

## **RESOLUTION OF WEYL MODULE IN THE CASE OF PARTITION (7, 6, 3)**

**HAYTHAM RAZOOKI HASSAN\***

**Department of Math science, Al-Mustansiriyah University / Science College, Baghdad, Iraq.**

**NAJAH MKHAIBER MOUSTAFA**

**Department of Math science, Al-Mustansiriyah/ Science College, Baghdad, Iraq.**

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### **ABSTRACT**

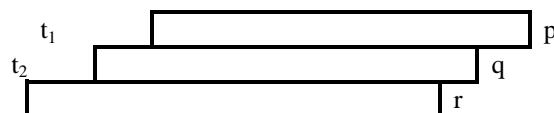
*In this paper we study the relation between the resolution of weyl module  $k_{(7,6,3)}F$  in characteristic-free mode and in the Lascoux mode (Characteristic zero), more precisely we obtain the Lascoux resolution of  $k_{(7,6,3)}F$  in characteristic zero as an application of the resolution of  $k_{(7,6,3)}F$  in characteristic - free.*

**Index Terms-** Resolution, weyl module, Lascoux module, divided power, characteristic free.

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### **I. INTRODUCTION**

Let  $R$  be commutative ring with 1 and  $F$  be free  $R$ -module by  $D_n F$  we mean the divided power of degree  $n$ . The resolution  $\text{Res}(p, q, r, t_1, t_2)$  of weyl module  $K_{\lambda/\mu} F$  associated to the three-rowed skew-shaps  $(p + t_1 + t_2, q + t_2, r) / (t_1 + t_2, t_2, 0)$  call the shape represented by the diagram



In general, the weyl module  $k_{\lambda/\mu} F$  is presented hby the "box" map

$$\begin{aligned} \sum_{k>0} D_{p+t_1+k} F \otimes D_{q-t_1-k} \otimes D_r F \\ \oplus \sum_{l>0} D_p F \otimes D_{q+t_2+l} \otimes D_{r-t_2-l} F \xrightarrow{n} D_p F \otimes D_q F \otimes D_r F \xrightarrow{d'_{\lambda/\mu}} k_{\lambda/\mu} \end{aligned}$$

Where the maps

$$\sum_{k>0} D_{p+t_1+k} F \otimes D_{q-t_1-k} \otimes D_r F \rightarrow D_p F \otimes D_q F \otimes D_r F$$

May be interpreted as  $k^{th}$  divided power of the place polarization from place one to place two (*i.e.*  $\partial_{21}^{(k)}$ ), the maps  $\sum_{l>0} D_p F \otimes D_{q+t_2-l} F \otimes D_{r-t_2-l} F \rightarrow D_p F \otimes D_q F \otimes D_r F$  may be place two interpreted as  $l^{th}$  divided power of the place polarization from place two to place three (*i.e.*  $\partial_{32}^{(l)}$ ) [2].

We have to mention that we shall use  $D_n$  instead of  $D_n F$  to refer to divided power algebra of degree n.

### **II. CHARACTRISTIC-FREE RESOLUTION OF THE PARTITION (7, 6, 3)**

We find the terms of the resolution of weyl module in the case of the partition (7, 6, 3), in general aterms of the resolution of weyl module in the case of a three-rowed partition  $(p, q, r)$  which appeared in [1] are

$$\begin{aligned} \text{Res}([p, q; 0]) \otimes D_r \oplus \sum_{l \geq 0} Z_{32}^{(l+1)} y \text{Res}([p, q+l+1; l+1]) \otimes D_{r-l-1} \oplus \\ \sum_{l_1 \geq 0, l_2 \geq l_1} Z_{32}^{(l_2+1)} y Z_{31}^{(l_1+1)} z \text{Res}([p+l_1+1, q+l_2+1, l_2-l_1]) \otimes D_{r-(l_1+l_2+2)} \end{aligned}$$

where  $x$ ,  $y$  and  $z$  stand for the separator variables, and the boundary map is  $\partial_x + \partial_y + \partial_z$ .

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**Corresponding Author: Haytham Razooki Hassan\***

**Department of Math science, Al-Mustansiriyah University / Science College, Baghdad, Iraq.**

Let again  $\text{Bar}(M, A; S)$  be the free bar module on the set  $S = \{x, y, z\}$  consisting of three separators  $x, y$  and  $z$ , where  $A$  is the free associative (non-commutative) algebra generated by  $Z_{21}, Z_{32}$  and  $Z_{31}$  and their divided powers with the following relation:

$$Z_{32}^{(a)} Z_{31}^{(b)} = Z_{31}^{(b)} Z_{32}^{(a)} \text{ and } Z_{21}^{(a)} Z_{31}^{(b)} = Z_{31}^{(b)} Z_{21}^{(a)}$$

and the module  $M$  is the direct sum of tensor products of divided power module  $D_{p_1} \otimes D_{p_2} \otimes D_{p_3}$  for suitable  $p_1, p_2$  and  $p_3$  with the action of  $Z_{21}, Z_{32}$  and  $Z_{31}$  and their divided powers. we will consider the case when  $p = 7, q = 6, \text{ and } r = 3$ .

We have:

$$\begin{aligned} & \mathbf{Res}([7,6,0]) \otimes D_3 \oplus \sum_{l \geq 0} Z_{32}^{(l+1)} y \mathbf{Res}([7,6+l+1;l+1]) \otimes D_{3-l-1} \oplus \\ & \sum_{l_1 \geq 0, l_2 \geq l_1} Z_{32}^{(l_2+1)} y Z_{31}^{(l_1+1)} z \mathbf{Res}([7+l_1+1,6+l_2+1;l_2-l_1]) \otimes D_{3-(l_1+l_2+2)} \end{aligned}$$

So,

$$\begin{aligned} & \sum_{l \geq 0} Z_{32}^{(l+1)} y \mathbf{Res}([7,6+l+1;l+1]) \otimes D_{3-l-1} = Z_{32} y \mathbf{Res}([7,7;1]) \otimes D_2 \oplus \\ & Z_{32}^{(2)} y \mathbf{Res}([7,8;2]) \otimes D_1 \oplus Z_{32}^{(3)} y \mathbf{Res}([7,9;3]) \otimes D_0 \end{aligned}$$

and

$$\begin{aligned} & \sum_{l_1 \geq 0, l_2 \geq l_1} Z_{32}^{(l_2+1)} y Z_{31}^{(l_1+1)} z \mathbf{Res}([7+l_1+1,6+l_2+1;l_2-l_1]) \otimes D_{3-(l_1+l_2+2)} \\ & = Z_{32} y Z_{31} z \mathbf{Res}([8,7;0]) \otimes D_1 \oplus Z_{32}^{(2)} y Z_{31} z \mathbf{Res}([8,8;1]) \otimes D_0 \end{aligned}$$

where  $Z_{32} y$  is the bar complex

$$0 \rightarrow Z_{32} y \xrightarrow{\partial_y} Z_{32} \rightarrow 0$$

$Z_{32}^{(2)} y$  is the bar complex

$$0 \rightarrow Z_{32} y Z_{32} y \xrightarrow{\partial_y} Z_{32}^{(2)} y \xrightarrow{\partial_y} Z_{32}^{(2)} \rightarrow 0$$

$Z_{32}^{(3)} y$  is the bar complex

$$\begin{aligned} & 0 \rightarrow Z_{32} y Z_{32} y Z_{32} y \xrightarrow{\partial_y} \frac{Z_{32}^{(2)} y Z_{32} y}{Z_{32} y Z_{32}^{(2)} y} \xrightarrow{\partial_y} Z_{32}^{(3)} y \rightarrow Z_{32}^{(3)} \rightarrow 0 \text{ and } Z_{31} z \text{ is the bar complex} \\ & 0 \rightarrow Z_{31} z \xrightarrow{\partial_z} Z_{31} \rightarrow 0 \end{aligned}$$

Then in this case we have the following terms:

