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COMPLEMENTARY EQUIVALENCE DOMINATING SETS IN GRAPHS

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ABSTRACT

Let $G=(V,E)$ be a simple finite undirected graph. A subset S of V(G) is called an equivalence set if every component of the induced sub graph $\langle S\rangle$ is complete. A graph G is an equivalence graph if every component of G is complete. A subset S of V(G) is called a complementary equivalence dominating set of G if $\langle V-S\rangle$ is an equivalence set of G and S is a dominating set of G. The minimum cardinality of a c-e-d set of G is denoted by $\gamma_{c-e}(G)$. In this paper, several results concerning complementary equivalence domination are derived Also Complementary equivalence irredundance is defined and relationship between the minimum cardinality of a maximal c-e irredundance set of G and $\gamma_{c-e}(G)$ are found. Further Independence c-e saturation parameter is also introduced.

Keywords: *Equivalence domination, Complementary equivalence domination, Complementary equivalence irredundance.*

1. INTRODUCTION

A subset S of V(G) is called an equivalence set if every component of the induced sub graph $\langle S\rangle$ is complete. A graph G is an equivalence graph if every component of G is complete. A sub set S of V(G) is called a complementary equivalence dominating set of G if $\langle V-S\rangle$ is an equivalence set of G and S is a dominating set of G. The minimum cardinality of a c-e-d set of G is denoted by $\gamma_{c-e}(G)$. The complementary equivalence number (upper complementary equivalence number) of G is defined and these parameters are found for standard graphs. Independent complementary equivalence sets are defined and two parameters $i_{c-e}(G)$ and $\beta_{c-e}(G)$ are introduced. These are determined for standard graphs. Several nice results involving c-e-d sets are derived, relationship with other graph parameters are found and inequality chain is established. Complementary equivalence irredundance is defined and relationship between the minimum cardinality of a maximal c-e irredundance set of G and $\gamma_{c-e}(G)$ are found. Independence c-e saturation parameter is also introduced.

2. COMPLEMENTARY EQUIVALENCE DOMINATING SETS IN GRAPHS

Definition 2.1 [3]: Let G = (V, E) be a simple graph. A subset S of V is called an equivalence set of G if the components of $\langle S\rangle$ are complete.

Definition 2.2: Let G = (V, E) be a simple graph. Then a subset S of V is called a complementary equivalence set if the components of $\langle V-S\rangle$ are complete.

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Definition 2.3: The complementary equivalence number of G (upper complimentary equivalence number of G) denoted by c-e(G) (C-E(G)) is defined as
c-e(G) = Min { │S│/S is a minimal c-e set of G}.
C-E(G)=Max{ │S│/S is a minimal c-e set of G}

Some Standard Results

$$C - E(K_n) = 0 = c - e(K_n)$$
$$C - E(C_n) = n - 1 = c - e(C_n)$$
$$C - E(P_n) = n - 1$$
$$c - e(P_n) = n - 1$$

Definition 2.4: $i_{c\text{-}e}(G)$=Min{ │S│/S is a maximal independent c-e set}

$$\beta_{c-e}(G) = Max\{|S|/S \text{ is a maximal independent c - e set }\}$$

Remark 2.5: A maximal independent c-e set of G is a c-e-dominating set of G.

Proof: Suppose S is a maximal independent c-e set of G. Let $x \in V - S$. Suppose x is not adjacent with any vertex of S. Then $S \cup \{x\}$ is an independent set of G. Also $V - (S \cup \{x\})$ is an equivalence set of G, since $V - (S \cup \{x\})$ is a subset of the equivalence set V-S. Therefore, $S \cup \{x\}$ is an independent c-e set of G, a contradiction, since S is a maximal independent c-e set of G. Therefore, S is a dominating set of G.

Some Standard Results

$$i_{c-e}(K_n) = 1$$
$$i_{c-e}(C_n) = \left\lceil \frac{n}{3} \right\rceil$$
$$i_{c-e}(P_n) = \left\lceil \frac{n}{3} \right\rceil$$
$$i_{c-e}(K_{1,n}) = 1$$
$$\beta_{c-e}(K_n) = 0$$
$$\beta_{c-e}(C_n) = \left\lfloor \frac{n}{2} \right\rfloor$$
$$\beta_{c-e}(P_n) = \left\lceil \frac{n}{2} \right\rceil$$
$$\beta_{c-e}(K_{1,n}) = n$$

Definition 2.6: Let G = (V, E) be a simple graph. Let S be a subset of V. S is called a complementary equivalence dominating set of G if S is a dominating set of G and V-S is an equivalence set of G and it is abbreviated as c-e-d set of G.

