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COMMON FIXED POINT THEOREM IN FUZZY METRIC SPACES FOR COMPATIBLE MAPS

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ABSTRACT

In this paper we prove a common fixed point theorem in fuzzy metric space for compatible maps Mathematics subject classification 47A62, 47A63

***Keywords**: fixed point, common fixed point, fuzzy set, fuzzy metric space compatible.*

1. INTRODUCTION

The concept of Fuzzy sets was initially investigated by Zadeh [51] as a new way to represent vagueness in everyday life. Subsequently, it was developed by many authors and used in various fields. To use this concept in Topology and Analysis, several researchers have defined Fuzzy metric space in various ways. In this paper we deal with the Fuzzy metric space defined by Kramosil and Michalek [58] and modified by George and Veeramani [29]. Recently, Grebiec [30] has proved fixed point results for Fuzzy metric space. In the sequel, Singh and Chauhan introduced the concept of compatible mappings of Fuzzy metric space and proved the common fixed point theorem. Jungck *et al.* [48] introduced the concept of compatible maps of type (A) in metric space and proved fixed point theorems. Cho [15, 16] introduced the concept of compatible maps of type (α) and compatible maps of type (β) in fuzzy metric space. Using the concept of compatible maps of type (A), Jain *et al.* [46] proved a fixed point theorem for six self maps in a fuzzy metric space. Using the concept of compatible maps of type (β), Jain *et al.* [47] proved a fixed point theorem in fuzzy metric space. In this paper, a fixed point theorem for six self maps has been established using the concept of compatible maps of type (β) and weak compatible maps, which generalizes the result of Cho [14].

For the sake of completeness, we recall some definition and known results in Fuzzy metric space, which are used in this chapter.

Definition 1.1: Let X be any set. A fuzzy set in X is a function with domain X and values in $[0,1]$.

Definition 1.2: A binary operation $\star : [0,1] \times [0,1] \to [0,1]$ is continuous $t-$norm if $\star$ is satisfying the following conditions:
1.1 (a) $\star$ is commutative and associative,
1.2 (b) $\star$ is continuous,
1.2 (c) $a \star 1 = a$ for all $a \in [0,1]$
1.2 (d) $a \star b \le c \star d$ whenever $a \le c$ and $b \le d$, for all $a, b, c, d \in [0,1]$

Examples of $t-$ norm are $a \star b = \min\{a, b\}$ and $a \star b = ab$.

Definition 1.3: A triplet $(X, M, \star)$ is a fuzzy metric space whenever X is an arbitrary set, $\star$ is continuous $t-$norm and M is fuzzy set on $X \times X \times [0, \infty^+)$ satisfying, for every $x, y, z \in X$ and $s, t > 0$, the following condition:
1.3 (a) $M(x,y,t) > 0$
1.3 (b) $M(x,y,0) = 0$
1.3 (c) $M(x,y,t) = 1$ iff $x = y$
1.3 (d) $M(x,y,t) = M(y,x,t)$
1.3 (e) $M(x,y,t) \star M(y,z,s) \le M(x,z,t+s)$
1.3 (f) $M(x,y,\cdot) : (0,\infty^+) \to [0,1]$ is continuous.

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We note that, $M(x, y, t)$ can be realized as the measure of nearness between x and y with respect to t. It is known that $M(x, y, \cdot)$ is non decreasing for all $x, y \in X$. Let $M(x, y, \star)$ be a fuzzy metric space for $t > 0$, the open ball

$$B(x, r, t) = \{y \in X: M(x, y, t) > 1 - r\}.$$

Now, the collection $\{B(x, r, t): x \in X, 0 < r < 1, t > 0\}$ is a neighborhood system for a topology τ on X induced by the fuzzy metric M. This topology is Housdroff and first countable.

Example 1.4: Let (X, d) be a metric space. Define $a \star b = \min\{a, b\}$ and $M(x, y, t) = \frac{t}{t + d(x, y)}$ for all $x, y \in X$ and all $t > 0$. Then $(X, M, \star)$ is a fuzzy metric space. It is called the fuzzy metric space induced by d.