- In dimension Zero ( $M_0$ ) we have  $D_7 \otimes D_6 \otimes D_3$
- In dimension one ( $M_1$ ) we have
- $Z_{21}^{(b)} x D_{7+b} \otimes D_{6-b} \otimes D_3; \text{ with } b = 1, 2, 3, 4, 5, 6$
- $Z_{32}^{(b)} x D_7 \otimes D_{6+b} \otimes D_{3-b}; \text{ with } b = 1, 2, 3$
- In dimension two ( $M_2$ ) we have the sum of the following terms:
- $Z_{21}^{(b_1)} x Z_{21}^{(b_2)} x D_{7+|b|} \otimes D_{6-|b|} \otimes D_3; \text{ with } |b| = b_1 + b_2 = 2, 3, 4, 5, 6$
- $Z_{32} y Z_{21}^{(b)} x D_{7+|b|} \otimes D_{7-b} \otimes D_2; \text{ with } b = 2, 3, 4, 5, 6, 7$
- $Z_{32}^{(2)} y Z_{21}^{(b)} x D_{7+b} \otimes D_{8-b} \otimes D_1; \text{ with } b = 3, 4, 5, 6$
- $Z_{32}^{(3)} y Z_{21}^{(b)} x D_{7+b} \otimes D_{9-b} \otimes D_0; \text{ with } b = 4, 5, 6, 7, 8, 9$
- $Z_{32}^{(b_1)} y Z_{32}^{(b_2)} y D_7 \otimes D_{6+|b|} \otimes D_{3-|b|}; \text{ with } |b| = b_1 + b_2 = 2, 3$
- $Z_{32}^{(b)} y Z_{31} z D_8 \otimes D_{6+b} \otimes D_{2-b}; \text{ with } b = 1, 2$
- In dimension three ( $M_3$ ) we have the sum of the following terms:
- $Z_{21}^{(b_1)} x Z_{21}^{(b_2)} x Z_{21}^{(b_3)} x D_{7+|b|} \otimes D_{6-|b|} \otimes D_3; \text{ with } |b| = b_1 + b_2 + b_3 = 3, 4, 5, 6 \text{ and } b_1 \geq 1$
- $Z_{32} y Z_{21}^{(b_1)} x Z_{21}^{(b_2)} x D_{7+|b|} \otimes D_{7-|b|} \otimes D_2; \text{ with } |b| = b_1 + b_2 = 2, 3, 4, 5, 6, 7 \text{ and } b_1 \geq 2$

- $Z_{21}^{(2)}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x D_{7+|b|} \otimes D_{8-|b|} \otimes D_1$ ; with  $|b| = b_1 + b_2 = 4,5,6,7,8$  and  $b_1 \geq 3$
- $Z_{32}y Z_{32}y Z_{21}^{(b)}x D_{7+b} \otimes D_{8-b} \otimes D_1$ ; with  $b = 3,4,5,6,7,8$
- $Z_{32}^{(3)}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x D_{7+|b|} \otimes D_{9-|b|} \otimes D_0$ ; with  $|b| = b_1 + b_2 = 5,6,7,8,9$  and  $b_1 \geq 4$
- $Z_{32}^{(c_1)}y Z_{32}^{(c_2)}y Z_{21}^{(b)}x D_{7+b} \otimes D_{9-b} \otimes D_0$ ; with  $c_1 + c_2 = 3$  and  $b = 4,5,6,7,8,9$
- $Z_{32}y Z_{32}y Z_{32}y D_7 \otimes D_9 \otimes D_0$
- $Z_{32}y Z_{31}z Z_{21}^{(b)}x D_{8+b} \otimes D_{7-b} \otimes D_1$ ; with  $1 \leq b = 1,2,3,4,5,6,7$
- $Z_{32}^{(2)}y Z_{31}z Z_{21}^{(b)}x D_{8+b} \otimes D_{8-b} \otimes D_0$ ; with  $b = 2,3,4,5,6,7,8$
- $Z_{32}y Z_{32}y Z_{31}z D_8 \otimes D_8 \otimes D_0$
  
- In dimension four ( $M_4$ ) we have the sum of the following terms:
- $Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x D_{7+|b|} \otimes D_{6-|b|} \otimes D_3$ ; with  $|b| = \sum_{i=1}^4 b_i = 4,5,6$  and  $b_1 \geq 1$
- $Z_{32}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x D_{7+|b|} \otimes D_{7-|b|} \otimes D_2$ ; with  $|b| = b_1 + b_2 + b_3 = 4,5,6,7$  and  $b_1 \geq 2$
- $Z_{32}^{(2)}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x D_{7+|b|} \otimes D_{8-|b|} \otimes D_1$ ; with  $|b| = b_1 + b_2 + b_3 = 5,6,7,8$  and  $b_1 \geq 3$
- $Z_{32}y Z_{32}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x D_{7+|b|} \otimes D_{8-|b|} \otimes D_1$ ; with  $|b| = b_1 + b_2 = 4,5,6,7,8$  and  $b_1 \geq 3$
- $Z_{32}^{(3)}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x D_{7+|b|} \otimes D_{9-|b|} \otimes D_0$ ; with  $|b| = b_1 + b_2 + b_3 = 6,7,8,9$  and  $b_1 \geq 4$
- $Z_{32}^{(c_1)}y Z_{32}^{(c_2)}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x D_{7+|b|} \otimes D_{9-|b|} \otimes D_0$ ; with  $b = c_1 + c_2 = 3$  and  $|b| = b_1 + b_2 = 5,6,7,8,9$  and  $b_1 \geq 4$
- $Z_{32}y Z_{32}y Z_{32}y Z_{21}^{(b)}x D_{7+b} \otimes D_{9-b} \otimes D_0$ ; with  $b = 4,5,6,7,8,9$
- $Z_{32}y Z_{31}z Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x D_{8+|b|} \otimes D_{7-|b|} \otimes D_1$ ; with  $|b| = b_1 + b_2 = 2,3,4,5,6,7$  and  $b_1 \geq 1$
- $Z_{32}^{(2)}y Z_{31}z Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x D_{8+|b|} \otimes D_{8-|b|} \otimes D_0$ ; with  $|b| = b_1 + b_2 = 3,4,5,6,7,8$  and  $b_1 \geq 2$
- $Z_{32}y Z_{32}y Z_{31}z Z_{21}^{(b)}x D_{8+b} \otimes D_{8-b} \otimes D_0$ ; with  $b = 2,3,4,5,6,7,8$
  
- In dimension five ( $M_5$ ) we have the sum of the following terms:
- $Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x Z_{21}^{(b_5)}x D_{7+|b|} \otimes D_{6-|b|} \otimes D_3$ ; with  $|b| = \sum_{i=1}^5 b_i = 5,6$  and  $b_1 \geq 1$
- $Z_{32}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x D_{7+|b|} \otimes D_{7-|b|} \otimes D_2$ ; with  $|b| = \sum_{i=1}^4 b_i = 5,6,7$  and  $b_1 \geq 2$
- $Z_{32}^{(2)}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x D_{7+|b|} \otimes D_{8-|b|} \otimes D_1$ ; with  $|b| = \sum_{i=1}^4 b_i = 6,7,8$  and  $b_1 \geq 3$
- $Z_{32}y Z_{32}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x D_{7+|b|} \otimes D_{8-|b|} \otimes D_1$ ; with  $|b| = b_1 + b_2 + b_3 = 5,6,7,8$  and  $b_1 \geq 3$
- $Z_{32}^{(3)}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x D_{7+|b|} \otimes D_{9-|b|} \otimes D_0$ ; with  $|b| = \sum_{i=1}^4 b_i = 7,8,9$  and  $b_1 \geq 4$
- $Z_{32}^{(c_1)}y Z_{32}^{(c_2)}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x D_{7+|b|} \otimes D_{9-|b|} \otimes D_0$ ;  
 $with  $c_1 + c_2 = 3$  and  $|b| = b_1 + b_2 + b_3 = 6,7,8,9$  and  $b_1 \geq 4$$
- $Z_{32}y Z_{32}y Z_{32}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x D_{7+|b|} \otimes D_{9-|b|} \otimes D_0$ ; with  $|b| = b_1 + b_2 = 5,6,7,8,9$  and  $b_1 \geq 4$
- $Z_{32}y Z_{31}z Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x D_{8+|b|} \otimes D_{7-|b|} \otimes D_1$ ; with  $|b| = b_1 + b_2 + b_3 = 3,4,5,6,7$  and  $b_1 \geq 1$
- $Z_{32}^{(2)}y Z_{31}z Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x D_{8+|b|} \otimes D_{8-|b|} \otimes D_0$ ; with  $|b| = b_1 + b_2 + b_3 = 4,5,6,7,8$  and  $b_1 \geq 2$
- $Z_{32}y Z_{32}y Z_{31}z Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x D_{8+|b|} \otimes D_{8-|b|} \otimes D_0$ ; with  $|b| = b_1 + b_2 = 3,4,5,6,7,8$  and  $b_1 \geq 2$
  
