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REGULAR WEAKY OPEN SETS AND REGULAR WEAKLY CLOSURE IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce and study the notions of Regular weakly open sets and Regular weakly closure in Topological spaces.

I. INTRODUCTION AND PRELIMINARIES

Regular open sets and strong regular open sets which were strong forms of open sets in topological spaces have been introduced and investigated by Stone [ST] and Tong [TO1] respectively. Semi open set, a weak form of open set was introduced by Levine [LN3]. Benchalli and Wali [BW] introduced regular weakly closed sets in topological spaces.

In this paper we introduce and sudy regular weakly open sets and regular weakly closure in topological spaces.

We recall the following definitions which are used in this paper.

Definition 1.1: A subset A of a topological space (X, τ) is called a

a) regular open if $A = int(cl(A))$ and regular closed if $A = cl(int(A))$.
b) pre-open if $A \subseteq int(cl(A))$ and preclosed if $cl(int(A)) \subseteq A$.
c) semiopen if $A \subseteq cl(int(A))$ and semiclosed if $int(cl(A)) \subseteq A$.
d) α-open if $A \subseteq int(cl(int(A)))$ and α -closed if $cl(int(cl(A))) \subseteq A$.
e) semi-preopen (= β-open) if $A \subseteq cl(int(cl(A)))$ and semi-preclosed (=β-closed) if $int(cl(int(A))) \subseteq A$.

Definition 1.2: A subset of a topological space (X, τ) is called regular weakly closed (briefly rw-closed) if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is regular semiopen in X.

II. REGULAR WEAKLY OPEN SETS IN TOPOLOGICAL SPACES

Definition 2.1: A subset A of a space X is called regular weakly (briefly rw-open) set if its complement is rw-closed. The family of all rw-open sets in X is denoted by RWO(X).

Theorem 2.2: A Subset A of (X, τ) is rw-open iff $U \subseteq int(A)$ whenever $U \subseteq A$ and U is regular semiclosed in X.

Proof: Suppose that $U \subseteq int(A)$, where U is regular semiclosed and $U \subseteq A$. Let $A^c \subseteq F$ and F is regular semiopen. Then $F^c \subseteq A$ and F^c is regular semiclosed. Therefore $F^c \subseteq int (A)$. Thus $cl(F) \subseteq F$. Hence A^c is rw-closed and therefore A is rw-open.

Conversely, let A be rw-open and $U \subseteq A$ where U is regular semiclosed. This implies U^c is regular semiopen and $A^c \subseteq U^c$. By assumption $cl(A^c) \subseteq U^c$. Thus $U \subseteq int(A)$.

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Theorem 2.3: For a space (X,τ),

(i) Every rw-open set is rg-open
(ii) Every open set is rw-open
(iii) Every rw-open set is gpr-open
(iv) Everyrw-open set is rwg-open
(v) Every ω-open set is rw-open

Proof: Obvious

Example 2.4: Let X={a,b,c,d} with topology $\tau = \{\phi,\{a\},\{b\},\{a,b\},\{a,b,c\},X\}$ then the set A={c,d} is rw-open but not open and the set B = {a,c} is rg-open , gpr-open and rwg-open but not rw-open set in X.

Example 2.5: Let X = {a,b,c} with topology $\tau=\{\phi,\{a\},\{b\},\{a,b\},X\}$ then the set A = {c} is rw-open but not ω-open set in X.

Theorem 2.6: If $int(A)\subseteq B\subseteq A$ and A is rw-open, then B is rw-open.

Proof: Let A be rw-open set and $int(A)\subseteq B\subseteq A$. Now $int(A)\subseteq B\subseteq A$ implies $A^c\subseteq B^c\subseteq [int(A)]^c$. That is $A^c\subseteq B^c\subseteq cl(A^c)$. Since A^c is rw-closed, B^c is rw-closed and B is rw-open.

Theorem 2.7: If $A\subseteq X$ is rw-closed then cl(A)-A is rw-open.

Proof: Let A be rw-closed. Let F be regular semiopen set such that $F\subseteq cl(A)-A$. Then by theorem 3.5 [12], $F=\phi$. So $F\subseteq int(cl(A)-A)$. This shows cl(A)-A is rw-open.

Remark 2.8: rw-open sets are independent with the notions of g-open ,mildly g-open, sg-open, gα-open, αg-open, gs-open, gp-open, wg-open, g^*-open.