Definition 1.5: A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, \star)$ is said t*o*

1. a converges to x iff for each $\varepsilon > 0$ and each $t > 0$, $n_0 \in N$ such that $M(x_n, x, t) > 1 - \varepsilon$ for all $n \geq n_0$.
2. a Cauchy sequence converges to x iff for each $\varepsilon > 0$ and each $t > 0$, $n_0 \in N$ such that $M(x_m, x_n, t) > 1 - \varepsilon$ for all $m, n \geq n_0$.
3. A fuzzy metric space $(X, M, \star)$ is said to be complete if every Cauchy sequence in it converges to a point in it.

Definition 1.6: Self mapping A and S of a fuzzy metric space $(X, M, \star)$ are said to be compatible if and only if $M(ASx_n, SAx_n, t) \to 1$ for all $t > 0$, where $\{x_n\}$ is a sequence in X such that Sx_n, $Ax_n \to p$ for some $p \in X$ as $n \to \infty$.

Definition 3.1.8: Self map A and S of a fuzzy metric space $(X, M, \star)$ are said

1. to be compatible of type (β) if and only if $M(AAx_n, SSx_n, t) \to 1$ for all $t > 0$, where $\{x_n\}$ is a sequence in X such that Sx_n, $Ax_n \to p$ for some $p \in X$ as $n \to \infty$.
2. A and S is said to be a weakly commuting if $M(ASx_n, SAx_n, t) \leq M(Sx_n, Ax_n, t)$ for all x \in X .
   It can be seen that commuting maps ($ASx = SAx \; \forall \; x \in X$) are weakly compatible but converse is not true.
3. Two maps A and B from a fuzzy metric space $(X, M, \star)$ into itself are said to be weakly compatible if they commute at their coincidence points i.e., $Ax = Bx$ implies $ABx = BAx$ for some $x \in X$.

Remark 1.7: The concept of compatible map of type (β) is more general then the concept of compatible map in fuzzy metric space.

Definition 1.8: Let A and S be two self maps of a fuzzy metric space $(X, M, \star)$ then

Lemma 1.9: In a fuzzy metric space $(X, M, \star)$ limit of a sequence is unique.

Lemma 1.10: Let $(X, M, \star)$ be a fuzzy metric space. Then for all $x, y \in X$ $M(x, y, .)$ is a non decreasing function.

Lemma 1.11: Let $(X, M, \star)$ be a fuzzy metric space. If there exists $k \in (0,1)$ such that for all $x, y \in X$, $M(x, y, kt) \geq M(x, y, t) \; \forall t > 0, then\ x = y$.

Lemma 1.12: Let $\{x_n\}$ be a sequence in a fuzzy metric space $(X, M, \star)$. If there exists a number $k \in (0,1)$ such that

$$M(x_{n+2}, x_{n+1}, kt) \geq M(x_{n+1}, x_n, t) \;\; \forall \;\; t > 0 \; and \; n \in N$$

Then $\{x_n\}$ is a Cauchy sequence in X.

Lemma 1.13: The only $t - $norm $\star$ satisfying $r \star r = r$ for all $r \in [0,1]$ is the minimum $t - $norm that is

$$a \star b = \min\{a, b\} \text{ for all } a, b \in [0,1].$$

2. MAIN THEOREM

Common Fixed Point Theorem for Compatible Maps of Type (β) and Type (α)

In this paper we prove a common fixed point theorem for compatible map of type (β) and type (α) in fuzzy metric space. In fact we prove the following theorem.

Theorem 2.1: Let $(X, M, \star)$ be a complete fuzzy metric space and let A, B, S, T, P and Q be mappings from X into itself such that the following conditions are satisfied:
2.1(a) $P(X) \subset ST(X)$ and $Q(X) \subset AB(X)$,
2.1(b) $AB = BA, ST = TS, PB = BP, QT = TQ$,
2.1(c) either P or AB is continuous,
2.1(d) (P, AB) is compatible of type (β) and (Q, ST) is weak compatible
2.1(e) there exists $k \in (0,1)$ such that for every $x, y \in X$ and $t > 0$

$$M^2(Px, Qy, kt) \geq \min\begin{Bmatrix} M^2(ABx, STy, t), M^2(Px, ABx, t) \\ M^2(Qy, STy, t), M^2(Px, STy, t) \end{Bmatrix}$$

Then A, B,S,T, P and Q have a unique common fixed point in X.

