SKOLEM DIFFERENCE FIBONACCI MEAN LABELLING OF SOME STANDARD GRAPHS

L. MEENAKSHI SUNDARAM*
Assistant professor, Department of Mathematics,
V. O. Chidambaram College, Thoothukudi, Tamil Nadu, India.

A. NAGARAJAN
Associate professor, Department of Mathematics,
V. O. Chidambaram College, Thoothukudi, Tamil Nadu, India.

(Received On: 15-11-16; Revised & Accepted On: 19-12-16)

Abstract

The concept of Skolem difference mean labelling was introduced by K. Murugan and A. Subramanian [6]. The concept of Fibonacci labelling was introduced by David W. Bange and Anthony E. Barkauskas [1] in the form Fibonacci graceful. This motivates us to introduce Skolem difference Fibonacci mean labelling and is defined as follows: “A graph G with p vertices and q edges is said to have Skolem difference Fibonacci mean labelling if it is possible to label the vertices \( x \in V \) with distinct elements \( f(x) \) from the set \( \{1, 2, ..., F_{p+q}\} \) in such a way that the edge \( e = uv \) is labelled with \[ \frac{|f(u) - f(v)|}{2} \] if \( |f(u) - f(v)| \) is even and \[ \frac{|f(u) - f(v)|+1}{2} \] if \( |f(u) - f(v)| \) is odd and the resulting edge labels are distinct and are from \( \{F_1, F_2,...,F_q\} \). A graph that admits Skolem difference Fibonacci mean labelling is called a Skolem difference Fibonacci mean graph”. In this paper, we prove that path, star, bistar, \( B(m,n,k) \) and union of stars are Skolem difference Fibonacci mean graphs.

Keywords: Skolem difference mean labelling, Fibonacci labelling, Skolem difference Fibonacci mean labelling.

1. INTRODUCTION

A graph \( G \) with \( p \) vertices and \( q \) edges is said to have Skolem difference Fibonacci mean labelling if it is possible to label the vertices \( x \in V \) with distinct elements \( f(x) \) from the set \( \{1, 2, ..., F_{p+q}\} \) in such a way that the edge \( e = uv \) is labelled with \[ \frac{|f(u) - f(v)|}{2} \] if \( |f(u) - f(v)| \) is even and \[ \frac{|f(u) - f(v)|+1}{2} \] if \( |f(u) - f(v)| \) is odd and the resulting edge labels are distinct and are from \( \{F_1, F_2,...,F_q\} \). A graph that admits Skolem difference Fibonacci mean labelling is called a Skolem difference Fibonacci mean graph.

Following definitions and notations are used in main results.

Definition 1.1: A path \( P_n \) with \( n \) points has \( V = \{v_1, v_2, ..., v_n\} \) for its vertex set and \( E(P_n) = \{v_1v_2, v_2v_3, ..., v_{n-1}v_n\} \) is its edge set. This path \( P_n \) is said to have length \( n-1 \).

Definition 1.2: A complete bigraph \( K_{1,n} \) is called a star.

Definition 1.3: The bistar \( B_{m,n} \) is obtained by joining the centre vertices of \( K_{1,m} \) and \( K_{1,n} \) with an edge.

Definition 1.4: The graph \( B_{m,n,k} \) is obtained from a path of length \( k \) by attaching the star \( K_{1,m} \) and \( K_{1,n} \) with its pendant vertices.

Definition 1.5: Let \( G_1(V_1, E_1) \) and \( G_2(V_2, E_2) \) be two graphs. Then their union \( G = G_1 \cup G_2 \) is a graph with vertex set \( V = V_1 \cup V_2 \) and edge set \( E = E_1 \cup E_2 \).

Corresponding Author: L. Meenakshi Sundaram*
Assistant professor, Department of Mathematics,
V. O. Chidambaram College, Thoothukudi, Tamil Nadu, India.
2. MAIN RESULT

**Theorem 2.1:** The path $P_n$ is skolem difference Fibonacci mean graph for all $n \geq 2$

**Proof:** Let $V(P_n) = \{v_i / 1 \leq i \leq n\}$

$E(P_n) = \{v_i v_{i+1}, v_n v_1 / 1 \leq i \leq n-1\}$

Then $|V(P_n)| = n$ and $|E(P_n)| = n-1$

Let $f: V \to \{1, 2, ..., F_{2n-1}\}$ be defined as follows

\[
f(v_i) = 2F_{i+1}, 1 \leq i \leq n
\]

\[
f^+(E) = \{f(v_i v_{i+1}) / i = 1, 2, ..., n-1\}
\]

\[= \left\{ \frac{2}{2}, \frac{2}{2}, \frac{2}{2}, ..., \frac{2}{2}, \frac{2}{2} \right\}
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\[= \left\{ \frac{2}{2}, \frac{2}{2}, \frac{2}{2}, ..., \frac{2}{2}, \frac{2}{2} \right\}
\]

Thus, the induced edge labels are distinct and are $F_1, F_2, ..., F_{n-1}$.

