



Theorem 3.7: If A is π -open and πgb - closed, then A is b -closed.

Proof: Let A is π -open and πgb -closed. Let $A \subset A$ where A is π -open. Since A is πgb - closed, $bcl(A) \subset A$. Then $A = bcl(A)$. Hence A is b -closed.

Theorem 3.8: Let A be πgb -closed in (X, τ) . Then $bcl(A) - A$ does not contain any non empty π -closed set.

Proof: Let F be a non empty π -closed set such that $F \subset bcl(A) - A$. Since A is πgb -closed, $A \subset X - F$ where $X - F$ is π -open implies $bcl(A) \subset X - F$. Hence $F \subset X - bcl(A)$. Now, $F \subset bcl(A) \cap (X - bcl(A))$ implies $F = \emptyset$ which is a contradiction. Therefore $bcl(A)$ does not contain any non empty π - closed set.

Corollary 3.9: Let A be πgb -closed in (X, τ) . Then A is b -closed iff $bcl(A) - A$ is π closed.

Proof: Let A be b -closed. Then $bcl(A) = A$. This implies $bcl(A) - A = \emptyset$ which is π - closed. Assume $bcl(A) - A$ is π -closed. Then $bcl(A) - A = \emptyset$. Hence, $bcl(A) = A$

Remark 3.10: Finite union of πgb -closed sets need not be πgb -closed.

Example 3.11: Consider $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Let $A = \{a\}$, $B = \{b\}$. Here A and B are πgb - closed but $A \cup B = \{a, b\}$ is not πgb -closed.

Remark 3.12: Finite intersection of πgb -closed sets need not be πgb - closed.

Example 3.13: Consider $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$. Let $A = \{a, b, c\}$, $B = \{a, b, d\}$. Here A and B are πgb -closed but $A \cap B = \{a, b\}$ is not πgb -closed.

Definition 3.14[5]: Let (X, τ) be a topological space $A \subset X$ and $x \in X$ is said to be b -limit point of A iff every b -open set containing x contains a point of A different from x .

Definition 3.15[5]: Let (X, τ) be a topological space, $A \subset X$. The set of all b -limit points of A is said to be b -derived set of A and is denoted by $D_b[A]$

Lemma 3.16[5]: If $D(A) = D_b(A)$, then we have $cl(A) = bcl(A)$

Lemma 3.17[5]: If $D(A) \subset D_b(A)$ for every subset A of X . Then for any subsets F and B of X , we have $bcl(F \cup B) = bcl(F) \cup bcl(B)$

Theorem 3.18: Let A and B be πgb - closed sets in (X, τ) such that $D[A] \subset D_b[A]$ and $D[B] \subset D_b[B]$. Then $A \cup B$ is πgb -closed

Proof: Let U be π -open set such that $A \cup B \subset U$. Since A and B are π gb-closed sets we have $\text{bcl}(A) \subset U$ and $\text{bcl}(B) \subset U$. Since $D[A] \subset D_b[A]$ and $D[B] \subset D_b[B]$, by lemma 3.16, $\text{cl}(A) = \text{bcl}(A)$ and $\text{cl}(B) = \text{bcl}(B)$. Thus $\text{bcl}(A \cup B) \subset \text{cl}(A \cup B) = \text{cl}(A) \cup \text{cl}(B) = \text{bcl}(A) \cup \text{bcl}(B) \subset U$. This implies $A \cup B$ is π gb-closed.

Theorem 3.19: If A is π gb-closed set and B is any set such that $A \subset B \subset \text{bcl}(A)$, then B is π gb-closed set.

Proof: Let $B \subset U$ and U be π -open. Given $A \subset B$. Then $A \subset U$

Since A is π gb- closed, $A \subset U$ implies $\text{bcl}(A) \subset U$. By assumption it follows that $\text{bcl}(B) \subset \text{bcl}(A) \subset U$. Hence B is a π gb-closed set.