Proof: Let $x_0 \in X$, then from 2.1(a) we have $x_1, x_2 \in X$ such that

$$Px_0 = STx_1 \text{ and } Qx_1 = ABx_2$$

Inductively, we construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that for $n \in N$

$$Px_{2n-2} = STx_{2n-1} = y_{2n-1} \text{ and } Qx_{2n-1} = ABx_{2n} = y_{2n}$$

Step-1: Put $x = x_{2n}$ and $y = x_{2n+1}$ in 2.1(e) then we have

$$M^2(Px_{2n}, Qx_{2n+1}, kt) \geq \min\begin{Bmatrix} M^2(ABx_{2n}, STx_{2n+1}, t), \\ M^2(Px_{2n}, ABx_{2n}, t), \\ M^2(Qx_{2n+1}, STx_{2n+1}, t), \\ M^2(Px_{2n}, STx_{2n+1}, t) \end{Bmatrix}$$

$$M^2(y_{2n+1}, y_{2n+2}, kt) \geq \min\begin{Bmatrix} M^2(y_{2n}, y_{2n+1}, t), M^2(y_{2n+1}, y_{2n}, t) \\ M^2(y_{2n+2}, y_{2n+1}, t), M^2(y_{2n+1}, y_{2n+1}, t) \end{Bmatrix}$$

$$M^2(y_{2n+1}, y_{2n+2}, kt) \geq \min\{M^2(y_{2n}, y_{2n+1}, t), M^2(y_{2n+2}, y_{2n+1}, t)\}$$

From lemma 1.13 and 1.14 we have

$$M^2(y_{2n+1}, y_{2n+2}, kt) \geq M^2(y_{2n}, y_{2n+1}, t)$$

That is

$$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t)$$

Similarly we have

$$M(y_{2n+2}, y_{2n+3}, kt) \geq M(y_{2n+1}, y_{2n+2}, t)$$

Thus we have

$$M(y_{n+1}, y_{n+2}, kt) \geq M(y_n, y_{n+1}, t)$$

$$M(y_{n+1}, y_{n+2}, t) \geq M\left(y_n, y_{n+1}, \frac{t}{k}\right)$$

$$M(y_n, y_{n+1}, t) \geq M\left(y_0, y_1, \frac{t}{k^n}\right) \to 1 \text{ as } n \to \infty,$$

and hance $M(y_n, y_{n+1}, t) \to 1$ as $n \to \infty$ for all $t > 0$.

For each $\epsilon > 0$ *and* $t > 0$, we can choose $n_0 \in N$ such that

$$M(y_n, y_{n+1}, t) > 1 - \epsilon \text{ for all } n > n_0.$$

For any $m, n \in N$ we suppose that $m \geq n$. Then we have

$$M(y_n, y_m, t) \geq M\left(y_n, y_{n+1}, \frac{t}{m-n}\right) \star M\left(y_{n+1}, y_{n+2}, \frac{t}{m-n}\right) \star \\ \dots \star M\left(y_{m-1}, y_m, \frac{t}{m-n}\right)$$

$$M(y_n, y_m, t) \geq (1-\epsilon) \star (1-\epsilon) \star \dots \dots \star (1-\epsilon)(m-n)\text{times}$$

$$M(y_n, y_m, t) \geq (1-\epsilon)$$

And hence $\{y_n\}$ is a Cauchy sequence in X.

Since $(X, M, \star)$ is complete, $\{y_n\}$ converges to some point $z \in X$. Also its subsequences converges to the same point $z \in X$.

That is

$$\{Px_{2n+2}\} \to z \text{ and } \{STx_{2n+1}\} \to z \quad 2.1\text{ (i)}$$

$$\{Qx_{2n+1}\} \to z \text{ and } \{ABx_{2n}\} \to z \quad 2.1\text{(ii)}$$

Case-1: Suppose AB is continuous

Since AB is continuous, we have

$$(AB)^2 x_{2n} \to ABz \text{ and } ABPx_{2n} \to ABz$$

As (P, AB) is compatible pair of type (β), we have

$$M(PPx_{2n}, (AB)(AB)x_{2n}, t) = 1, \quad \text{for all } t > 0$$

Or

$$M(PPx_{2n}, ABz, t) = 1$$

Therefore, $PPx_{2n} \to ABz$.