Example 2.9: In Example 2.1.4, the set A = {b,c} isg^*-open , mildly g-open, g-open, wg-open,αg-open, sg-open,gs-open,gp-open sets but not rw-open and the set B={a, b, d}is gα-open but not rw-open . Moreover, the set C={d} is rw-open but not g^*-open , mildly g-open, g-open, wg-open,αg-open, sg-open,gs-open,gp-open sets,gα-open.

Theorem 2.10: If A and B are rw-open sets in a space X. Then $A\cap B$ is also rw-open set in X.

Proof: If A and B are rw-open sets in a space X. Then A^c and B^c are rw-closed sets in a space X. Then by theorem $(A^c)\cup(B^c)$ is also rw-closed set in X. Therefore $A\cap B$ is rw-open set in X.

Remark 2.11: The Union of two rw-open sets in X is need not be rw-open in X.

Example 2.12: Let X={a,b,c,d} be a topological space with topology $\tau=\{\phi,\{a\},\{b\},\{a, b\},\{a, b, c\}, X\}$. Then the set A={a,b}and B={d} are rw-open set in X but $A\cup B$={a,b,d} is not rw-open in X.

Definition 2.13: Let X be a topological space and let $x\in X$. A subset N of X is said to be a rw-neighbourhood of x iff there exists a rw-open set U such that $x\in U \subseteq N$.

Theorem 2.14: Everyneighbourhood N of $x\in X$ is a rw-nbhd of X but not conversely.

Proof: Let N be a neighbourhood of point $x\in X$. Then there exists an open set U such that $x\in U\subseteq N$. Since every open set is rw-open, U is arw-open set such that $x\in U \subseteq N$. This implies N is rw-neighbourhood of x.

Example 2.15: Let X={a,b,c,d} be a topological space with topology $\tau=\{\phi,\{a\},\{b\},\{a,b\},\{a,b,c\},X\}$. Then the set {c, d} is rw-neighbourhood of the point $c\in X$, but not the neighbourhood of the point c.

Theorem 2.16: Every rw-open set is rw-neighbourhood of each of its points but not conversely.

Proof: Let N berw-open and $x\in N$. For N is a rw-open set such that $x\in N\subseteq N$. Since x is an arbitrary point of N, it follows that N is arw-neighbourhood of each of its points.

Example 2.17: Let X={a, b, c, d} be a topological space with topology $\tau=\{\phi, \{a\},\{b\},\{a, b\},\{a, b, c\}, X\}$. Then the set {b, d} is a rw-neighbourhood of each of its points but not a rw-open set in X.

Theorem 2.18: Let Abe a subset of (X,τ). Then $x\in$rw-cl(A) if and only if for any rw-neighbourhood N of x in (X,τ), $A \cap N \neq \phi$.

Proof: Necessary- Assume $x\in$rw-cl(A). Suppose that there is a rw-neighbourhood N of x in $x\in(X,\tau)$ such that $A\cap N \neq\phi$. Since N is a rw-neighbourhood of x in (X,τ), there exists a rw-open set U such that $x\in U \subseteq N$. Therefore we have $U \cap A = \phi$. and so $A\subseteq U^c$. Since U^c is a rw-closed set containing A, then we have rw-cl(A)$\subseteq U^c$ and therefore $x\notin$rw-cl(A) which is a contradiction.

Sufficiency: Assume for each rw-neighbourhood N of x in (X,τ), $A \cap N \neq \phi$. Suppose that $x\notin$rw-cl(A). Then there exists a rw-closed set F of X such that $A\subseteq F$ and $x\in F$. Thus $x\in F^c$ and F^c is rw-open in X and hence F^c is arw-neighbourhood of x in X. But $(A \cap F^c) = \phi$, a contradiction.

Theorem 2.19: If F is a rw-closed subset of X and $x\in F^c$ then there exists a rw-neighbourhood N of X such that $N \cap F \neq \phi$.

Proof: Let F be rw-closed subset of X and $x\in F^c$. Then F^c is rw-open set of X. So by theorem 2.1.14 F^c rw-neighbourhood of each of its points. Put $F^c = N$, it follows that N is arw-neighbourhood of x such that $N \cap F=F^c\cap F = \phi$.

III.REGULAR WEAKLY CLOSURE AND ITS PROPERTIES

Definition 3.1: For every set $F \subseteq X$, we define the rw- closure of F to be the intersection of all rw - closed sets containing F.i.e., rw-cl(A)= $\cap$ {F:A $\subseteq$ F, F is rw-closed in X}.