Remark 2.7:
1. c-e-d property is super hereditary.
2. The minimum cardinality of a c-e-d set is called the c-e domination number and it is denoted by $\gamma_{c-e}(G)$.

Characterization of minimal c-e-d set.

Theorem 2.8: Let S be a c-e-d set of G. S is minimal if and only if for any $u \in S$, one of the following holds:
i) $pn[u, S] \neq \phi$.
ii) At least one component of V-S has two or more elements and in each component there exists a vertex which is not adjacent to u.

Proof: Let S be a minimal c-e-d set of G. Let $u \in S$. Then S-{u} is not a dominating set of G. Suppose S-{u} is not a dominating set of G. Then either u is an isolate of G or u has a private neighbour with respect to S in V-S. That is, condition (i) holds.

Suppose S-{u} is a dominating set but its complement is not an equivalence set of G. That is, V-(S-{u}) is not an equivalence set of G. But V-S is an equivalence set of G. Let $T_1, T_2, \ldots, T_k$ be the components of $\langle V-S \rangle$. If u is adjacent with every vertex of a component T_i $(1 \le i \le k)$ then $\langle (V-S) \cup \{u\} \rangle$ is an equivalence set of G, a contradiction. Therefore, in each component of $\langle V-S \rangle$, there exists a vertex which is not adjacent with u. Suppose every component of $\langle V-S \rangle$ is singleton. Suppose u is not adjacent with the vertex in each component. Then $(V-S) \cup \{u\}$ is an equivalence set of G, a contradiction. Therefore, at least one component of $\langle V-S \rangle$ contains two or more elements. Therefore, condition (ii) holds.

Conversely, suppose S is a c-e-d set of G and every vertex $u \in S$ satisfies one of the two conditions. Suppose u satisfies (i). Then $pn[u,S] \ne \varphi$.Therefore, S-{u} is not a dominating set of G. Suppose u satisfies (ii). Then $(V-S) \cup \{u\}$ is not an equivalence set of G. Therefore, S-{u} is not complementary equivalence set of G. Thus if u satisfies (i) or (ii), then S-{u} is not a c-e-d set of G. Therefore, S is 1-minimal c-e-d set of G. Since c-e-d property is super-hereditary, S is a minimal c-e-d set of G.

Definition 2.9: The upper c-e domination number $\Gamma_{c-e}(G)$ is the maximum cardinality of a minimal c-e-d set.

Some Standard Results

$$\gamma_{c-e}(K_n) = 1$$

$$\gamma_{c-e}(C_n) = \left\lceil \frac{n}{3} \right\rceil$$

$$\gamma_{c-e}(P_n) = \left\lceil \frac{n}{3} \right\rceil$$

$$\gamma_{c-e}(K_{1,n}) = 1$$

Observation 2.10: For any graph G with out isolates, $\gamma_{c-e}(G) \le \alpha(G)$.

Proof: Let S be a maximum independent set of a graph G with out isolates. Then V-S is a dominating set and is also a complementary equivalence set. Therefore, $|V-S| \ge \gamma_{c-e}(G)$. That is, $n-|S| \ge \gamma_{c-e}(G)$. That is, $n-\beta(G) \ge \gamma_{c-e}(G)$. That is, $\alpha(G) \ge \gamma_{c-e}(G)$.

Definition 2.11 [15]: A dominating set S of a graph G is a non split dominating set of G if $\langle V-S \rangle$ is connected. The minimum cardinality of a non split dominating set of G is denoted by $\gamma_{ns}(G)$ and is called the non split domination number of G.

Definition 2.12 [12]: A dominating set S of a graph G is called a strong non split dominating set of G if $\langle V-S \rangle$ is complete. The minimum cardinality of a strong non split dominating set of G is denoted by $\gamma_{sns}(G)$ and is called the strong non split domination number of G.

Observation 2.13: $\gamma(G) \le \gamma_{c-e}(G) \le \gamma_{sns}(G)$.