Step-2: Put $x = (AB)x_{2n}$ and $y = x_{2n+1}$ in 3.2.1(e) we have

$$M^2(P(AB)x_{2n}, Qy, kt) \geq \min\left\{\begin{matrix} M^2(AB(AB)x_{2n}, STx_{2n+1}, t), \\ M^2(P(AB)x_{2n}, AB(AB)x_{2n}, t), \\ M^2(Qx_{2n+1}, STx_{2n+1}, t), \\ M^2(P(AB)x_{2n}, STx_{2n+1}, t) \end{matrix}\right\}$$

Taking $n \to \infty$ we get

$$M^2\big((AB)z, z, kt\big) \geq \min\left\{\begin{matrix} M^2\big((AB)z, z, t\big), M^2\big((AB)z, (AB)z, t\big), \\ M^2\big((AB)z, z, t\big), M^2\big((AB)z, z, t\big) \end{matrix}\right\}$$

$$M^2\big((AB)z, z, kt\big) \geq \min\{M^2\big((AB)z, z, t\big), M^2\big((AB)z, z, t\big)\}$$

That is $M\big((AB)z, z, kt\big) \geq M\big((AB)z, z, t\big)$

Therefore by lemma 1.14 we have

$$ABz = z. \qquad 2.1(iii)$$

Step-3: Put $x = z$ and $y = x_{2n+1}$ in 3.2.1(e) we have

$$M^2(Pz, Q\,x_{2n+1}, kt) \geq \min\left\{\begin{matrix} M^2(ABz, ST\,x_{2n+1}, t), M^2(Pz, ABz, t), \\ M^2(Q\,x_{2n+1}, ST\,x_{2n+1}, t), M^2(Pz, ST\,x_{2n+1}, t) \end{matrix}\right\}$$

Taking $n \to \infty$ and using equation 2.1 (i) we have

$$M^2(Pz, z, kt) \geq \min\left\{\begin{matrix} M^2(ABz, z, t), M^2(Pz, ABz, t), \\ M^2(z, z, t), M^2(Pz, z, t) \end{matrix}\right\}$$

That is $M^2(Pz, z, kt) \geq M^2(Pz, z, t)$

And hence $M(Pz, z, kt) \geq M(Pz, z, t)$

Therefore by using lemma1.14, we get

$$Pz = z$$

So we have $ABz = Pz = z$.

Step-4: Putting $x = Bz$ and $y = x_{2n+1}$ in $2.1(e)$, we get

$$M^2(PBz, Qx_{2n+1}, kt) \geq \min\left\{\begin{matrix} M^2(ABBz, STx_{2n+1}, t), \\ M^2(PBz, ABBz, t), \\ M^2(Qx_{2n+1}, STx_{2n+1}, t), \\ M^2(PBz, STx_{2n+1}, t) \end{matrix}\right\}$$

As $BP = PB$ and $AB = BA$, so we have
$P(Bz) = B(Pz) = Bz$ and $(AB)(Bz) = (BA)(Bz) = B(ABz) = Bz$.

Taking $n \to \infty$ and using 2.1(i) we get

$$M^2(Bz, z, kt) \geq \min\left\{\begin{matrix} M^2(Bz, z, t), M^2(Bz, Bz, t), \\ M^2(z, z, t), M^2(Bz, z, t) \end{matrix}\right\}$$

$$M^2(Bz, z, kt) \geq M^2(Bz, z, t)$$

That is $M(Bz, z, kt) \geq M(Bz, z, t)$

Therefore by Lemma 1.14 we have $Bz = z$

And also we have $ABz = z$ implies $Az = z$

Therefore $Az = Bz = Pz = z$. 2.1 (iv)

Step–5: As $P(X) \subset ST(X)$ there exists $u \in X$ such that

$$z = Pz = STu$$

Putting $x = x_{2n}$ and $y = u$ in 2.1(e) we get

$$M^2(Px_{2n}, Qu, kt) \geq \min\left\{\begin{matrix} M^2(ABx_{2n}, STu, t), M^2(Px_{2n}, ABx_{2n}, t), \\ M^2(Qu, STu, t), M^2(Px_{2n}, STu, t) \end{matrix}\right\}$$

Taking $n \to \infty$ and using 2.1(i) and 2.1(ii) we get

$$M^2(z, Qu, kt) \geq \min\begin{Bmatrix} M^2(z, STu, t), M^2(z, z, t), \\ M^2(Qu, STu, t), M^2(z, STu, t) \end{Bmatrix}$$

$$M^2(z, Qu, kt) \geq M^2(z, Qu, t)$$

That is $\quad M(z, Qu, kt) \geq M(z, Qu, t)$

Therefore by using Lemma 1.13 we have $Qu = z$

Hence $STu = z = Qu$.