- In dimension six ( $M_6$ ) we have the sum of the following terms:
- $Z_{21}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x D_{13} \otimes D_0 \otimes D_3$
- $Z_{32}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x Z_{21}^{(b_5)}x D_{7+|b|} \otimes D_{7-|b|} \otimes D_2$ ; with  $|b| = \sum_{i=1}^5 b_i = 6,7$  and  $b_1 \geq 2$
- $Z_{32}^{(2)}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x Z_{21}^{(b_5)}x D_{7+|b|} \otimes D_{8-|b|} \otimes D_1$ ; with  $|b| = \sum_{i=1}^5 b_i = 7,8$  and  $b_1 \geq 3$
- $Z_{32}y Z_{32}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x D_{7+|b|} \otimes D_{8-|b|} \otimes D_1$ ; with  $|b| = \sum_{i=1}^4 b_i = 6,7,8$  and  $b_1 \geq 3$
- $Z_{32}^{(3)}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x Z_{21}^{(b_5)}x D_{7+|b|} \otimes D_{9-|b|} \otimes D_0$ ; with  $|b| = \sum_{i=1}^5 b_i = 8,9$  and  $b_1 \geq 4$
- $Z_{32}^{(c_1)}y Z_{32}^{(c_2)}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x D_{7+|b|} \otimes D_{9-|b|} \otimes D_0$ ; with  $c_1 + c_2 = 3$   
 $and  $|b| = \sum_{i=1}^4 b_i = 7,8,9$  and  $b_1 \geq 4$$
- $Z_{32}y Z_{32}y Z_{32}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x D_{7+|b|} \otimes D_{9-|b|} \otimes D_0$ ; with  $|b| = b_1 + b_2 + b_3 = 6,7,8,9$  and  $b_1 \geq 4$
- $Z_{32}y Z_{31}z Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x D_{8+|b|} \otimes D_{7-|b|} \otimes D_1$ ; with  $|b| = \sum_{i=1}^4 b_i = 4,5,6,7$  and  $b_1 \geq 1$
- $Z_{32}^{(2)}y Z_{31}z Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x D_{8+|b|} \otimes D_{8-|b|} \otimes D_0$ ; with  $|b| = \sum_{i=1}^4 b_i = 5,6,7,8$  and  $b_1 \geq 2$
- $Z_{32}y Z_{32}y Z_{31}z Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x D_{8+|b|} \otimes D_{8-|b|} \otimes D_0$ ; with  $|b| = b_1 + b_2 + b_3 = 4,5,6,7,8$  and  $b_1 \geq 2$
  
- In dimension seven ( $M_7$ ) we have the sum of the following terms:
- $Z_{32}y Z_{21}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x D_{14} \otimes D_0 \otimes D_2$
- $Z_{32}^{(2)}y Z_{21}^{(3)}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x D_{15} \otimes D_0 \otimes D_1$
- $Z_{32}y Z_{32}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x Z_{21}^{(b_5)}x D_{7+|b|} \otimes D_{8-|b|} \otimes D_1$ ; with  $|b| = \sum_{i=1}^5 b_i = 7,8$  and  $b_1 \geq 3$

- $Z_{32}^{(3)}y Z_{21}^{(4)}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(c_1)}y Z_{32}^{(c_2)}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x Z_{21}^{(b_5)}x D_{7+|b|} \otimes D_{9-|b|} \otimes D_0;$   
 $\quad \quad \quad \text{with } c_1 + c_2 = 3 \text{ and } |b| = \sum_{i=1}^5 b_i = 8,9 \text{ and } b_1 \geq 4$
- $Z_{32}y Z_{32}y Z_{32}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x D_{7+|b|} \otimes D_{9-|b|} \otimes D_0; \text{with } |b| = \sum_{i=1}^4 b_i = 7,8,9 \text{ and } b_1 \geq 4$
- $Z_{32}y Z_{32}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x D_{8+|b|} \otimes D_{7-|b|} \otimes D_1; \text{with } |b| = \sum_{i=1}^5 b_i = 5,6,7,8 \text{ and } b_1 \geq 1$
- $Z_{32}^{(2)}y Z_{31}z Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x Z_{21}^{(b_5)}x D_{8+|b|} \otimes D_{8-|b|} \otimes D_1; \text{with } |b| = \sum_{i=1}^5 b_i = 6,7,8 \text{ and } b_1 \geq 2$
- $Z_{32}y Z_{32}y Z_{31}z Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x D_{8+|b|} \otimes D_{8-|b|} \otimes D_0; \text{with } |b| = \sum_{i=1}^4 b_i = 5,6,7,8 \text{ and } b_1 \geq 2$
- In dimension eight ( $M_8$ ) we have the sum of the following terms:
  - $Z_{32}y Z_{32}y Z_{21}^{(3)}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x D_{15} \otimes D_0 \otimes D_1$
  - $Z_{32}^{(2)}y Z_{32}y Z_{21}^{(4)}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x D_{16} \otimes D_0 \otimes D_0$
  - $Z_{32}y Z_{32}^{(2)}y Z_{21}^{(4)}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x D_{16} \otimes D_0 \otimes D_0$
  - $Z_{32}y Z_{32}y Z_{32}y Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x Z_{21}^{(b_5)}x D_{7+|b|} \otimes D_{9-|b|} \otimes D_0; \text{with } |b| = \sum_{i=1}^5 b_i = 8,9 \text{ and } b_1 \geq 4$
  - $Z_{32}y Z_{31}z Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x Z_{21}^{(b_6)}x D_{8+|b|} \otimes D_{7-|b|} \otimes D_1; \text{with } |b| = \sum_{i=1}^6 b_i = 6,7 \text{ and } b_1 \geq 1$
  - $Z_{32}^{(2)}y Z_{31}z Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x Z_{21}^{(b_5)}x Z_{21}^{(b_6)}x D_{8+|b|} \otimes D_{8-|b|} \otimes D_0; \text{with } |b| = \sum_{i=1}^6 b_i = 7,8 \text{ and } b_1 \geq 2$
  - $Z_{32}y Z_{32}y Z_{31}z Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x Z_{21}^{(b_5)}x D_{8+|b|} \otimes D_{8-|b|} \otimes D_0; \text{with } |b| = \sum_{i=1}^5 b_i = 6,7,8 \text{ and } b_1 \geq 2$
- In dimension nine ( $M_9$ ) we have the sum of the following terms:
  - $Z_{32}y Z_{32}y Z_{32}y Z_{21}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x D_{16} \otimes D_0 \otimes D_0$
  - $Z_{32}y Z_{31}z Z_{21}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x D_{15} \otimes D_0 \otimes D_1$
  - $Z_{32}^{(2)}y Z_{31}z Z_{21}^{(2)}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x D_{16} \otimes D_0 \otimes D_0$
  - $Z_{32}y Z_{32}y Z_{31}z Z_{21}^{(b_1)}x Z_{21}^{(b_2)}x Z_{21}^{(b_3)}x Z_{21}^{(b_4)}x Z_{21}^{(b_5)}x Z_{21}^{(b_6)}x D_{8+|b|} \otimes D_{8-|b|} \otimes D_0;$   
 $\quad \quad \quad \text{with } |b| = \sum_{i=1}^6 b_i = 7,8 \text{ and } b_1 \geq 2$

Finally in dimension ten ( $M_{10}$ ) we have

- $Z_{32}y Z_{32}y Z_{31}z Z_{21}^{(2)}Z_{21}x Z_{21}x Z_{21}x Z_{21}x Z_{21}x D_{16} \otimes D_0 \otimes D_0$

In [1], it is necessary to introduce a quotient of bar complex module the caplli identities relations; the proof these relation are compatible with the boundary map  $\partial_x + \partial_y + \partial_z$  is complicated [1].