Definition 2.14 [13]: A dominating set S of a graph G is called a split dominating set of G if $\langle V-S \rangle$ is disconnected. The minimum cardinality of a split dominating set of G is denoted by $\gamma_s(G)$ and is called the split domination number of G.

Definition 2.15 [14]: A dominating set S of a graph G is called a strong split dominating set of G if $\langle V-S \rangle$ is totally disconnected with at least two vertices. The minimum cardinality of a strong split dominating set of G is denoted by $\gamma_{ss}(G)$ and is called the strong split domination number of G.

Observation 2.16: If $\gamma_{c-e}(G) < \gamma_{sns}(G)$ then $\gamma_s(G) \le \gamma_{c-e}(G) \le \gamma_{ss}(G)$.

Definition 2.17: A dominating set S of a graph G is a complementary strong split dominating set of G if $\langle S \rangle$ is totally disconnected. The complementary strong split domination number $\gamma_{c-ss}(G)$ of G is the minimum cardinality of a complementary strong split dominating set of G.

Definition 2.18: The upper complementary strong split domination number $\Gamma_{c-ss}(G)$ is the maximum cardinality of a minimal complementary strong split dominating set of G.

Definition 2.19: A subset S of V of G is called complementary independent if $\langle V-S \rangle$ is totally disconnected. S is also called a covering set of G. The minimum cardinality of S such that V-S is an independent set is called complement maximum independent set or a minimum covering set of G.

Observation 2.20: Let G be an isolate free graph. Suppose S is a subset of V(G) such that V-S is a maximal independent set. Then S is a minimal complementary strong split dominating set.

Proof: Let $u \in V-S$ **.** Then u is an isolate in $\langle V-S \rangle$. Since u is not an isolate of G, u is adjacent with some vertex of S. Therefore, S is a complementary strong split dominating set. Suppose S is not minimal. Then there exists some $u \in S$ such that S-{u}is a complementary strong split dominating set. Therefore, $(V-S) \cup \{u\}$ is totally disconnected set, contradicting the maximality of V-S. Therefore, S is a minimal complementary strong split dominating set.

The following inequality chain is observed.

$$ir(G) \le \gamma(G) \le \gamma_{c-e}(G) \le i_{c-e}(G) \le \beta_{c-e}(G) \le \Gamma_{c-e}(G) \le \Gamma(G) \le IR(G).$$

Also if G has no isolates then $\gamma_{c-e}(G) \le \alpha_0(G)$, $\mathrm{i}(G) \le \mathrm{i}_{c\text{-}e}(G)$, $\beta_{c-e}(G) \le \beta_0(G)$

3. COMPLEMENTARY EQUIVALENCE C-E IRREDUNDANCE IN GRAPHS

Definition 3.1: A subset S of V is called c-e irredundant set if for each $u \in S$, one of the following holds.

i) $pn[u,S] \ne \phi$ where $pn[u,S] = N[u] - N[S-\{u\}]$

ii) In every component of V-S of order ≥ 2 there exists w_1 such that w_1 is not adjacent to u and there exists w_2 such that w_2 is adjacent with u.

Definition 3.2: The minimum (maximum) cardinality of a maximum c-e-irredundant set of a graph G is called c-e irredundance number of G (upper c-e-irredundance number of G) and is denoted by $ir_{c-e}(G)\,(IR_{c-e}(G))$.

Some Standard Results

$$ir_{c-e}(K_n) = 1$$

$$ir_{c-e}(K_{1,n}) = 2$$

$$ir_{c-e}(C_n) = \left\lceil \frac{n}{3} \right\rceil$$

$$ir_{c-e}(P_n) = \left\lceil \frac{n}{3} \right\rceil$$

Proposition 3.3: c-e irreddundance is hereditary**.**

Proof: Let S be a c-e ireredundance set of G and let T be a subset of S. Let $u \in T$. Then $u \in S$. **(**Suppose u satisfies the condition that every component of V-S, there exists w such that w is not adjacent to u). Suppose $pn[u,T] = \phi$. Then $pn[u,S] = \phi$. Then V-S has a component say X of order≥2 and u is adjacent with at least one vertex of X and not adjacent with a vertex of X. Hence $\langle X \cup \{u\} \rangle$ is non complete component of V-T. Hence T is c-e irredundent.

Theorem 3.4: Any minimal c-e-d set is a maximal c-e irredundent set.