Hence (Q, ST) is weak compatible, therefore, we have

$$QSTu = STQu$$

Thus $Qz = STz$.

Step–6: Putting $x = x_{2n}$ and $y = z$ in 2.1(e) we get

$$M^2(Px_{2n}, Qz, kt) \geq \min\begin{Bmatrix} M^2(ABx_{2n}, STz, t), M^2(Px_{2n}, ABx_{2n}, t), \\ M^2(Qz, STz, t), M^2(Px_{2n}, STz, t) \end{Bmatrix}$$

Taking $n \to \infty$ and using 2.1(ii) and step 5 we get

$$M^2(z, Qz, kt) \geq \min\begin{Bmatrix} M^2(z, STz, t), M^2(z, z, t), \\ M^2(Qz, STz, t), M^2(z, STz, t) \end{Bmatrix}$$

$$M^2(z, Qz, kt) \geq M^2(z, Qz, t)$$

And hence $\quad M(z, Qz, kt) \geq M(z, Qz, t)$

Therefore by using Lemma 1.13 we get $Qz = z$.

Step–7: Putting $x = x_{2n}$ and $y = Tz$ in 2.1(e) we get

$$M^2(Px_{2n}, QTz, kt) \geq \min\begin{Bmatrix} M^2(ABx_{2n}, STTz, t), M^2(Px_{2n}, ABx_{2n}, t), \\ M^2(QTz, STTz, t), M^2(Px_{2n}, STTz, t) \end{Bmatrix}$$

As $QT = TQ$ and $ST = TS$ we have

$$QTz = TQz = Tz$$

And $ST(Tz) = T(STz) = TQz = Tz$.

Taking $n \to \infty$ we get

$$M^2(z, Tz, kt) \geq \min\begin{Bmatrix} M^2(z, Tz, t), M^2(z, z, t), \\ M^2(Tz, Tz, t), M^2(z, Tz, t) \end{Bmatrix}$$

$$M^2(z, Tz, kt) \geq M^2(z, Tz, t)$$

Therefore $\quad M(z, Tz, kt) \geq M(z, Tz, t)$

Therefore by Lemma 1.13 we have $Tz = z$

Now $STz = Tz = z$ implies $Sz = z$.

Hence

$$Sz = Tz = Qz = z \tag{2.1(v)}$$

Combining 2.1(iv) and 2.1(v) we have

$$Az = Bz = Pz = Sz = Tz = Qz = z$$

Hence z is the common fixed point of A, B, S, T, P and Q.

Case – II: suppose P is continuous

As P is continuous $P^2x_{2n} \to Pz$ and $P(AB)x_{2n} \to Pz$

As (P,AB) is compatible pair of type (β),

$$M(PPx_{2n}, (AB)(AB)x_{2n}, t) = 1 \text{ forall } t > 0$$

Or $\quad M(Pz, (AB)(AB)x_{2n}, t) = 1$

Therefore $\quad (AB)^2x_{2n} \to Pz$.

Step-8: Putting $x = Px_{2n}$ and $y = x_{2n+1}$ in 3.2.1(e) then we get

$$M^2(PPx_{2n}, Qx_{2n+1}, kt) \geq \min \begin{Bmatrix} M^2(ABPx_{2n}, STx_{2n+1}, t), \\ M^2(PPx_{2n}, ABPx_{2n}, t), \\ M^2(Qx_{2n+1}, STx_{2n+1}, t), \\ M^2(PPx_{2n}, STx_{2n+1}, t) \end{Bmatrix}$$

