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SKOLEM DIFFERENCE FIBONACCI MEAN LABELLING OF SOME STANDARD GRAPHS

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ABSTRACT

The concept of Skolem difference mean labelling was introduced by K. Murugan and A. Subramanian [6]. The concept of Fibonacci labelling was introduced by David W. Bange and Anthony E. Barkauskas [1] in the form Fibonacci graceful. This motivates us to introduce Skolem difference Fibonacci mean labelling and is defined as follows: "A graph G with p vertices and q edges is said to have Skolem difference Fibonacci mean labelling if it is possible to label the vertices x∈ V with distinct elements f(x) from the set {1,2,...,F_{p+q}} in such a way that the edge e = uv is labelled with $\left|\frac{f(u)-f(v)}{2}\right|$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd and the resulting edge labels are distinct and are from {F_1, F_2,...,F_q}. A graph that admits ***Skolem difference Fibonacci mean labelling*** *is called a Skolem difference Fibonacci mean graph". In this paper, we prove that path, star, bistar, B (m, n, k) and union of stars are Skolem difference Fibonacci mean graphs.*

Keywords: *Skolem difference mean labelling, Fibonacci labelling, Skolem difference Fibonacci mean labelling.*

1. INTRODUCTION

A graph G with p vertices and q edges is said to have Skolem difference Fibonacci mean labelling if it is possible to label the vertices x∈ V with distinct elements f(x) from the set {1,2,...,F_{p+q}} in such a way that the edge e = uv is labelled with $\left|\frac{f(u)-f(v)}{2}\right|$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd and the resulting edge labels are distinct and are from {F_1, F_2,...,F_q}. A graph that admits Skolem difference Fibonacci mean labelling is called a Skolem difference Fibonacci mean graph. It was found that some special class of trees [7], H- class of graphs [8], some special class of graphs [9] and path related graphs [10] are Skolem difference Fibonacci mean graphs. The following definitions and notations are used in main results.

Definition 1.1: A *path* P_n with n points has V = {v_1, v_2, ..., v_n} for its vertex set and E (P_n) = {v_1v_2, v_2v_3, ..., $v_{n-1}v_n$} is its edge set. This path P_n is said to have length n-1.

Definition 1.2: A complete bigraph $K_{1,n}$, is called a *star*.

Definition 1.3: The bistar $B_{m,n}$ is obtained by joining the centre vertices of $K_{1,m}$ and $K_{1,n}$ with an edge.

Definition 1.4: The graph B (m,n,k) is obtained from a path of length k by attaching the star $K_{1,m}$ and $K_{1,n}$ with its pendant vertices.

Definition 1.5: Let G_1 (V_1, E_1) and G_2 (V_2, E_2) be two graphs. Then their union G = $G_1 \cup G_2$ is a graph with vertex set V = V_1 ∪ V2 and edge set E = $E_1 \cup E_2$.

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2. MAIN RESULT

Theorem 2.1: The path P_n is skolem difference Fibonacci mean graph for all n$\geq$ 2

Proof: Let V (P_n) = {v_i / $1 \leq i \leq n$}
E (P_n) = {v_iv_{i+1}, v_nv_1 /$1 \leq i \leq n-1$}

Then $|V(P_n)|$ = n and $|E(P_n)|$ = n-1

Let f: V $\rightarrow$ {1, 2,..., F_{2n-1}} be defined as follows
f (v_i) = $2F_{i+1}$, $1 \leq i \leq n$

$$f^+(E) = \{f(v_iv_{i+1})/ i=1,2,...,n-1\}$$
$$= \{f(v_1v_2), f(v_2v_3), ..., f(v_{n-1}v_n)\}$$
$$= \left\{\left|\frac{f(v_1)-f(v_2)}{2}\right|, \left|\frac{f(v_2)-f(v_3)}{2}\right|, \ldots, \left|\frac{f(v_{n-1})-f(v_n)}{2}\right|\right\}$$
$$= \left\{\left|\frac{2F_2-2F_3}{2}\right|, \left|\frac{2F_3-2F_4}{2}\right|, \ldots, \left|\frac{2F_n-2F_{n+1}}{2}\right|\right\}$$
$$= \left\{2\frac{|F_2-F_3|}{2}, 2\frac{|F_3-F_4|}{2}, \ldots, 2\frac{|F_n-F_{n+1}|}{2}\right\}$$
$$= \{F_1, F_2,..., F_{n-1}\}$$

Thus, the induced edge labels are distinct and are F_1, F_2,..., F_{n-1}.