### III. LASCOUX RESOLUTION OF THE PARTITION (7, 6, 3)

The Lasoux resolution of the weyl module associated to the partition (7, 6, 3) looks like this

$$\begin{array}{ccccccc}
 D_8 F \otimes D_7 F \otimes D_1 F & & D_7 F \otimes D_7 F \otimes D_2 F \\
 0 \rightarrow D_9 F \otimes D_6 F \otimes D_1 F \rightarrow & \oplus & \rightarrow & \oplus & \rightarrow D_7 F \otimes D_6 F \otimes D_3 F \rightarrow 0 \\
 D_9 F \otimes D_5 F \otimes D_2 F & & D_8 F \otimes D_5 F \otimes D_3 F
 \end{array}$$

Were the positions of the terms of the complex are determined by the length of the permutation to which they correspond. The correspondence between the terms of the resolution above and permutations is as follows:

$$D_7 F \otimes D_6 F \otimes D_3 F \leftrightarrow \text{identity}$$

$$D_8 F \otimes D_5 F \otimes D_3 F \leftrightarrow (12)$$

$$D_7 F \otimes D_7 F \otimes D_2 F \leftrightarrow (23)$$

$$D_9 F \otimes D_5 F \otimes D_2 F \leftrightarrow (123)$$

$$D_8 F \otimes D_7 F \otimes D_1 F \leftrightarrow (132)$$

$$D_9 F \otimes D_6 F \otimes D_1 F \leftrightarrow (13)$$

Now, the terms can be presented as below, following Buchsbaum method [2].

$$M_0 = A_0$$

$$M_1 = A_1 \oplus B_1$$

$$M_2 = A_2 \oplus B_2$$

$$M_3 = A_3 \oplus B_3$$

$$M_j = B_j; \quad \text{for } j = 4,5,6,7,8,9,10.$$

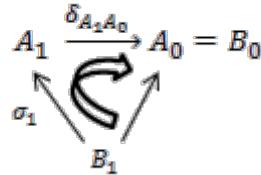
Where the  $A_S$  are the sums of the lascoux terms, and the  $B_S$  are the sums of the others.

Now, we define the map  $\sigma_1$  from  $B_1$  to  $A_1$  as follows:

- $Z_{21}^{(2)}x(v) \mapsto \frac{1}{2}Z_{21}x\partial_{21}(v); \text{ where } v \in D_9 \otimes D_4 \otimes D_3$
- $Z_{21}^{(3)}x(v) \mapsto \frac{1}{3}Z_{21}x\partial_{21}^{(2)}(v); \text{ where } v \in D_{10} \otimes D_3 \otimes D_3$
- $Z_{21}^{(4)}x(v) \mapsto \frac{1}{4}Z_{21}x\partial_{21}^{(3)}(v); \text{ where } v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}^{(5)}x(v) \mapsto \frac{1}{5}Z_{21}x\partial_{21}^{(4)}(v); \text{ where } v \in D_{12} \otimes D_1 \otimes D_3$
- $Z_{21}^{(6)}x(v) \mapsto \frac{1}{6}Z_{21}x\partial_{21}^{(5)}(v); \text{ where } v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{32}^{(2)}y(v) \mapsto \frac{1}{2}Z_{32}y\partial_{32}(v); \text{ where } v \in D_7 \otimes D_8 \otimes D_1$
- $Z_{32}^{(3)}y(v) \mapsto \frac{1}{3}Z_{32}y\partial_{32}^{(2)}(v); \text{ where } v \in D_7 \otimes D_9 \otimes D_0$

We should point out that the map  $\sigma_1$  satisfies the identity:

$$\delta_{A_1 A_0} \sigma_1 = \delta_{B_1 B_0}. \quad (3.1)$$



Where by  $\delta_{A_1 A_0}$  we mean the component of the boundary of the fat complex which conveys  $A_1$  to  $A_0$ . We will use notation  $\delta_{A_{i+1} A_i}$ ,  $\delta_{A_{i+1} B_i}$  etc.

Then we can define  $\partial_1: A_1 \rightarrow A_0$  as  $\partial_1 = \delta_{A_1 A_0}$ . It is easy to show that  $\partial_1$  which we defined above satisfies (3.1) for example:

$$\begin{aligned} (\delta_{A_1 A_0} o \sigma_1)(Z_{21}^{(4)}x(v)) &= \delta_{A_1 A_0}\left(\frac{1}{4}Z_{21}x\partial_{21}^{(3)}(v)\right) \\ &= \frac{1}{4}(\partial_{21}\partial_{21}^{(3)}(v)) = \partial_{21}^{(4)}(v) = \delta_{B_1 B_0}(Z_{21}^{(4)}x(v)) \end{aligned}$$

At this point we are in position to define

$$\partial_2: A_2 \rightarrow A_1 \quad \text{as} \quad \partial_2 = \delta_{A_2 A_1} + \sigma_1 \delta_{A_2 B_1}$$

Proposition (3.1)

The composition  $\partial_1 o \partial_2 = 0$

**Proof:** [2], [3]

$$\begin{aligned} \partial_1 o \partial_2(b) &= \delta_{A_1 A_0} o (\delta_{A_2 A_1}(b) + \sigma_1 o \delta_{A_2 B_1}(b)) \\ &= \delta_{A_1 A_0} o \delta_{A_2 A_1}(b) + \delta_{A_1 A_0} o \sigma_1 \delta_{A_2 B_1}(b) \end{aligned}$$

but  $\delta_{A_1 A_0} o \sigma_1 = \delta_{B_1 B_0}$  we have

$$\partial_1 o \partial_2(b) = \delta_{A_1 A_0} o (\delta_{A_2 A_1}(b) + \delta_{B_1 B_2} o \delta_{A_2 B_1}(b))$$

Which equal to zero because the properties of the boundary map  $\delta$  [2], so we get  $\partial_1 o \partial_2 = 0$

Now, we define map  $\sigma_2: B_2 \rightarrow A_2$  such that

$$\delta_{A_2 A_1} + \sigma_1 o \delta_{B_2 B_1} = (\delta_{A_2 A_1} + \sigma_1 o \delta_{A_2 B_1}) o \sigma_2 \quad (3.2)$$

We define this map as follows:

- $Z_{21}x Z_{21}x(v) \mapsto 0; \quad \text{where } v \in D_9 \otimes D_4 \otimes D_3$
- $Z_{21}^{(2)}x Z_{21}x(v) \mapsto 0; \quad \text{where } v \in D_{10} \otimes D_3 \otimes D_3$
- $Z_{21}x Z_{21}^{(2)}x(v) \mapsto 0; \quad \text{where } v \in D_{10} \otimes D_3 \otimes D_3$
- $Z_{21}^{(3)}x Z_{21}x(v) \mapsto 0; \quad \text{where } v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}^{(2)}x Z_{21}^{(2)}x(v) \mapsto 0; \quad \text{where } v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}x Z_{21}^{(3)}x(v) \mapsto 0; \quad \text{where } v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}^{(4)}x Z_{21}x(v) \mapsto 0; \quad \text{where } v \in D_{12} \otimes D_1 \otimes D_3$
- $Z_{21}^{(3)}x Z_{21}^{(2)}x(v) \mapsto 0; \quad \text{where } v \in D_{12} \otimes D_1 \otimes D_3$