Proof: Let S be a minimal c-e-d set. Then S is a c-e irredundent set. Suppose S is not a maximal c-e irredundent set. Then there exists $u \in V - S$ such that $S \cup \{u\}$ is a c-e irredundent set.

Case I: Suppose $pn[u, S \cup \{u\}] \neq \phi$. Therefore, there exists $v \in V - (S \cup \{u\})$ such that v is adjacent only with u with respect to $S \cup \{u\}$. That is, v is not adjacent with any vertex of S. Therefore, S is not a dominating set, a contradiction. Therefore, S is a maximal c-e -irredundent set.

Case II: Suppose in every component of $V - (S \cup \{u\})$, there exists w such that w is not adjacent with u. If X_1, X_2,...,X_r be the components of $V - (S \cup \{u\})$ then u is not adjacent to some vertex in each component of $V - (S \cup \{u\})$. That is, V-S is not component wise complete, a contradiction, since S is a c-e-d set. Therefore, S is a maximal c-e irredundent set.

Remark 3.5: $ir_{c-e}(G) \leq \gamma_{c-e}(G)$.

Example 3.6:

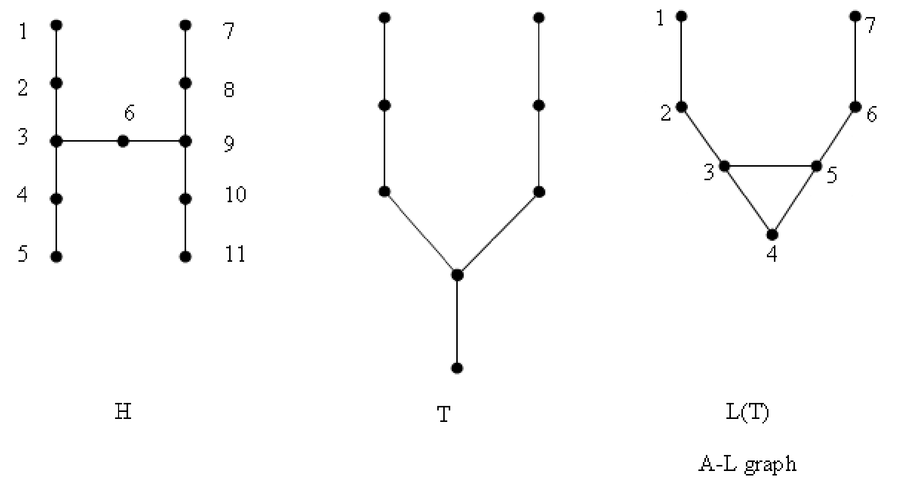

Figure 3.1: Graphs for which $ir_{c-e}(G) < \gamma_{c-e}(G)$

For the graph H, S={2,3,8,9} is a c-e irredundent set .
[Because pn(2) =1; pn(3) = 4; pn(8) = 7; pn(9) = 10; V-S = {1,4,5,6,7,10,11} Each component in the induced subgraph of V-S is complete].
S' = {2, 4, 6, 8, 10} is a c-e dominating set.
That is, $ir_{c-e}(H) = 4$ and $\gamma_{c-e}(H) = 5$.

For the A-L graph (L(T)),
$ir_{c-e}(L(T)) = 2$ and $\gamma_{c-e}(L(T)) = 3$.
S = {3, 6} is a c-e irredundant set.
S' = {2, 6, 3} is a c-e dominating set.

Theorem 3.7: $\gamma_{c-e}(G)/2 < ir_{c-e}(G) \leq \gamma_{c-e}(G) \leq 2ir_{c-e}(G) - 1$

Proof: Let ir_{c-e} (G)=k. Let S={v_1, v_2, . . . v_k} be an ir c-e set of G. Since S is irredundent, $pn[v_i, S] \neq \phi$ or there exist a component of V-S of order greater than or equal to two and v_i is adjacent with a vertex of the component and not adjacent with another vertex of that component. Let S'={u_1, u_2 . . . ,u_s} where $u_i \in pn[v_i, S]$ if v_i has a private neighbor and u_i is one of the vertices in a component of V-S of order greater than or equal to two adjacent with v_i.