Hence the path P_n is skolem difference Fibonacci mean graph for all n$\geq$ 2.

Example 2.2:

Figure-1

Theorem 2.3: The graph $K_{1,n}$ is skolem difference Fibonacci mean graph for all n$\geq$ 1

Proof: Let V ($K_{1,n}$) = {u, u_i / $1 \leq i \leq n$}
E ($K_{1,n}$) = {uu_i /$1 \leq i \leq n$}

Then $|V(K_{1,n})|$ = n+1 and $|E(K_{1,n})|$ = n

Let f: V $\rightarrow$ {1, 2,..., F_{2n+1}} be defined as follows
f (u) = 1

f (u_i) = $2F_i+1$, $1 \leq i \leq n$

$$f^+(E) = \{f(uu_i)/ 1 \leq i \leq n\}$$
$$= \{f(uu_1), f(uu_2), ..., f(uu_n)\}$$
$$= \left\{\left|\frac{f(u)-f(u_1)}{2}\right|, \left|\frac{f(u)-f(u_2)}{2}\right|, \ldots, \left|\frac{f(u)-f(u_n)}{2}\right|\right\}$$
$$= \left\{\left|\frac{1-2F_1-1}{2}\right|, \left|\frac{1-2F_2-1}{2}\right|, \ldots, \left|\frac{1-2F_n-1}{2}\right|\right\}$$
$$= \{F_1, F_2,..., F_n\}$$

Thus, the induced edge labels are distinct and are F_1, F_2,..., F_n.

Hence the graph $K_{1,n}$ is skolem difference Fibonacci mean graph for all n$\geq$ 1.

Example 2.4: The Skolem difference Fibonacci mean labelling of the star graph $K_{1,5}$ is

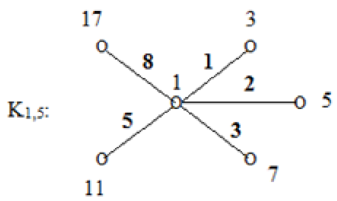

Figure-2

Theorem 2.5: The bistar $B_{m,n}$ is skolem difference Fibonacci mean graph for all m, n$\geq$ 1.

Proof: Let V ($B_{m,n}$) = {u, u_i, v, v_j / $1 \leq i \leq m$ and $1 \leq j \leq n$}
E ($B_{m,n}$) = {uv, uu_i, vv_j /$1 \leq i \leq m$ and $1 \leq j \leq n$}

Then $|V(B_{m,n})|$ = m+ n+2 and $|E(B_{m,n})|$ = m+n+1

Let f: V $\rightarrow$ {1, 2,..., $F_{2m+2n+3}$} be defined as follows

f (u) = 1

f (u_i) = $2F_i+1$, $1 \leq i \leq m$

f (v) = $2F_{m+1}+1$

f (v_j) = $2F_{m+1+j}+f(v)$, $1 \leq j \leq n$

$f^+(E)$ = { f(uv), f(uu_i), f(vv_j) /$1 \leq i \leq m$ and $1 \leq j \leq n$}

$$= \{f(uv), f(uu_1), f(uu_2), ..., f(uu_m), f(vv_1), f(vv_2), ..., f(vv_n)\}$$

$$= \left\{\left|\frac{f(u)-f(v)}{2}\right|, \left|\frac{f(u)-f(u_1)}{2}\right|, \left|\frac{f(u)-f(u_2)}{2}\right|, \ldots, \left|\frac{f(u)-f(u_m)}{2}\right|, \left|\frac{f(v)-f(v_1)}{2}\right|, \left|\frac{f(v)-f(v_2)}{2}\right|, \ldots, \left|\frac{f(v)-f(v_n)}{2}\right|\right\}$$