- $Z_{21}^{(2)}x Z_{21}^{(3)}x(v) \mapsto 0;$  where  $v \in D_{12} \otimes D_1 \otimes D_3$
- $Z_{21}x Z_{21}^{(4)}x(v) \mapsto 0;$  where  $v \in D_{12} \otimes D_1 \otimes D_3$
- $Z_{21}^{(5)}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}^{(4)}x Z_{21}^{(2)}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}^{(3)}x Z_{21}^{(3)}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)}x Z_{21}^{(4)}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}x Z_{21}^{(5)}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{32}y Z_{21}^{(3)}x(v) \mapsto \frac{1}{3}Z_{32}y Z_{21}^{(2)}x \partial_{21}(v);$  where  $v \in D_{10} \otimes D_4 \otimes D_2$
- $Z_{32}y Z_{21}^{(4)}x(v) \mapsto \frac{1}{6}Z_{32}y Z_{21}^{(2)}x \partial_{21}^{(2)}(v);$  where  $v \in D_{11} \otimes D_3 \otimes D_2$
- $Z_{32}y Z_{21}^{(5)}x(v) \mapsto \frac{1}{10}Z_{32}y Z_{21}^{(2)}x \partial_{21}^{(3)}(v);$  where  $v \in D_{12} \otimes D_2 \otimes D_2$
- $Z_{32}y Z_{21}^{(6)}x(v) \mapsto \frac{1}{15}Z_{32}y Z_{21}^{(2)}x \partial_{21}^{(4)}(v);$  where  $v \in D_{13} \otimes D_1 \otimes D_2$
- $Z_{32}y Z_{21}^{(7)}x(v) \mapsto \frac{1}{21}Z_{32}y Z_{21}^{(2)}x \partial_{21}^{(5)}(v);$  where  $v \in D_{14} \otimes D_0 \otimes D_2$
- $Z_{32}y Z_{21}^{(8)}x(v) \mapsto \frac{1}{3}Z_{32}y Z_{21}^{(2)}x \partial_{21}^{(2)}(v) - \frac{1}{3}Z_{32}y Z_{31}z \partial_{21}^{(2)}(v);$  where  $v \in D_{10} \otimes D_5 \otimes D_1$
- $Z_{32}y Z_{21}^{(9)}x(v) \mapsto -\frac{1}{12}Z_{32}y Z_{21}^{(2)}x \partial_{21}^{(2)} \partial_{32}(v) - \frac{1}{2}Z_{32}y Z_{31}z \partial_{21}^{(3)}(v);$  where  $v \in D_{11} \otimes D_4 \otimes D_1$
- $Z_{32}y Z_{21}^{(10)}x(v) \mapsto -\frac{1}{30}Z_{32}y Z_{21}^{(2)}x \partial_{21}^{(3)} \partial_{32}(v) - \frac{1}{3}Z_{32}y Z_{31}z \partial_{21}^{(4)}(v);$  where  $v \in D_{12} \otimes D_3 \otimes D_1$
- $Z_{32}y Z_{21}^{(11)}x(v) \mapsto -\frac{1}{60}Z_{32}y Z_{21}^{(2)}x \partial_{21}^{(4)} \partial_{32}(v) - \frac{1}{4}Z_{32}y Z_{31}z \partial_{21}^{(5)}(v);$  where  $v \in D_{13} \otimes D_2 \otimes D_1$
- $Z_{32}y Z_{21}^{(12)}x(v) \mapsto -\frac{1}{105}Z_{32}y Z_{21}^{(2)}x \partial_{21}^{(5)} \partial_{32}(v) - \frac{1}{5}Z_{32}y Z_{31}z \partial_{21}^{(6)}(v);$  where  $v \in D_{14} \otimes D_1 \otimes D_1$
- $Z_{32}y Z_{21}^{(13)}x(v) \mapsto \frac{1}{42}Z_{32}y Z_{21}^{(2)}x \partial_{21}^{(5)} \partial_{31}(v);$  where  $v \in D_{15} \otimes D_0 \otimes D_1$
- $Z_{32}y Z_{21}^{(14)}x(v) \mapsto 0; \text{ where } v \in D_7 \otimes D_8 \otimes D_1$
- $Z_{32}y Z_{21}^{(15)}x(v) \mapsto -\frac{1}{6}Z_{32}y Z_{21}^{(2)}x \partial_{21}^{(2)} \partial_{32}^{(2)}(v) - \frac{1}{3}Z_{32}y Z_{31}z \partial_{21}^{(3)} \partial_{32}(v) + \frac{1}{3}Z_{32}y Z_{21}^{(2)}x \partial_{31}^{(2)}(v);$  where  $v \in D_{11} \otimes D_5 \otimes D_0$
- $Z_{32}y Z_{21}^{(16)}x(v) \mapsto \frac{1}{30}Z_{21}^{(2)}x Z_{31}z \partial_{32} \partial_{21}^{(2)} \partial_{31}(v) + \frac{1}{30}Z_{32}y Z_{21}^{(2)}x \partial_{21}^{(3)} \partial_{32}^{(2)}(v) - \frac{1}{15}Z_{32}y Z_{31}z \partial_{21}^{(3)} \partial_{31}(v);$  where  $v \in D_{12} \otimes D_4 \otimes D_0$
- $Z_{32}y Z_{21}^{(17)}x(v) \mapsto \frac{1}{36}Z_{32}y Z_{21}^{(2)}x \partial_{32} \partial_{21}^{(3)} \partial_{31}(v) - \frac{1}{36}Z_{32}y Z_{31}z \partial_{21}^{(5)} \partial_{32}(v) + \frac{1}{90}Z_{32}y Z_{21}^{(2)}x \partial_{21}^{(4)} \partial_{32}^{(2)}(v);$  where  $v \in D_{13} \otimes D_3 \otimes D_0$
- $Z_{32}y Z_{21}^{(18)}x(v) \mapsto \frac{1}{210}Z_{32}y Z_{21}^{(2)}x \partial_{21}^{(3)} \partial_{31}^{(2)}(v) + \frac{1}{42}Z_{32}y Z_{31}z \partial_{21}^{(6)} \partial_{32}(v) - \frac{1}{14}Z_{32}y Z_{31}z \partial_{32} \partial_{21}^{(6)}(v);$  where  $v \in D_{14} \otimes D_2 \otimes D_0$
- $Z_{32}y Z_{21}^{(19)}x(v) \mapsto \frac{1}{420}Z_{32}y Z_{21}^{(2)}x \partial_{21}^{(6)} \partial_{32}^{(2)}(v) - \frac{1}{3}Z_{32}y Z_{31}z \partial_{32} \partial_{21}^{(7)}(v) - \frac{1}{30}Z_{32}y Z_{31}z \partial_{21}^{(6)} \partial_{31}(v);$  where  $v \in D_{15} \otimes D_1 \otimes D_0$
- $Z_{32}y Z_{21}^{(20)}x(v) \mapsto -\frac{1}{18}Z_{32}y Z_{31}z \partial_{21}^{(7)} \partial_{31}(v); \text{ where } v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}y Z_{21}^{(21)}x(v) \mapsto 0; \text{ where } v \in D_7 \otimes D_9 \otimes D_0$
- $Z_{32}y Z_{32}^{(2)}y(v) \mapsto 0; \text{ where } v \in D_7 \otimes D_9 \otimes D_0$
- $Z_{32}y Z_{32}^{(2)}z(v) \mapsto 0; \text{ where } v \in D_8 \otimes D_8 \otimes D_0$

It is easy to show that  $\sigma_2$  which is defined above satisfies the condition (3.2), for example we chose one of them

- $(\delta_{B_2 A_1} + \sigma_1 \delta_{B_2 B_1})(Z_{32}y Z_{21}^{(5)}x(v)); \text{ where } v \in D_{12} \otimes D_2 \otimes D_2$ 

$$= \sigma_1(Z_{21}^{(5)}x \partial_{32}(v)) + \sigma_1(Z_{21}^{(4)}x \partial_{31}(v)) - Z_{32}y \partial_{21}^{(5)}(v)$$

$$= \frac{1}{5}Z_{21}x \partial_{21}^{(4)} \partial_{32}(v) + \frac{1}{4}Z_{21}x \partial_{21}^{(3)} \partial_{31}(v) - Z_{32}y \partial_{21}^{(5)}(v)$$

and

$$\begin{aligned}
 (\delta_{A_2 A_1} + \sigma_2 \delta_{A_2 B_1}) & \left( \frac{1}{10}Z_{32}y Z_{21}^{(2)}x \partial_{21}^{(3)}(v) \right) \\
 & = \sigma_1 \left( \frac{1}{10}Z_{21}^{(2)}x \partial_{32} \partial_{32}^{(3)}(v) \right) + \frac{1}{10}Z_{21}x \partial_{31} \partial_{21}^{(3)}(v) - \frac{1}{10}Z_{32}y \partial_{21}^{(2)} \partial_{21}^{(3)}(v) \\
 & = \frac{1}{5}Z_{21}x \partial_{21}^{(4)} \partial_{32}(v) + \frac{3}{20}Z_{21}x \partial_{21}^{(3)} \partial_{31}(v) + \frac{1}{10}Z_{21}x \partial_{21}^{(3)} \partial_{31}(v) - Z_{32}y \partial_{21}^{(5)}(v) \\
 & = \frac{1}{5}Z_{21}x \partial_{21}^{(4)} \partial_{32}(v) + \frac{1}{4}Z_{21}x \partial_{21}^{(3)} \partial_{31}(v) - Z_{32}y \partial_{21}^{(5)}(v)
 \end{aligned}$$