4. π gb- open sets

Definition 4.1: A set $A \subset X$ is called π gb-open if and only if its complement is π gb- closed.

Remark 4.2: $\text{bcl}(X-A) = X - \text{bint}(A)$

By $\pi\text{GBO}(\tau)$ we mean the family of all π gb-open subsets of the space (X, τ) .

Theorem 4.3: If $A \subset X$ is π gb-open iff $F \subset \text{bint}(A)$ whenever F is π -closed and $F \subset A$

Proof: Necessity: Let A be π gb-open. Let F be π -closed and $F \subset A$. Then $X-A \subset X-F$ where $X-F$ is π -open. By assumption, $\text{bcl}(X-A) \subset X-F$. By remark 4.2, $X - \text{bint}(A) \subset X-F$. Thus $F \subset \text{bint}(A)$.

Sufficiency: Suppose F is π - closed and $F \subset A$ such that $F \subset \text{bint}(A)$. Let $X-A \subset U$ where U is π -open. Then $X-U \subset A$ where $X-U$ is π -closed. By hypothesis, $X-U \subset \text{bint}(A)$. $X - \text{bint}(A) \subset U$. $\text{bcl}(X-A) \subset U$. Thus $X-A$ is π gb-closed and A is π gb-open.

Theorem 4.4: If $\text{bint}(A) \subset B \subset A$ and A is π gb-open then B is π gb-open.

Proof: Let $\text{bint}(A) \subset B \subset A$. Thus $X-A \subset X-B \subset \text{bcl}(X-A)$. Since $X-A$ is π gb- closed, by theorem 3.19, $(X-A) \subset (X-B) \subset \text{bcl}(X-A)$ implies $(X-B)$ is π gb- closed.

Remark 4.5: For any $A \subset X$, $\text{bint}(\text{bcl}(A)-A) = \Phi$

Theorem 4.6: If $A \subset X$ is π gb- closed, then $\text{bcl}(A)-A$ is π gb- open.

Proof: Let A be π gb- closed let F be π -closed set. $F \subset \text{bcl}(A)-A$. By theorem 3.8, $F = \Phi$. By remark 4.5, $\text{bint}(\text{bcl}(A)-A) = \Phi$. Thus $F \subset \text{bint}(\text{bcl}(A)-A)$. Thus $\text{bcl}(A)-A$ is π gb- open.

Lemma 4.7[24]: Let $A \subset X$. If A is open or dense, then $\pi O(A, \tau/A) = V \cap A$ such that $V \in \pi O(X, \tau)$.

Theorem 4.8: Let $B \subset A \subset X$ where A is π gb-closed and π -open set. Then B is π gb- closed relative to A iff B is π gb-closed in X .

Proof: Let $B \subset A \subset X$, where A is π gb-closed and π -open set. Let B be π gb-closed in A . Let $B \subset U$ where U is π -open in X . Since $B \subset A$, $B = B \cap A \subset U \cap A$, this implies $\text{bcl}(B) = \text{bcl}_A(B) \subset U \cap A \subset U$. Hence, B is π gb-closed in X .

Let B be π gb-closed in X . Let $B \subset O$ where O is π -open in A . Then $O = U \cap A$ where U is π -open in X . This implies $B \subset O = U \cap A \subset U$. Since B is π gb-closed in X , $\text{bcl}(B) \subset U$. Thus $\text{bcl}_A(B) = A \cap \text{bcl}(B) \subset U \cap A = O$. Hence, B is π gb-closed relative to A .

Corollary 4.9: Let A be π -open, π gb-closed set. Then $A \cap F$ is π gb-closed whenever $F \in \text{bC}(X)$

Proof: Since A is π gb- closed and π -open, then $\text{bcl}(A) \subset A$ and thus A is b -closed. Hence $A \cap F$ is b -closed in X which implies $A \cap F$ is π gb-closed in X .