$$= \left\{\begin{matrix}\left|\frac{1-2F_{m+n}-1}{2}\right|, \left|\frac{1-2F_1-1}{2}\right|, \left|\frac{1-2F_2-1}{2}\right|, \ldots, \left|\frac{1-2F_m-1}{2}\right|, \left|\frac{f(v)-2F_{m+2}-f(v)}{2}\right|, \left|\frac{f(v)-2F_{m+3}-f(v)}{2}\right|, \ldots, \\ \left|\frac{f(v)-2F_{m+n+1}-f(v)}{2}\right|\end{matrix}\right\}$$

$$= \{F_{m+1}, F_1, F_2, ..., F_m, F_{m+2}, F_{m+3}, ..., F_{m+n+1}\}$$

$$= \{F_1, F_2, ..., F_m, F_{m+1}, F_{m+2}, F_{m+3}, ..., F_{m+n+1}\}$$

Thus, the induced edge labels are distinct and are $F_1, F_2, ..., F_{m+n+1}$.

Hence the graph $B_{m,n}$ is skolem difference Fibonacci mean graph for all m, n$\geq$ 1.

Example 2.6: The Skolem difference Fibonacci mean labelling of the bistar graph $B_{4,3}$ is

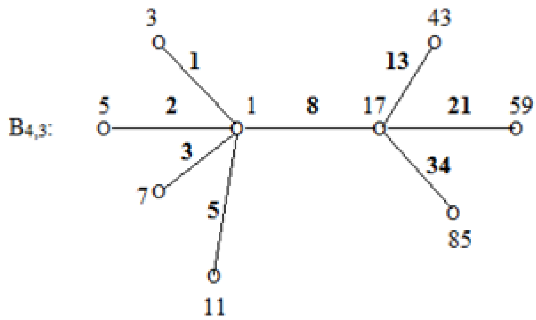

Figure-3

Corollary 2.7: The bistar $B_{n,n}$ is Skolem difference Fibonacci mean graph for all n$\geq$ 1.

Theorem 2.8: The graph B (m,n,k) is Skolem difference Fibonacci mean graph for all m, n, k$\geq$ 1(or) $K_{1,m}@ P_k@ K_{1,n}$ is skolem difference Fibonacci mean graph for all m, n, k$\geq$ 1 .

Proof: Let V (B (m, n, k)) = {u_i, v_j, w_s / $1 \leq i \leq m$, $1 \leq j \leq n$ and $1 \leq s \leq k+1$}
E (B (m, n, k)) = {w_1u_i, w_sw_{s+1}, $w_{k+1}v_j$ /$1 \leq i \leq m$, $1 \leq j \leq n$ and $1 \leq s \leq k$}

Then $|V(\mathrm{B(m,n,k)})|$ = k+m+ n+1 and $|E(\mathrm{B(m,n,k)})|$ = k+m+n

Let f: V $\rightarrow$ {1, 2,..., $F_{2k+2m+2n+1}$} be defined as follows

f (w_s) = $2F_{s+1}$, $1 \leq s \leq k+1$

f (u_i) = $2F_{k+i}+f(w_1)$, $1 \leq i \leq m$

f (v_j) = $2F_{m+k+j}+f(w_{k+1})$, $1 \leq j \leq n$

f^+ (E) = {f(w_1u_i), f(w_sw_{s+1}), f($w_{k+1}v_j$) /$1 \leq i \leq m$,$1 \leq j \leq n$ and $1 \leq s \leq k$}