Lets S"=S ∪ S'. Suppose S" is not a dominating set. Then there exist w ∈ V-S" such that w is not adjacent to u_i as well as vi, $1 \leq i \leq k$. Therefore, w does not belongs to N [x] for any vertex x in S". $pn[w, S \cup \{u\}] \neq \phi$. Since $w \notin N[x]$ for any x ∈ S", w is not adjacent with any u_i. Therefore, $p_n[u_i, S \cup \{w\}] \neq \phi$.

Case I: $u_i = v_i$. Then v_i is a private neighbor of S. v_i is not adjacent with w. Therefore, v_i is an isolate of $S \cup \{w\}$. $S \cup \{w\}$ is complementary componentwise complete.

Case II: $u_i \neq v_i$. u_i is not adjacent with w. If u_i is a private neighbor of v_i then pn[u_i, $S \cup \{w\}$]$\neq \phi$.

Case III: $u_i \neq v_i$, and u_i is not a private neighbor of v_i. Then u_i is a vertex in a component of V-S such that v_i is adjacent with u_i and v_i is not adjacent with a vertex of the component. Since $S \cup \{w\}$ is complementary componentwise complete, u_i is not a private neighbor of v_i but u_i is adjacent with v_i in a component containing v_i having at least two elements.

From case I, case II, case III, S ∪{w} is a c-e irredundent set, contradicting the maximality of S. Therefore, S" is a dominating set. Also S" is a complementary component wise complete. Therefore, S" is a c-e-d set.

Suppose S" is a minimal c-e-d set. Then S" is a maximal c-e irredundent set containing S, a contradiction. Therefore, $\gamma_{c-e}(G) < |S''| = 2ir_{c-e}(G)$. Hence $\gamma_{c-e}(G) \leq 2ir_{c-e}(G) - 1$.

$\gamma_{c-e}(G)/2 \leq ir_{c-e}(G) - 1/2$.

$\gamma_{c-e}(G)/2 < ir_{c-e}(G)$.

Therefore, $r_{c-e}(G)/2 < ir_{c-e}(G) \leq \gamma_{c-e}(G) \leq 2ir_{c-e}(G) - 1$

Theorem 3.8: Let S be a β_{c-e} set of G. Then S is dominating set.

Proof: Suppose S is not a dominating set. Then there exists a vertex $u \in V - S$ such that u is not adjacent with any vertex of S. Therefore, $S \cup \{u\}$ is an independent set and complement of $S \cup \{u\}$ is componentwise complete. This contradicts the fact that S is a maximum independent set with complement componentwise complete. Therefore, S is a dominating set. That is, S is a c-e-d set.

Remark 3.9: A β_0 -set of a graph need not be a c-e set.

For example, let G = A-L-graph (Figure 3.1). {1, 7, 3} is a maximum independent set. The complement is not componentwise complete.

Definition 3.10:
Independent c-e saturation parameter (I-c-e Saturation parameter)
Let G be an i-c-e excellent graph. Let $u \in V(G)$ .Then
i-c-e-s(u)=Maximum{ | S | :S is a independent c-e set containing u}
i-c-e-s(G)=Minimum{i-c-e-s(u): $u \in V(G)$ }

Remark 3.11:

1. Let S be a maximum i-c-e-s(u) set. Then S is a dominating set.
2. $i_{c-e}(G) \leq i-c-e-s(G) \leq \beta_{c-e}(G)$

4. RELATIONSHIP BETWEEN OTHER GRAPH PARAMETERS

Proposition 4.1: Given positive integers a,b and c such that $a \leq b \leq c$, there exists a connected graph G with $\gamma(G) = a, \gamma_{c-e}(G) = b$ and $\gamma_{sns}(G) = c$.

Proof: Let a, b and c be three positive integers such that $a \leq b \leq c$. Consider K_b. Let $\{u_1, u_2, u_3, ..., u_a, ..., u_b\}$ be the vertex set of K_b. Add c vertices $\{u_1', u_2', u_3', ..., u_c'\}$. Attach each $u_i', 1 \leq i \leq c$, as a pendant vertex to some $u_j, 1 \leq j \leq a$ such that each u_j has at least one pendant vertex. Join u_{a+k} with u_{a+k}', $1 \leq k \leq b - a$. Let G be the resulting graph. Then $\{u_1, u_2, u_3, ..., u_a\}$ is a minimum dominating set of G, $\{u_1, u_2, u_3, ..., u_a, ..., u_b\}$ is a minimum c-e dominating set of G and $\{u_1', u_2', u_3', ..., u_c'\}$ is a minimum sns-dominating set of G.