Definition 4.10: A space (X, τ) is called a πgb - $T_{1/2}$ space if every πgb - closed set is b-closed.

Theorem 4.11:

(i) $BO(\tau) \subset \pi GBO(\tau)$

(ii) A space (X, τ) is πgb - $T_{1/2}$ iff $BO(\tau) = \pi GBO(\tau)$.

Proof: (i) Let A be b-open, then $X-A$ is b-closed so $X-A$ is πgb -closed. Thus A is πgb -open. Hence $BO(\tau) \subset \pi GBO(\tau)$

(ii) **Necessity:** Let (X, τ) be πgb - $T_{1/2}$ space. Let $A \in \pi GBO(\tau)$. Then $X-A$ is πgb - closed. By hypothesis, $X-A$ is b-closed thus $A \in BO(\tau)$. Thus $\pi GBO(\tau) = BO(\tau)$.

Sufficiency: Let $BO(\tau) = \pi GBO(\tau)$. Let A be πgb -closed. Then $X-A$ is πgb - open. $X-A \in \pi GBO(\tau)$. $X-A \in BO(\tau)$. Hence A is b-closed. This implies (X, τ) is πgb - $T_{1/2}$ space.

Lemma 4.12: Let A be a subset of (X, τ) and $x \in X$. Then $x \in bcl(A)$ iff $V \cap A \neq \emptyset$ for every b-open set V containing x .

Theorem 4.13: For a topological space (X, τ) the following are equivalent

(i) X is πgb - $T_{1/2}$ space.

(ii) Every singleton set is either π -closed or b-open.

Proof: To prove (i) \Rightarrow (ii): Let X be a πgb - $T_{1/2}$ space. Let $x \in X$ and assuming that $\{x\}$ is not π - closed. Then clearly $X-\{x\}$ is not π - open. Hence $X-\{x\}$ is trivially a πgb - closed. Since X is πgb - $T_{1/2}$ space, $X-\{x\}$ is b-closed. Therefore $\{x\}$ is b-open.

(ii) \Rightarrow (i): Assume every singleton of X is either π -closed or b-open. Let A be a πgb -closed set. Let $\{x\} \in bcl(A)$.

Case (i): Let $\{x\}$ be π - closed. Suppose $\{x\}$ does not belong to A . Then $\{x\} \in bcl(A)-A$. By theorem 3.8, $\{x\} \in A$. Hence $bcl(A) \subset A$.

Case (ii): Let $\{x\}$ be b-open. Since $\{x\} \in bcl(A)$, we have $\{x\} \cap A \neq \emptyset$ implies $\{x\} \in A$. Therefore $bcl(A) \subset A$. Therefore A is b-closed.

5. πgb - continuous and πgb - irresolute functions

Definition 5.1: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called πgb - continuous if every $f^{-1}(V)$ is πgb - closed in (X, τ) for every closed set V of (Y, σ) .

Definition 5.2: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called πgb - irresolute if $f^{-1}(V)$ is πgb - closed in (X, τ) for every g -closed set V in (Y, σ)

Proposition 5.3: Every πgb - irresolute function is πgb - continuous.

Remark 5.4: Converse of the above need not be true as seen in the following example.

Example 5.5: Consider $X = \{a, b, c\}$, $\tau = \{ \emptyset, \{a\}, \{b\}, \{a, b\}, X \}$, $\sigma = \{ \emptyset, \{a\}, X \}$. Let $f: (X, \tau) \rightarrow (X, \sigma)$ be the identity function. Then f is πgb -continuous but not πgb - irresolute.