Hence the path $P_n$ is skolem difference Fibonacci mean graph for all $n \geq 2$.

**Example 2.2:**

![Figure-1](image1.png)

**Theorem 2.3:** The graph $K_{1,n}$ is skolem difference Fibonacci mean graph for all $n \geq 1$

**Proof:** Let $V(K_{1,n}) = \{u, u_i / 1 \leq i \leq n\}$

$E(K_{1,n}) = \{uu_i / 1 \leq i \leq n\}$

Then $|V(K_{1,n})| = n+1$ and $|E(K_{1,n})| = n$

Let $f: V \to \{1, 2, ..., F_{2n+1}\}$ be defined as follows

\[f(u) = 1
\]

\[f(u_i) = 2F_{i+1}, 1 \leq i \leq n
\]

\[f^+(E) = \{f(uu_i) / 1 \leq i \leq n\}
\]

\[= \left\{ \frac{2}{2}, \frac{2}{2}, \frac{2}{2}, ..., \frac{2}{2}, \frac{2}{2} \right\}
\]

\[= \left\{ \frac{2}{2}, \frac{2}{2}, \frac{2}{2}, ..., \frac{2}{2}, \frac{2}{2} \right\}
\]

\[= \left\{ \frac{2}{2}, \frac{2}{2}, \frac{2}{2}, ..., \frac{2}{2}, \frac{2}{2} \right\}
\]

Thus, the induced edge labels are distinct and are $F_1, F_2, ..., F_{n+1}$.

Hence the graph $K_{1,n}$ is skolem difference Fibonacci mean graph for all $n \geq 1$.

**Example 2.4:** The Skolem difference Fibonacci mean labelling of the star graph $K_{1,5}$ is

![Figure-2](image2.png)
Theorem 2.5: The bistar $B_{m,n}$ is skolem difference Fibonacci mean graph for all $m, n \geq 1$.

Proof: Let $V(B_{m,n}) = \{u, u_i, v, v_j \mid 1 \leq i \leq m$ and $1 \leq j \leq n\}$

$E(B_{m,n}) = \{uv, uu_i, vv_j \mid 1 \leq i \leq m$ and $1 \leq j \leq n\}$

Then $|V(B_{m,n})| = m+n+2$ and $|E(B_{m,n})| = m+n+1$

Let $f: V \rightarrow \{1, 2, ..., F_{2m+2n+3}\}$ be defined as follows

$f(u) = 1$

$f(u_i) = 2F_i+1, 1 \leq i \leq m$

$f(v) = 2F_{m+1}+1$

$f(v_j) = 2F_{m+1+j+f(v)}, 1 \leq j \leq n$

$f^+(E) = \{f(uv), f(uu_1), f(uu_2), ..., f(uu_m), f(vv_1), f(vv_2), ..., f(vv_n)\}$

$= \{\sqrt{f(v)-2F_{m+n+1}-f(v)}, \sqrt{f(v)-2F_{m+n+1}-f(v)}, ..., \sqrt{f(v)-2F_{m+n+1}-f(v)}\}$

$= \{F_{m+1}, F_{1}, F_{2}, ..., F_{m}, F_{m+1}, F_{m+2}, F_{m+3,2}, ..., F_{m+n+1}\}$

Thus, the induced edge labels are distinct and are $F_1, F_2, ..., F_{m+n+1}$.

Hence the graph $B_{m,n}$ is skolem difference Fibonacci mean graph for all $m, n \geq 1$.

Example 2.6: The Skolem difference Fibonacci mean labelling of the bistar graph $B_{4,3}$ is

![Figure-3](image)

Corollary 2.7: The bistar $B_{n,n}$ is Skolem difference Fibonacci mean graph for all $n \geq 1$.