Taking $n \to \infty$, we get

$$M^2(Pz, z, kt) \geq \min \begin{Bmatrix} M^2(Pz, z, t), M^2(Pz, Pz, t), \\ M^2(z, z, t), M^2(Pz, z, t) \end{Bmatrix}$$

$$M^2(Pz, z, kt) \geq M^2(Pz, z, t)$$

Hence $M(Pz, z, kt) \geq M(Pz, z, t)$

Therefore by Lemma 1.13 we get $Pz = z$

Step-9: Put $x = ABx_{2n}$ and $y = x_{2n+1}$ in 2.1(e) then we get

$$M^2(P\,ABx_{2n}, Qx_{2n+1}, kt) \geq \min \begin{Bmatrix} M^2(AB\,ABx_{2n}, STx_{2n+1}, t), \\ M^2(P\,ABx_{2n}, AB\,ABx_{2n}, t), \\ M^2(Qx_{2n+1}, STx_{2n+1}, t), \\ M^2(P\,ABx_{2n}, STx_{2n+1}, t) \end{Bmatrix}$$

Taking $n \to \infty$ we get

$$M^2(ABz, z, kt) \geq \min \begin{Bmatrix} M^2(AB\,z, z, t), M^2(ABz, z, t), \\ M^2(z, z, t), M^2(ABz, z, t) \end{Bmatrix}$$

Therefore $M^2(ABz, z, kt) \geq M^2(ABz, z, t)$
And hence

$$M(ABz, z, kt) \geq M(ABz, z, t)$$

By Lemma 1.13 we get $ABz = z$

By applying step 4, 5, 6, 7, 8 we get

$$Az = Bz = Sz = Tz = Pz = Qz = z.$$

That is z is a common fixed point of A, B, S, T, P, Q in X.

Uniqueness: Let u be another common fixed point of A, B, S, T, P and Q. Then

$$Au = Bu = Su = Tu = Pu = Qu = u$$

Putting $x = u$ and $y = z$ in 2.1(e) then we get

$$M^2(Pu, Qz, kt) \geq \min \begin{Bmatrix} M^2(ABu, STz, t), M^2(Pu, ABu, t), \\ M^2(Qz, STz, t), M^2(Pu, STz, t) \end{Bmatrix}$$

Taking limit both side then we get

$$M^2(u, z, kt) \geq \min \begin{Bmatrix} M^2(u, z, t), M^2(u, u, t), \\ M^2(z, z, t), M^2(u, z, t) \end{Bmatrix}$$

$$M^2(u, z, kt) \geq M^2(u, z, t)$$

And hence $M(u, z, kt) \geq M(u, z, t)$

By lemma 1.13 we get $z = u$.

That is z is a unique common fixed point of A,B, S, T, P and Q in X.

Remark 2.2: If we take $B = T = I$ identity map on X in Theorem .2.1 then condition 2.2.1(b) is satisfy trivially and we get following Corollary

Corollary 2.3: Let $(X, M, \star)$ be a complete fuzzy metric space and let A, S, P and Q be mappings from X into itself such that the following conditions are satisfied:
2.3(a) $P(X) \subset S(X)$ and $Q(X) \subset A(X)$,
2.3(b) either P or A is continuous,
2.3(c) (P, A) is compatible of type (β) and (Q, S) is weak compatible,
2.3(d) there exists $k \in (0,1)$ such that for every $x, y \in X$ and $t > 0$

$$M^2(Px, Qy, kt) \geq \min\begin{Bmatrix} M^2(Ax, Sy, t), M^2(Px, Ax, t) \\ M^2(Qy, Sy, t), M^2(Px, Sy, t) \end{Bmatrix}$$

Then A, S, P and Q have a unique common fixed point in X.

Remark 2.4: If we take the pair (P, AB) is weakly compatible in place of compatible type of (β) in Theorem 2.1 then we get the following result.

Corollary 2.5: Let $(X, M, \star)$ be a complete fuzzy metric space and let A, B, S, T, P and Q be mappings from X into itself such that the following conditions are satisfied:
2.5(a) $P(X) \subset ST(X)$ and $Q(X) \subset AB(X)$,
2.5(b) $AB = BA, ST = TS, PB = BP, QT = TQ$,
2.5(c) either P or AB is continuous,
2.5(d) (P, AB) and (Q, ST) is weak compatible,
2.5(e) there exists $k \in (0,1)$ such that for every $x, y \in X$ and $t > 0$

$$M^2(Px, Qy, kt) \geq \min\begin{Bmatrix} M^2(ABx, STy, t), M^2(Px, ABx, t) \\ M^2(Qy, STy, t), M^2(Px, STy, t) \end{Bmatrix}$$

Then A, B, S, T, P and Q have a unique common fixed point in X.

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