Remark 5.6: Composition of two πgb -continuous functions need not be πgb -continuous

Example 5.7: Let $X = \{a, b, c, d\}$, $\tau = \{ \emptyset, \{b\}, \{c\}, \{b, c\}, X \}$, $\sigma = \{ \emptyset, \{a, b, d\}, X \}$, $\eta = \{ \emptyset, \{a, d\}, X \}$. Define $f: (X, \tau) \rightarrow (X, \sigma)$ by $f(a) = a$, $f(b) = c$, $f(c) = b$, $f(d) = d$. Define $g: (X, \sigma) \rightarrow (X, \eta)$ by $g(a) = d$, $g(b) = c$, $g(c) = b$, $g(d) = a$. Then f and g are πgb -continuous but $g \circ f$ is not πgb -continuous.

Definition 5.8: A function $f: X \rightarrow Y$ is said to be pre b-closed if $f(U)$ is b-closed in Y for each b-closed set in X .

Proposition 5.9: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be π - irresolute and pre b-closed map. Then $f(A)$ is πgb - closed in Y for every πgb - closed set A of X

Proof: Let A be πgb -closed in X . Let $f(A) \subset V$ where V is π - open in Y . Then $A \subset f^{-1}(V)$ and A is πgb -closed in X implies $bcl(A) \subset f^{-1}(V)$. Hence, $f(bcl(A)) \subset V$. Since f is pre b -closed, $bcl(f(A)) \subset bcl(f(bcl(A))) = f(bcl(A)) \subset V$. Hence $f(A)$ is πgb - closed in Y .

Definition 5.10: A topological space X is a πgb - space if every πgb - closed set is closed.

Proposition 5.11: Every πgb -space is $\pi gb-T_{1/2}$ space.

Theorem 5.12: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function.

- (1) If f is πgb - irresolute and X is $\pi gb-T_{1/2}$ space, then f is b -irresolute.
- (2) If f is πgb - continuous and X is $\pi gb-T_{1/2}$ space, then f is b -continuous

Proof: (1) Let V be b -closed in Y . Since f is πgb -irresolute, $f^{-1}(V)$ is πgb -closed in X . Since X is $\pi gb-T_{1/2}$ space, $f^{-1}(V)$ is b -closed in X . Hence f is b -irresolute.

(2) Let V be closed in Y . Since f is πgb -continuous, $f^{-1}(V)$ is πgb - closed in X . By assumption, it is b -closed. Therefore f is b -continuous.

Definition 5.13[14]: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is π -open map if $f(F)$ is π -open map in Y for every π -open in X .

Theorem 5.14: If the bijective $f: (X, \tau) \rightarrow (Y, \sigma)$ is b -irresolute and π -open map, then f is πgb - irresolute.

Proof: Let V be πgb -closed in Y . Let $f^{-1}(V) \subset U$ where U is π - open in X . Then $V \subset f(U)$ and $f(U)$ is π -open implies $bcl(V) \subset f(U)$. Since f is b -irresolute, $(f^{-1}(bcl(V)))$ is b -closed. Hence $bcl(f^{-1}(V)) \subset bcl(f^{-1}(bcl(V))) = f^{-1}(bcl(V)) \subset U$. Therefore, f is πgb - irresolute.

Theorem 5.15: If $f: X \rightarrow Y$ is π -open, b -irresolute, pre b -closed surjective function. If X is $\pi gb-T_{1/2}$ space, then Y is $\pi gb-T_{1/2}$ space.

Proof: Let F be a πgb -closed set in Y . Let $f^{-1}(F) \subset U$ where U is π - open in X . Then $F \subset f(U)$ and F is a πgb -closed set in Y implies $bcl(F) \subset f(U)$. Since f is b -irresolute, $bcl(f^{-1}(F)) \subset bcl(f^{-1}(bcl(F))) = f^{-1}(bcl(F)) \subset U$. Therefore $f^{-1}(F)$ is πgb -closed in X . Since X is $\pi gb-T_{1/2}$ space, $f^{-1}(F)$ is b -closed in X . Since f is pre b -closed, $f(f^{-1}(F)) = F$ is b -closed in Y . Hence Y is $\pi gb-T_{1/2}$ space.

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