$= \{f(w_1u_1), f(w_1u_2),..., f(w_1u_m), f(w_1,w_2), f(w_2w_3),..., f(w_kw_{k+1}), f(w_{k+1}v_1), f(w_{k+1}v_2),..., f(w_{k+1}v_n)\}$

$$= \left\{\begin{array}{c} \left|\frac{f(w_1)-f(u_1)}{2}\right|, \left|\frac{f(w_1)-f(u_2)}{2}\right|, ..., \left|\frac{f(w_1)-f(u_m)}{2}\right|, \left|\frac{f(w_1)-f(w_2)}{2}\right|, \left|\frac{f(w_2)-f(w_3)}{2}\right|, ..., \left|\frac{f(w_k)-f(w_{k+1})}{2}\right|, \\ \left|\frac{f(w_{k+1})-f(v_1)}{2}\right|, \left|\frac{f(w_{k+1})-f(v_2)}{2}\right|, ..., \left|\frac{f(w_{k+1})-f(v_n)}{2}\right| \end{array}\right\}$$

$$= \left\{\begin{array}{c} \left|\frac{f(w_1)-2F_{k+1}-f(w_1)}{2}\right|, \left|\frac{f(w_1)-2F_{k+2}-f(w_1)}{2}\right|, ..., \left|\frac{f(w_1)-2F_{k+m}-f(w_1)}{2}\right|, \left|\frac{2F_2-2F_3}{2}\right|, \left|\frac{2F_3-2F_4}{2}\right|, ..., \\ \left|\frac{2F_{k+1}-2F_{k+2}}{2}\right|, \left|\frac{f(w_{k+1})-2F_{m+k+1}-f(w_{k+1})}{2}\right|, \left|\frac{f(w_{k+1})-2F_{m+k+2}-f(w_{k+1})}{2}\right|, ..., \left|\frac{f(w_{k+1})-2F_{m+k+n}-f(w_{k+1})}{2}\right| \end{array}\right\}$$

$= \{F_{k+1}, F_{k+2},..., F_{k+m}, |F_2 - F_3|, |F_3 - F_4|,..., |F_{k+1} - F_{k+2}|, F_{m+k+1}, F_{m+k+2},..., F_{m+k+n}\}$

$= \{F_{k+1}, F_{k+2},..., F_{k+m}, |F_1|, |F_2|,..., |F_k|, F_{m+k+1}, F_{m+k+2},..., F_{m+k+n}\}$

$= \{F_1, F_2, ..., F_k, F_{k+1}, F_{k+2},..., F_{k+m}, F_{m+k+1}, F_{m+k+2},..., F_{m+k+n}\}$

Thus, the induced edge labels are distinct and are $F_1, F_2,..., F_{m+n+k}$.

Hence the graph B (m, n, k) is Skolem difference Fibonacci mean graph for all m, n, k $\geq$ 1.

Example 2.9: The Skolem difference Fibonacci mean labelling of the graph B(2,3,3) is

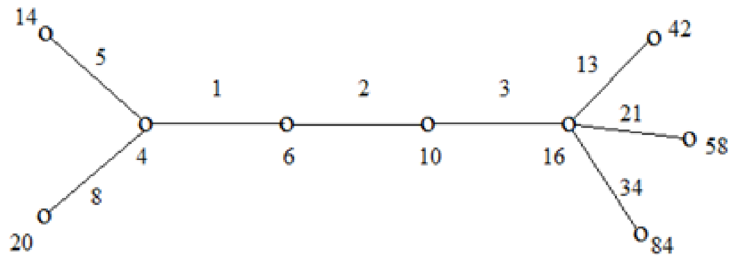

Figure-4

Definition 2.10: The coconut tree graph is obtained by identifying the central vertex of $K_{1,m}$ with a pendant vertex of the path P_n.

Corollary 2.11: The coconut tree graph B (1, n-1, m) is Skolem difference Fibonacci mean graph.

Corollary 2.12: The graph obtained by the subdivision of the central edge of the bistar $B_{m,n}$ is Skolem difference Fibonacci mean graph for all m, n $\geq$ 1.

Proof: Note that G ≅ B(m,n,2).

Hence G Skolem difference Fibonacci mean graph.