**Proposition (3.2):** we have exactness at  $A_i$

**Proof:** see [2] and [3]

Now by using  $\sigma_2$  we can also define

$$\partial_3: A_3 \rightarrow A_2 \text{ by } \delta_{A_3 A_2} + \sigma_2 \circ \delta_{A_3 B_2}$$

**Proposition (3.3):**

The composition  $\partial_2 \circ \partial_3 = 0$

**Proof:** the same way used in proposition (3.1)

We need to definition of map  $\sigma_3: B_3 \rightarrow A_3$  such that

$$\delta_{B_3 A_3} + \sigma_2 \delta_{B_3 B_2} = (\delta_{A_3 A_2} + \sigma_2 \delta_{A_3 B_2}) \sigma_3 \quad (3.3)$$

As follows:

- $Z_{21}x Z_{21}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{10} \otimes D_3 \otimes D_3$
- $Z_{21}^{(2)}x Z_{21}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}x Z_{21}^{(2)}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}x Z_{21}x Z_{21}^{(2)}x(v) \mapsto 0;$  where  $v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}^{(3)}x Z_{21}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{12} \otimes D_1 \otimes D_3$
- $Z_{21}^{(2)}x Z_{21}^{(2)}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{12} \otimes D_1 \otimes D_3$
- $Z_{21}^{(2)}x Z_{21}x Z_{21}^{(2)}x(v) \mapsto 0;$  where  $v \in D_{12} \otimes D_1 \otimes D_3$
- $Z_{21}x Z_{21}^{(3)}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}x Z_{21}^{(2)}x Z_{21}^{(2)}x(v) \mapsto 0;$  where  $v \in D_{12} \otimes D_1 \otimes D_3$
- $Z_{21}x Z_{21}x Z_{21}^{(3)}x(v) \mapsto 0;$  where  $v \in D_{11} \otimes D_2 \otimes D_3$
- $Z_{21}^{(4)}x Z_{21}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}^{(3)}x Z_{21}^{(2)}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}^{(3)}x Z_{21}x Z_{21}^{(2)}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)}x Z_{21}^{(3)}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)}x Z_{21}^{(2)}x Z_{21}^{(2)}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)}x Z_{21}x Z_{21}^{(3)}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}x Z_{21}^{(4)}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}x Z_{21}^{(3)}x Z_{21}^{(2)}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}x Z_{21}^{(2)}x Z_{21}^{(3)}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{21}x Z_{21}x Z_{21}^{(4)}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_0 \otimes D_3$
- $Z_{32}y Z_{21}^{(2)}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{10} \otimes D_4 \otimes D_2$
- $Z_{32}y Z_{21}^{(3)}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{11} \otimes D_3 \otimes D_2$
- $Z_{32}y Z_{21}^{(2)}x Z_{21}^{(2)}x(v) \mapsto 0;$  where  $v \in D_{11} \otimes D_3 \otimes D_2$
- $Z_{32}y Z_{21}^{(4)}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{12} \otimes D_2 \otimes D_2$
- $Z_{32}y Z_{21}^{(3)}x Z_{21}^{(2)}x(v) \mapsto 0;$  where  $v \in D_{12} \otimes D_2 \otimes D_2$
- $Z_{32}y Z_{21}^{(2)}x Z_{21}^{(3)}x(v) \mapsto 0;$  where  $v \in D_{12} \otimes D_2 \otimes D_2$
- $Z_{32}y Z_{21}^{(5)}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_1 \otimes D_2$
- $Z_{32}y Z_{21}^{(4)}x Z_{21}^{(2)}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_1 \otimes D_2$
- $Z_{32}y Z_{21}^{(3)}x Z_{21}^{(3)}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_1 \otimes D_2$
- $Z_{32}y Z_{21}^{(2)}x Z_{21}^{(3)}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_1 \otimes D_2$
- $Z_{32}y Z_{21}^{(6)}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{14} \otimes D_0 \otimes D_2$
- $Z_{32}y Z_{21}^{(5)}x Z_{21}^{(2)}x(v) \mapsto 0;$  where  $v \in D_{14} \otimes D_0 \otimes D_2$
- $Z_{32}y Z_{21}^{(4)}x Z_{21}^{(3)}x(v) \mapsto 0;$  where  $v \in D_{14} \otimes D_0 \otimes D_2$
- $Z_{32}y Z_{21}^{(3)}x Z_{21}^{(4)}x(v) \mapsto 0;$  where  $v \in D_{14} \otimes D_0 \otimes D_2$
- $Z_{32}y Z_{21}^{(2)}x Z_{21}^{(5)}x(v) \mapsto 0;$  where  $v \in D_{14} \otimes D_0 \otimes D_2$
- $Z_{32}y Z_{21}^{(2)}x Z_{21}x(v) \mapsto \frac{1}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(2)}(v);$  where  $v \in D_{11} \otimes D_4 \otimes D_1$

- $Z_{32}^{(2)}y Z_{21}^{(4)}x Z_{21}x(v) \mapsto -\frac{1}{12}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(3)}(v);$  where  $v \in D_{12} \otimes D_3 \otimes D_1$
- $Z_{32}^{(2)}y Z_{21}^{(3)}x Z_{21}x(v) \mapsto \frac{1}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(3)}(v);$  where  $v \in D_{12} \otimes D_3 \otimes D_1$
- $Z_{32}^{(2)}y Z_{21}^{(5)}x Z_{21}x(v) \mapsto -\frac{1}{30}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(4)}(v);$  where  $v \in D_{13} \otimes D_2 \otimes D_1$
- $Z_{32}^{(2)}y Z_{21}^{(4)}x Z_{21}x(v) \mapsto -\frac{1}{4}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(4)}(v);$  where  $v \in D_{13} \otimes D_2 \otimes D_1$
- $Z_{32}^{(2)}y Z_{21}^{(3)}x Z_{21}x(v) \mapsto \frac{1}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(4)}(v);$  where  $v \in D_{13} \otimes D_2 \otimes D_1$
- $Z_{32}^{(2)}y Z_{21}^{(6)}x Z_{21}x(v) \mapsto -\frac{1}{60}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(5)}(v);$  where  $v \in D_{14} \otimes D_1 \otimes D_1$
- $Z_{32}^{(2)}y Z_{21}^{(5)}x Z_{21}x(v) \mapsto -\frac{2}{15}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(5)}(v);$  where  $v \in D_{14} \otimes D_1 \otimes D_1$
- $Z_{32}^{(2)}y Z_{21}^{(4)}x Z_{21}x(v) \mapsto -\frac{1}{2}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(5)}(v);$  where  $v \in D_{14} \otimes D_1 \otimes D_1$
- $Z_{32}^{(2)}y Z_{21}^{(3)}x Z_{21}x(v) \mapsto \frac{1}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(5)}(v);$  where  $v \in D_{14} \otimes D_1 \otimes D_1$
- $Z_{32}^{(2)}y Z_{21}^{(7)}x Z_{21}x(v) \mapsto -\frac{1}{5}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)}(v);$  where  $v \in D_{15} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)}y Z_{21}^{(6)}x Z_{21}x(v) \mapsto -\frac{3}{4}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)}(v);$  where  $v \in D_{15} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)}y Z_{21}^{(5)}x Z_{21}x(v) \mapsto -\frac{5}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)}(v);$  where  $v \in D_{15} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)}y Z_{21}^{(4)}x Z_{21}x(v) \mapsto -\frac{5}{2}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)}(v);$  where  $v \in D_{15} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)}y Z_{21}^{(3)}x Z_{21}x(v) \mapsto -Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)}(v);$  where  $v \in D_{15} \otimes D_0 \otimes D_1$
- $Z_{32}y Z_{32}y Z_{21}^{(3)}x(v) \mapsto -\frac{1}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{21}(v);$  where  $v \in D_{10} \otimes D_5 \otimes D_1$
- $Z_{32}y Z_{32}y Z_{21}^{(4)}x(v) \mapsto -\frac{1}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(2)}(v);$  where  $v \in D_{11} \otimes D_4 \otimes D_1$
- $Z_{32}y Z_{32}y Z_{21}^{(5)}x(v) \mapsto -\frac{1}{6}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(3)}(v);$  where  $v \in D_{12} \otimes D_3 \otimes D_1$
- $Z_{32}y Z_{32}y Z_{21}^{(6)}x(v) \mapsto -\frac{1}{10}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(4)}(v);$  where  $v \in D_{13} \otimes D_2 \otimes D_1$
- $Z_{32}y Z_{32}y Z_{21}^{(7)}x(v) \mapsto -\frac{1}{15}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(5)}(v);$  where  $v \in D_{14} \otimes D_1 \otimes D_1$
- $Z_{32}y Z_{32}y Z_{21}^{(8)}x(v) \mapsto 0;$  where  $v \in D_{15} \otimes D_0 \otimes D_1$
- $Z_{32}^{(3)}y Z_{21}^{(4)}x Z_{21}x(v) \mapsto -\frac{1}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(3)} \partial_{32}(v);$  where  $v \in D_{12} \otimes D_4 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(5)}x Z_{21}x(v) \mapsto \frac{2}{15}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(4)} \partial_{32}(v)$ 