Definition 3.2: Let τ_{rw} be the topology on X generated by rw-closure in the usual manner.
i.e., τ_{rw} = {U:rw-cl(X - U) = X - U}

Remark 3.3: If $A \subseteq X$ is rw-closed then rw-cl(A)=A but the converse is not true.

Example 3.4: Let X= {a, b, c, d} with topology τ = {ϕ,{a},{a,b},{b},{a,b,c}, X}. Let A={a}, then rw-cl(A)=A, but A is not rw-closed.

Theorem 3.5: If A is rw-closed in (X,τ), then A is closed in (X,τ_{rw}).

Proof: Since A is rw-closed in (X,τ), rw-cl(A) = A. This implies $X – A\in\tau_{rw}$. That is X - A is open in (X,τ_{rw}). Hence A is closed in in (X,τ_{rw}).

Remark 3.6:
(i) rw-cl(ϕ) = ϕ, rw-cl(X) = X
(ii) For any $A \subseteq X$, $A\subseteq$rw-cl(A)$\subseteq$ cl(A)
(iii) For any A, B $\subseteq$ X and $A \subseteq B$, rw – cl(A) $\subseteq$rw – cl(B)

Theorem 3.7: For any $x\in X$, $x\in$rw-cl(A) if and only if $V\cap A\neq\phi$ for every rw-open set V containing x.

Proof: Necessity- Let $x \in$rw - cl(A) for any $x \in X$. Suppose there exists a rw-open set U containing x such that $U \cap A =\phi$. Then $A \subseteq X - U$, rw-cl(A) $\subseteq$ X - U implies $x\notin$rw-cl(A), a contradiction. Thus $U \cap A \neq\phi$ for every rw – open set U containing x.

Sufficiency: Let $U \cap A \neq\phi$for every rw – open set U containing x.Suppose $x \notin$rw-cl(A), then there exists a rw-closed subset F containing A such that $x\notin F$. Then $x \in X$ -F and X - F is rw-open. Also $(X - F)\cap A = \phi$, a contradiction. Thus $x \in$rw - cl(A)

Theorem 3.8: Let A and B be subsets of X, then rw-cl($A\cap B$)$\subseteq$rw-cl(A)$\cap$rw-cl(B).

Proof: Since $A\cap B\subseteq A$ and $A \cap B\subseteq B$, by Remark 2.2.7 (iii), rw-cl($A\cap B$)$\subseteq$rw-cl(A)andrw-cl($A\cap B$)$\subseteq$rw-cl(B).Thus rw-cl($A\cap B$)$\subseteq$rw-cl(A)$\cap$rw-cl(B).

Theorem 3.9: If A and B are rw-closed sets then rw-cl($A\cup B$)=rw-cl(A)$\cup$rw-cl(B).

Proof: Let A and B be rw-closed in X. Then $A\cup B$ is also rw-closed[12]. Then rw-cl($A\cup B$)=$A\cup B$=rw-cl(A)$\cup$rw-cl(B).

Definition 3.10: For any $A \subseteq X$, rw - int(A) is defined as the union of all rw-open sets contained in A.

i.e., rw - int(A) = $\cup$ {F : F $\subseteq$ A, F is rw-open in X}.

Lemma 3.11: For any $A \subseteq X$, int(A)$\subseteq$rw-int(A)$\subseteq$ A.

Proof: Follows from the Theorem 2.1.3(ii).

Theorem 3.11: X -rw - int(A) =rw - cl(X - A)

Proof: Let x$\in$X - rw-int(A), then x$\notin$rw-int(A). Thus every rw-open set U containing x such that U$\not\subset$ A. This implies every rw-open set U containing x such that U $\cap$ $A^c$$\neq\phi$. By theorem 2.2.8, x$\in$rw - cl(X - A). Hence X - rw-int(A) $\subseteq$rw-cl(X - A).

Conversely, let x $\in$rw-cl(X - A). Then by theorem 2.2.8, every rw-open set U containing x such that U $\cap$ $A^c$$\neq\phi$. That is every rw-open set U containing x such that U$\not\subset$A, implies x$\notin$rw-int(A). i.e., x$\in$(rw-int(A))^c . Hencerw-cl(X - A) $\subseteq$ (X - rw-int(A)). Thus (X - rw-int(A))=rw-cl(X - A).

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