Example 2.13: The Skolem difference Fibonacci mean labelling of the subdivision of the central edge of the bistar $B_{6,4}$ is

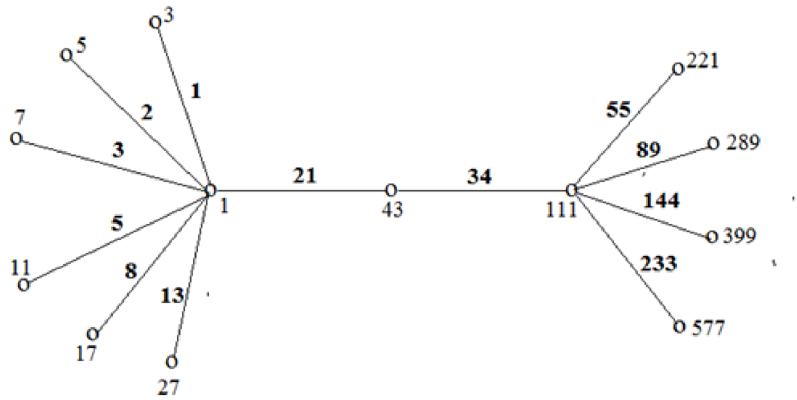

Figure-5

Theorem 2.14: The graph $\bigcup_{i=1}^{r} K_{1,l_i}$ is skolem difference Fibonacci mean graph.

Proof: Let V $(\bigcup_{i=1}^{r} K_{1,l_i})$ = $\{u_i / 1 \leq i \leq r\} \cup \{u_{ij} / 1 \leq i \leq r$ and $1 \leq j \leq \ell_i\}$
E $(\bigcup_{i=1}^{r} K_{1,l_i})$ = $\{u_i\, u_{ij} / 1 \leq i \leq r$ and $1 \leq j \leq \ell_i\}$

Then $\left|V(\bigcup_{i=1}^{r} K_{1,l_i})\right|$ = r+ ℓ_1+ ℓ_2+...+ ℓ_r and $\left|E(\bigcup_{i=1}^{r} K_{1,l_i})\right|$ = ℓ_1+ ℓ_2+...+ ℓ_r

Let f: V (G) → $\{1,2,\ldots,F_{2(\ell1+\ell2+\ldots+\ell r)+r}\}$ be defined as follows

$f(u_1) = 1$, $f(u_2) = 2$

$f(u_i) = F_{i+2}$, $3 \le i \le r$

$f(u_{1j}) = 2F_j+1$, $1 \le j \le \ell_1$

$f(u_{ij}) = 2F_{\left(\sum_{k=2}^{i} l_{k-1}\right)+j} + f(u_i)$, $2 \le i \le r$ and $1 \le j \le \ell_r$

$$\begin{aligned}
f^{+}(E) &= \{f(u_iu_{ij}) / 1 \le i \le r \text{ and } 1 \le j \le \ell_i\} \\
&= \{f(u_1u_{11}), f(u_1u_{12}),\ldots, f(u_1u_{1\ell 1}), f(u_2u_{21}), f(u_2u_{22}),\ldots, f(u_2u_{2\ell 2}),\ldots, f(u_ru_{r1}), f(u_ru_{r2}),\ldots, f(u_ru_{r\ell r})\} \\
&= \left\{\begin{array}{c}\left|\frac{f(u_1)-f(u_{11})}{2}\right|, \left|\frac{f(u_1)-f(u_{12})}{2}\right|, \ldots, \left|\frac{f(u_1)-f(u_{1l_1})}{2}\right|, \left|\frac{f(u_2)-f(u_{21})}{2}\right|, \left|\frac{f(u_2)-f(u_{22})}{2}\right|, \ldots, \left|\frac{f(u_2)-f(u_{2l_2})}{2}\right|, \ldots, \\ \left|\frac{f(u_r)-f(u_{r1})}{2}\right|, \left|\frac{f(u_r)-f(u_{r2})}{2}\right|, \ldots, \left|\frac{f(u_r)-f(u_{rl_r})}{2}\right|\end{array}\right\} \\
&= \left\{\begin{array}{c}\left|\frac{1-2F_1-1}{2}\right|, \left|\frac{1-2F_2-1}{2}\right|, \ldots, \left|\frac{1-2F_{l_1}-1}{2}\right|, \left|\frac{f(u_2)-2F_{l_1+1}-f(u_2)}{2}\right|, \left|\frac{f(u_2)-2F_{l_1+2}-f(u_2)}{2}\right|, \ldots, \left|\frac{f(u_2)-2F_{l_1+l_2}-f(u_2)}{2}\right|, \\ \ldots, \left|\frac{f(u_r)-2F_{l_1+l_2+\cdots+l_{r-1}+1}-f(u_r)}{2}\right|, \left|\frac{f(u_r)-2F_{l_1+l_2+\cdots+l_{r-1}+2}-f(u_r)}{2}\right|, \ldots, \left|\frac{f(u_r)-2F_{l_1+l_2+\cdots+l_{r-1}+l_r}-f(u_r)}{2}\right|\end{array}\right\} \\
&= \{F_1, F_2,\ldots, F_{l_1}, F_{l_1+1}, F_{l_1+2},\ldots, F_{l_1+l_2},\ldots, F_{l_1+l_2+\cdots+l_{r-1}+1}, F_{l_1+l_2+\cdots+l_{r-1}+2},\ldots, F_{l_1+l_2+\cdots+l_{r-1}+l_r}\} \\
&= \{F_1, F_2,\ldots, F_{l_1+l_2+\cdots+l_{r-1}+l_r}\}
\end{aligned}$$