$$+ \frac{1}{30}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(3)} \partial_{31}(v) - \frac{1}{10}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(4)}(v);$$
 where  $v \in D_{13} \otimes D_3 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(4)}x Z_{21}x(v) \mapsto -\frac{1}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(4)} \partial_{32}(v)$ 

$$- \frac{1}{4}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(4)}(v) - \frac{1}{12}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(3)} \partial_{31}(v);$$
 where  $v \in D_{13} \otimes D_3 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(6)}x Z_{21}x(v) \mapsto \frac{2}{45}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(5)}(v)$ 

$$+ \frac{1}{20}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(4)} \partial_{31}(v) - \frac{1}{60}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(5)} \partial_{32}(v);$$
 where  $v \in D_{14} \otimes D_2 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(5)}x Z_{21}x(v) \mapsto \frac{1}{6}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(5)}(v);$  where  $v \in D_{14} \otimes D_2 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(7)}x Z_{21}x(v) \mapsto -\frac{1}{105}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(6)}(v) + \frac{2}{105}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(5)} \partial_{31}(v);$  where  $v \in D_{15} \otimes D_1 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(6)}x Z_{21}x(v) \mapsto \frac{17}{60}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(6)}(v)$ 

$$- \frac{7}{30}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{32}(v);$$
 where  $v \in D_{15} \otimes D_1 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(5)}x Z_{21}x(v) \mapsto \frac{2}{5}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(6)}(v)$ 

$$- \frac{2}{15}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{32}(v);$$
 where  $v \in D_{15} \otimes D_1 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(4)}x Z_{21}x(v) \mapsto -\frac{1}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(6)}(v)$ 

$$- Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{32}(v);$$
 where  $v \in D_{15} \otimes D_1 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(8)}x Z_{21}x(v) \mapsto 0;$  where  $v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(7)}x Z_{21}x(v) \mapsto \frac{1}{42}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{31}(v);$  where  $v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(6)}x Z_{21}x(v) \mapsto \frac{7}{12}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{31}(v);$  where  $v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(5)}x Z_{21}x(v) \mapsto \frac{2}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{31}(v);$  where  $v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(4)}x Z_{21}x(v) \mapsto -\frac{2}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{31}(v);$  where  $v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(2)}y Z_{32}y Z_{21}^{(4)}x(v) \mapsto -\frac{1}{6}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(2)} \partial_{32}(v) + \frac{1}{6}Z_{32}y Z_{31}z Z_{21}x \partial_{21} \partial_{31}(v);$  where  $v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(2)}y Z_{32}y Z_{21}^{(5)}x(v) \mapsto \frac{1}{20}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(3)}(v)$ 

$$+ \frac{1}{30}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(2)} \partial_{31}(v) + \frac{1}{30}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(3)} \partial_{32}(v);$$
 where  $v \in D_{12} \otimes D_4 \otimes D_0$



- $Z_{32}y Z_{32}y Z_{32}y(v) \mapsto 0;$  where  $v \in D_7 \otimes D_9 \otimes D_0$
- $Z_{32}y Z_{31}z Z_{21}^{(2)}x(v) \mapsto \frac{1}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{21}(v);$  where  $v \in D_{10} \otimes D_5 \otimes D_0$
- $Z_{32}y Z_{31}z Z_{21}^{(3)}x(v) \mapsto 0;$  where  $v \in D_{11} \otimes D_4 \otimes D_1$
- $Z_{32}^{(3)}y Z_{21}^{(6)}x Z_{21}^{(2)}x(v) \mapsto \frac{17}{60}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(6)}(v) - \frac{7}{30}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{32}(v);$  where  $v \in D_{15} \otimes D_1 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(5)}x Z_{21}^{(3)}x(v) \mapsto \frac{2}{5}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(6)}(v) - \frac{2}{15}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{32}(v);$  where  $v \in D_{15} \otimes D_1 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(4)}x Z_{21}^{(4)}x(v) \mapsto -\frac{1}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(6)}(v) - Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{32}(v);$  where  $v \in D_{15} \otimes D_1 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(8)}x Z_{21}x(v) \mapsto 0; \text{ where } v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(7)}x Z_{21}^{(2)}x(v) \mapsto \frac{1}{42}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{31}(v);$  where  $v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(6)}x Z_{21}^{(3)}x(v) \mapsto \frac{7}{12}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{31}(v);$  where  $v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(5)}x Z_{21}^{(4)}x(v) \mapsto \frac{2}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{31}(v);$  where  $v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)}y Z_{21}^{(4)}x Z_{21}^{(5)}x(v) \mapsto -\frac{2}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{31}(v);$  where  $v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(2)}y Z_{32}y Z_{21}^{(4)}x(v) \mapsto -\frac{1}{6}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(2)} \partial_{32}(v) + \frac{1}{6}Z_{32}y Z_{31}z Z_{21}x \partial_{21} \partial_{31}(v); \text{ where } v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}^{(2)}y Z_{32}y Z_{21}^{(5)}x(v) \mapsto \frac{1}{20}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(3)}(v)$   
 $\quad \quad \quad + \frac{1}{20}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(2)} \partial_{31}(v) + \frac{1}{30}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(3)} \partial_{32}(v);$  where  $v \in D_{12} \otimes D_4 \otimes D_0$
- $Z_{32}^{(2)}y Z_{32}y Z_{21}^{(6)}x(v) \mapsto \frac{1}{60}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(4)} \partial_{32}(v)$   
 $\quad \quad \quad + \frac{1}{60}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(4)}(v) + \frac{1}{15}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(3)} \partial_{31}(v);$  where  $v \in D_{13} \otimes D_3 \otimes D_0$
- $Z_{32}^{(2)}y Z_{32}y Z_{21}^{(7)}x(v) \mapsto \frac{1}{105}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(5)}(v)$   
 $\quad \quad \quad - \frac{1}{420}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(4)} \partial_{31}(v);$  where  $v \in D_{14} \otimes D_2 \otimes D_0$
- $Z_{32}^{(2)}y Z_{32}y Z_{21}^{(8)}x(v) \mapsto -\frac{1}{70}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{32}(v);$  where  $v \in D_{15} \otimes D_1 \otimes D_0$
- $Z_{32}^{(2)}y Z_{32}y Z_{21}^{(9)}x(v) \mapsto -\frac{1}{42}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{31}(v);$  where  $v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}y Z_{32}^{(2)}y Z_{21}^{(4)}x(v) \mapsto -\frac{1}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(2)} \partial_{32}(v);$  where  $v \in D_{11} \otimes D_5 \otimes D_0$
- $Z_{32}y Z_{32}^{(2)}y Z_{21}^{(5)}x(v) \mapsto \frac{1}{15}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(3)} \partial_{32}(v) - \frac{1}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(3)}(v);$  where  $v \in D_{12} \otimes D_4 \otimes D_0$
- $Z_{32}y Z_{32}^{(2)}y Z_{21}^{(6)}x(v) \mapsto -\frac{1}{60}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(4)} \partial_{32}(v); \text{ where } v \in D_{13} \otimes D_3 \otimes D_0$
- $Z_{32}y Z_{32}^{(2)}y Z_{21}^{(7)}x(v) \mapsto \frac{1}{42}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(5)} \partial_{32}(v)$   
 $\quad \quad \quad + \frac{1}{210}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(4)} \partial_{31}(v) - \frac{1}{21}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(5)}(v);$  where  $v \in D_{14} \otimes D_2 \otimes D_0$
- $Z_{32}y Z_{32}^{(2)}y Z_{21}^{(8)}x(v) \mapsto -\frac{1}{60}Z_{32}y Z_{31}z Z_{21}x \partial_{32} \partial_{21}^{(6)}(v)$   
 $\quad \quad \quad - \frac{1}{60}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(5)} \partial_{31}(v) + \frac{1}{420}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)} \partial_{32}(v);$  where  $v \in D_{15} \otimes D_1 \otimes D_0$
- $Z_{32}y Z_{32}y Z_{32}y(v) \mapsto 0;$  where  $v \in D_7 \otimes D_9 \otimes D_0$
- $Z_{32}y Z_{31}z Z_{21}^{(2)}x(v) \mapsto \frac{1}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{21}(v);$  where  $v \in D_{10} \otimes D_5 \otimes D_0$
- $Z_{32}y Z_{31}z Z_{21}^{(3)}x(v) \mapsto 0;$  where  $v \in D_{11} \otimes D_4 \otimes D_1$
- $Z_{32}y Z_{31}z Z_{21}^{(5)}x(v) \mapsto 0;$  where  $v \in D_{13} \otimes D_2 \otimes D_1$
- $Z_{32}y Z_{31}z Z_{21}^{(6)}x(v) \mapsto 0;$  where  $v \in D_{14} \otimes D_1 \otimes D_1$
- $Z_{32}y Z_{31}z Z_{21}^{(7)}x(v) \mapsto \frac{1}{7}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(6)}(v);$  where  $v \in D_{15} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)}y Z_{31}z Z_{21}^{(2)}x(v) \mapsto \frac{1}{3}Z_{32}y Z_{31}z Z_{21}x \partial_{31}(v);$  where  $v \in D_{10} \otimes D_6 \otimes D_0$
- $Z_{32}^{(2)}y Z_{31}z Z_{21}^{(3)}x(v) \mapsto -\frac{1}{6}Z_{32}y Z_{31}z Z_{21}x \partial_{32}(v) + \frac{1}{6}Z_{32}y Z_{31}z Z_{21}x \partial_{21} \partial_{31}(v);$  where  $v \in D_{11} \otimes D_5 \otimes D_0$
- $Z_{32}^{(2)}y Z_{31}z Z_{21}^{(4)}x(v) \mapsto \frac{1}{15}Z_{32}y Z_{31}z Z_{21}x \partial_{21} \partial_{31}(v);$  where  $v \in D_{12} \otimes D_4 \otimes D_0$
- $Z_{32}^{(2)}y Z_{31}z Z_{21}^{(5)}x(v) \mapsto \frac{1}{12}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(3)} \partial_{31}(v);$  where  $v \in D_{13} \otimes D_3 \otimes D_0$
- $Z_{32}^{(2)}y Z_{31}z Z_{21}^{(6)}x(v) \mapsto \frac{1}{105}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(4)} \partial_{31}(v) - \frac{1}{105}Z_{32}y Z_{31}z Z_{21}x \partial_{21}^{(5)} \partial_{32}(v);$  where  $v \in D_{14} \otimes D_2 \otimes D_0$
- $Z_{32}^{(2)}y Z_{31}z Z_{21}^{(7)}x(v) \mapsto 0;$  where  $v \in D_{15} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)}y Z_{31}z Z_{21}^{(8)}x(v) \mapsto 0;$  where  $v \in D_{16} \otimes D_0 \otimes D_0$
- $Z_{32}y Z_{32}y Z_{31}z(v) \mapsto 0;$  where  $v \in D_8 \otimes D_8 \otimes D_0$