Theorem 2.8: The graph $B(m,n,k)$ is Skolem difference Fibonacci mean graph for all $m, n, k \geq 1$ (or) $K_{1,m} \oplus P_k \oplus K_{1,n}$ is skolem difference Fibonacci mean graph for all $m, n, k \geq 1$.

Proof: Let $V(B(m,n,k)) = \{u_i, v_j, w_s \mid 1 \leq i \leq m, 1 \leq j \leq n$ and $1 \leq s \leq k+1\}$

$E(B(m,n,k)) = \{w_1u_i, w_sw_s+1, w_{k+1}v_j \mid 1 \leq i \leq m, 1 \leq j \leq n$ and $1 \leq s \leq k\}$

Then $|V(B(m,n,k))| = k+m+n+1$ and $|E(B(m,n,k))| = k+m+n$

Let $f: V \rightarrow \{1, 2, ..., F_{2k+2m+2n+1}\}$ be defined as follows

$f(w_i) = 2F_{k+i}+f(w_i), 1 \leq i \leq k+1$

$f(v_j) = 2F_{m+k+j}+f(w_{k+1}), 1 \leq j \leq n$

$f^+(E) = \{f(w_1u_i), f(w_sw_s+1), f(w_{k+1}v_j) \mid 1 \leq i \leq m, 1 \leq j \leq n$ and $1 \leq s \leq k\}$
Thus, the induced edge labels are distinct and are $F_1, F_2, \ldots, F_{m+n+k}$.

Hence the graph $B(m, n, k)$ is Skolem difference Fibonacci mean graph for all $m, n, k \geq 1$.

**Example 2.9:** The Skolem difference Fibonacci mean labelling of the graph $B(2,3,3)$ is

![Figure-4](image)

**Definition 2.10:** The coconut tree graph is obtained by identifying the central vertex of $K_{1,m}$ with a pendant vertex of the path $P_n$.

**Corollary 2.11:** The coconut tree graph $B(1, n-1, m)$ is Skolem difference Fibonacci mean graph.

**Corollary 2.12:** The graph obtained by the subdivision of the central edge of the bistar $B_{m,n}$ is Skolem difference Fibonacci mean graph for all $m, n \geq 1$.

**Proof:** Note that $G \cong B(m,n,2)$.

Hence $G$ Skolem difference Fibonacci mean graph.

**Example 2.13:** The Skolem difference Fibonacci mean labelling of the subdivision of the central edge of the bistar $B_{6,4}$ is

![Figure-5](image)

**Theorem 2.14:** The graph $\bigcup_{i=1}^{\ell} K_{1,i}$ is skolem difference Fibonacci mean graph.

**Proof:** Let $V(U_{i=1}^{r} K_{1,i}) = \{u_i / 1 \leq i \leq r\}$ and $E(U_{i=1}^{r} K_{1,i}) = \{u_i u_j / 1 \leq i \leq r \text{ and } 1 \leq j \leq \ell_i\}$

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Then $\left| V \left( \bigcup_{i=1}^{r} K_{1,i} \right) \right| = r + \ell_1 + \ell_2 + \ldots + \ell_r$ and $\left| E \left( \bigcup_{i=1}^{r} K_{1,i} \right) \right| = \ell_1 + \ell_2 + \ldots + \ell_r$.

Let $f\colon V (G) \to \{1, 2, \ldots, F_{\ell_1+\ell_2+\ldots+\ell_r}+r\}$ be defined as follows

- $f(u_i) = 1$, $f(u_2) = 2$,
- $f(u_{i}) = F_{i+2}$, $3 \leq i \leq r$,
- $f(u_{i_j}) = 2F_{j+1}$, $1 \leq j \leq \ell_1$,
- $f(u_{ij}) = 2F_{\sum_{k=2}^{i} \ell_k} - f(u_{i-1})$,

Thus, the induced edge labels are distinct and are $F_1, F_2, \ldots, F_{\ell_1+\ell_2+\ldots+\ell_r-1+i}$. Hence, $\bigcup_{i=1}^{r} K_{1,i}$ is a Skolem difference Fibonacci mean graph.

**Example 2.15:** Skolem difference Fibonacci mean labelling of the graph $k_{1,3} \cup k_{1,5} \cup k_{1,7} \cup k_{1,4}$ is

![Figure-6](image-url)
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Source of support: Nil, Conflict of interest: None Declared.

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