Thus, the induced edge labels are distinct and are $F_1, F_2,\ldots, F_{l_1+l_2+\cdots+l_{r-1}+l_r}$.

Hence, $\bigcup_{i=1}^{r} K_{1,l_i}$ is a Skolem difference Fibonacci mean graph.

Example 2.15: Skolem difference Fibonacci mean labelling of the graph $k_{1,3} \cup k_{1,5} \cup k_{1,7} \cup k_{1,4}$ is

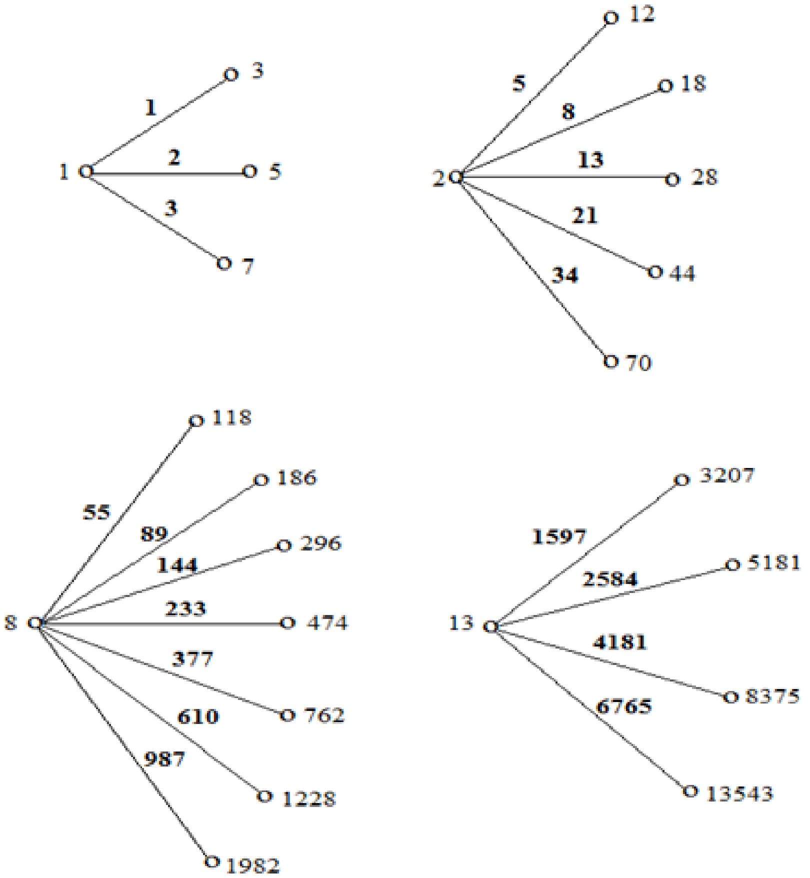

Figure-6

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