Therefore, $\gamma(G)=a, \gamma_{c-e}(G)=b$ and $\gamma_{\text{sns}}(G)=c$.

Remark 4.2: $\gamma(G)\le n-2$ if and only if $\gamma_{c-e}(G)\le n-2$.

Proof: Let S be a dominating set of G. Then any super set of S containing n-2 vertices is a complementary equivalence dominating set of G. Therefore, $\gamma_{c-e}(G)\le n-2$.The converse is also true (since, $\gamma(G)\le\gamma_{c-e}(G)\le n-2$).

Observation 4.3: If G is a graph without isolates and of order greater than or equal to 4 and if $\gamma_{c-e}(G)=n-2$, then G is a triangle free graph.

Proof: Let $\gamma_{c-e}(G)=n-2$. Since G has no isolates, $\gamma(G)\le\frac{n}{2}$. Let S be a dominating set of G. If $\langle V-S\rangle$ contains a triangle say x, y, z , then $S\cup((V-S)-\{x,y,z\})$ is a complementary equivalence dominating set of G. Therefore, $\gamma_{c-e}(G)\le n-3$, a contradiction. Suppose $\langle S\rangle$ contains a triangle x, y, z. Since S is a minimum dominating set and since x, y, z are not isolates of $\langle S\rangle$, each of them has a private neighbour in $\langle V-S\rangle$. Let $S_1=V-\{x,y,z\}$. Then S_1 is a complementary equivalence dominating set of G. Therefore, $\gamma_{c-e}(G)\le n-3$, a contradiction. Suppose G has a triangle with one vertex in S and two vertices in V-S (or) two vertices in S and one vertex in V-S.

Sub Case I: $\langle x,y,z\rangle$ is a triangle in G with $x\in S$ and $y,z\in V-S$.

If x is an isolate of $\langle S\rangle$, then x has private neighbour in V-S. If x is not an isolate of $\langle S\rangle$, then there exists vertices in S which are adjacent to x. Therefore, $V-\{x,y,z\}$ is a complementary equivalence dominating set of G. Therefore, $\gamma_{c-e}(G)\le n-3$, a contradiction.

Sub Case II: $x,y\in S$ and $z\in V-S$.

Then x and y are not isolates of $\langle S\rangle$ and hence $V-\{x,y,z\}$ is complementary equivalence dominating set of G. Therefore, $\gamma_{c-e}(G)\le n-3$. Therefore, G has no triangle.

Remark 4.4: The converse of the above result is not true.

Consider $K_{m,n}$. Then $\gamma_{c-e}(K_{m,n})=\min\{\mathrm{m},\mathrm{n}\}<\mathrm{m}+\mathrm{n}-2$ if $m,n\ge 3$. Also $K_{m,n}$ is triangle free.

Remark 4.5: Let G be a connected graph of even order. If $\gamma(G)=\frac{n}{2}$, then $\gamma_{c-e}(G)=\frac{n}{2}$.

Proof: Since G is connected with $\gamma(G)=\frac{n}{2}$, G is either C_4 or H^+ where H is connected graph. $\gamma_{c-e}(C_4)=\gamma(C_4)$ and $\gamma_{c-e}(H^+)=\gamma(H^+)$. Therefore, $\gamma_{c-e}(G)=\gamma(G)=\frac{n}{2}$.

Remark 4.6: Let G be a complete bipartite graph. Then $\gamma_{c-e}(G)=\gamma(G)$ if and only if G is either a star or $\overline{K_{n+2}}+\overline{K_2}$.

Proof: If G is a complete bipartite graph with m,n as the orders of the partition, then

$$\gamma(G)=\begin{cases}2 \text{ if } \mathrm{m},\mathrm{n}\ge 2\\ 1 \text{ if } \mathrm{m}=1 \text{ or } \mathrm{n}=1\end{cases},\ \gamma_{c-e}(G)=\min\{\mathrm{m},\mathrm{n}\}.$$

Therefore, $\gamma(G)=\gamma_{c-e}(G)$ if and only if min{m, n}=2 or min{m, n}=1. That is, G is either a star or $\overline{K_{n+2}}+\overline{K_2}$.

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