A gain we can show that  $\sigma_3$  which defined above satisfies the condition (3.3), and here we chose one of them as an example.

- $(\delta_{B_3A_2} + \sigma_2 \delta_{B_3B_2}) (Z_{32}^{(2)} y Z_{21}^{(3)} Z_{21}x(v))$ ; where  $v \in D_{11} \otimes D_4 \otimes D_1$

$$\begin{aligned}
 &= \sigma_2 (Z_{21}^{(3)} x Z_{21}x \partial_{32}^{(2)}(v)) + \sigma_2 (Z_{21}^{(2)} x Z_{21}x \partial_{32} \partial_{31}(v)) \\
 &\quad + \sigma_2 (Z_{21} x Z_{21} x \partial_{31}^{(2)} x(v)) - \sigma_2 (4 Z_{32}^{(2)} y Z_{21}^{(4)} x(v)) + \sigma_2 (Z_{32}^{(2)} y Z_{21}^{(3)} x \partial_{21}) \\
 &= \frac{1}{3} Z_{32} y Z_{21}^{(2)} x \partial_{21}^{(2)} \partial_{32}(v) + 2 Z_{32} y Z_{31} z \partial_{21}^{(3)}(v) + \frac{1}{3} Z_{32} y Z_{21}^{(2)} x \partial_{21} \partial_{31}(v) - Z_{32} y Z_{31} z \partial_{21}^{(3)}(v) \\
 &= \frac{1}{3} Z_{32} y Z_{21}^{(2)} x \partial_{21}^{(2)} \partial_{32}(v) + Z_{32} y Z_{31} z \partial_{21}^{(3)}(v) + \frac{1}{3} Z_{32} y Z_{21}^{(2)} x \partial_{21} \partial_{32}(v).
 \end{aligned}$$

And

$$\begin{aligned}
 &(\delta_{A_3A_2} + \sigma_2 \delta_{A_3B_2}) \left( \frac{1}{3} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(2)}(v) \right) \\
 &= -\sigma_2 \left( \frac{1}{3} Z_{21} x Z_{21} x \partial_{32}^{(2)} \partial_{21}^{(2)}(v) \right) + \frac{1}{3} Z_{32} y Z_{21}^{(2)} x \partial_{32} \partial_{21}^{(2)}(v) \\
 &\quad - \sigma_2 \left( \frac{1}{3} Z_{32} y Z_{32} y \partial_{21}^{(2)} \partial_{21}^{(2)}(v) \right) + Z_{32} y Z_{31} z \partial_{21}^{(3)}(v) \\
 &= \frac{1}{3} Z_{32} y Z_{21}^{(2)} x \partial_{32} \partial_{21}^{(2)}(v) + Z_{32} y Z_{31} z \partial_{21}^{(3)}(v) \\
 &= \frac{1}{3} Z_{32} y Z_{21}^{(2)} x \partial_{21}^{(2)} \partial_{32}(v) + Z_{32} y Z_{31} z \partial_{21}^{(3)}(v) + \frac{1}{3} Z_{32} y Z_{21}^{(2)} x \partial_{21} \partial_{31}(v).
 \end{aligned}$$

So from all we have done above we the complex.

$$0 \rightarrow A_3 \xrightarrow{\partial_3} A_2 \xrightarrow{\partial_2} A_1 \xrightarrow{\partial_1} A_0 \quad (3.4)$$

Where  $\partial_i$  defied as follows:

- $\partial_1(Z_{21}x(v)) = \partial_{21}(v)$
- $\partial_1(Z_{32}y(v)) = \partial_{32}(v)$
- $\partial_2(Z_{32} y Z_{21}^{(2)} xv) = \frac{1}{2} Z_{21} x \partial_{21} \partial_{32}(v) + Z_{21} x \partial_{31}(v) - Z_{32} y \partial_{21}^{(2)}(v)$
- $\partial_2(Z_{32} y Z_{31} z v) = \frac{1}{2} Z_{32} y \partial_{32} \partial_{21}(v) + Z_{21} x \partial_{32}^{(2)}(v) - Z_{32} y \partial_{32}^{(2)}(v)$  finally, we defined the map  $\hat{\partial}_3$  by :
- $\partial_3(Z_{32} y Z_{31} z Z_{21} xv) = Z_{32} y Z_{21}^{(2)} x \partial_{32}(v) + Z_{32} y Z_{31} z \partial_{21}(v)$  Proposition (3.4).

The complex  $0 \rightarrow A_3 \xrightarrow{\partial_3} A_2 \xrightarrow{\partial_2} A_1 \xrightarrow{\partial_1} A_0 \rightarrow k_{(7,6,3)}$  is exact

**Prove:** see [2] and [3]

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