CERTAIN COEFFICIENT INEQUALITIES FOR SAKAGUCHI TYPE FUNCTIONS
AND APPLICATIONS TO FRACTIONAL DERIVATIVES

B. Srutha Keerthi*

Department of Applied Mathematics Sri Venkateswara College of Engineering
Sriperumbudur, Chennai –602105, India

E-mail: laya@svce.ac.in, sruthilaya06@yahoo.co.in

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ABSTRACT

In the present paper, sharp upper bounds of \( a_2 \) for the functions \( f(z) = z + a_2 z^2 + a_3 z^3 + \cdots \) belonging to a new subclass of Sakaguchi type functions are obtained. Also, application of our results for subclass of functions defined by convolution with a normalized analytic function are given. In particular, Fekete-Szegö inequalities for certain classes of functions defined through fractional derivatives are obtained.

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1. Introduction

Let \( \mathcal{A} \) be the class of analytic functions of the form

\[
 f(z) = z + \sum_{n=2}^{\infty} a_n z^n \quad (z \in \Delta := \{ z \in \mathbb{C} : |z| < 1 \}) \tag{1.1}
\]

and \( \mathcal{S} \) be the subclass of \( \mathcal{A} \) consisting of univalent functions. For two functions \( f, g \in \mathcal{A} \), we say that the function \( f(z) \) is subordinate to \( g(z) \) in \( \Delta \) and write \( f \prec g \) or \( f(z) \prec g(z) \), if there exists an analytic function \( w(z) \) with \( w(0) = 0 \) and \( |w(z)| < 1 \) \( (z \in \Delta) \), such that \( f(z) = g(w(z)) \), \( (z \in \Delta) \). In particular, if the function \( g \) is univalent in \( \Delta \), the above subordination is equivalent to \( f(0) = g(0) \) and \( f(\Delta) \subset g(\Delta) \).

A function \( f(z) \in \mathcal{A} \) is said to be in the class \( M(\lambda, \alpha, t) \) if it satisfies

\[
 \text{Re} \left\{ \frac{(1-t)[\lambda z^2 f'(z) + zf(z)]}{(1-\lambda)[f(z)-zf'(zt)] + \lambda z[f'(z)-tf'(zt)]} \right\} > \alpha, \quad \left| t \right| \leq 1, t \neq 0 \leq \alpha \leq 1 \tag{1.2}
\]

For the case \( \lambda = 0 \) in (1.2) we get a Sakaguchi type class \( \mathcal{S}(\alpha, t) \). A function \( f(z) \in \mathcal{A} \) is said to be in the class \( \mathcal{S}(\alpha, t) \) if it satisfies

\[
 \text{Re} \left\{ \frac{(1-t)zf'(z)}{f(z)-zf(zt)} \right\} > \alpha, \quad \left| t \right| \leq 1, t \neq 1 \tag{1.3}
\]

which was introduced and studied by Owa et al. [9, 10]. For some \( \alpha \in [0, 1) \) and for all \( z \in \Delta \). For \( \lambda = 0, \alpha = 0 \) and \( t = -1 \) in \( M(\lambda, \alpha, t) \), we get the class the class \( \mathcal{S}(0, -1) \) studied by Sakaguchi [11]. A function \( f(z) \in \mathcal{S}(\alpha, -1) \) is called Sakaguchi function of order \( \alpha \).

*Corresponding author: B. Srutha Keerthi*, *E-mail: laya@svce.ac.in*
In this paper, we define the following class $M(\lambda, \phi, t)$, which is generalization of the class $M(\alpha, \phi, t)$.

**Definition 1.1** Let $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots$ be univalent starlike function with respect to ‘1’ which maps the unit disk $\Delta$ onto a region in the right half plane which is symmetric with respect to the real axis, and let $B_1 > 0$. Then function $f \in A$ is in the class $M(\lambda, \phi, t)$ if

$$
\left\{ \frac{(1-t)[\phi(z) - \phi(tz) + \lambda z(f(z) - tf(tz))]}{(1-\lambda)(f(z) + f(tz))} \right\} < \phi(z),
$$

\( |t| \leq 1, t \neq 0, 0 \leq \lambda \leq 1 \) \hspace{1cm} (1.4)

For $\lambda = 0$ in (1.4) we get the class $S(\phi, t)$ which was defined by Goyal and Goswami [3].

**Definition 1.2** Let $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots$ be univalent starlike function with respect to ‘1’ which maps the unit disk $\Delta$ onto a region in the right half plane which is symmetric with respect to the real axis, and let $B_1 > 0$. Then function $f \in A$ is in the class $S(\phi, t)$ if

$$
\left\{ \frac{(1-t)zf'(z)}{f(z) - f(tz)} \right\} < \phi(z), \hspace{1cm} |t| \leq 1, t \neq 1
$$

(1.5)

Again $T(\phi, t)$ denote the subclass of $A$ consisting functions $f(z)$ such that $zf'(z) \in S(\phi, t)$.

When $\phi(z) = (1+Az)/(1+Bz), (-1 \leq B < A \leq 1)$, we denote the subclasses $S(\phi, t)$ and $T(\phi, t)$ by $S[A, B, t]$ and $T[A, B, t]$ respectively.

Obviously $S(\phi, 0) = S(\phi)$. When $t = -1$, then $S(\phi, -1) = S(\phi)$, which is a known class studied by Shanmugam et al. [12]. For $t = 0$ and $\phi(z) = (1+Az)/(1+Bz), (-1 \leq B < A \leq 1)$, the subclass $S(\phi, t)$ reduces to the class $S[A, B]$ studied by Janowski [4]. For $0 \leq \alpha < 1$ let $S'(\alpha, t) := S[1-2\alpha, -1; t]$, which is a known class studied by Owa et al. [10]. Also, for $t = -1$ and $\phi(z) = \frac{1 + (1-2\alpha)z}{1-z}$, our class reduces to a known class $S(\alpha, -1)$ studied by Cho et al. ([1], see also [10]).

In the present paper, we obtain the Fekete-Szegö inequality for the functions in the subclass $M(\lambda, \phi, t)$. We also give application of our results to certain functions defined through convolution (or Hadamard product) and in particular, we consider the class $M(\lambda, \phi, t)$ defined by fractional derivatives.

To prove our main results, we need the following lemma:

**Lemma 1.3** [6] If $p(z) = 1 + c_1 z + c_2 z^2 + \cdots$ is an analytic function with positive real part in $\Delta$, then

$$
|c_2 - \nu c_1| \leq \begin{cases} 
-4\nu + 2 & \text{if } \nu \leq 0, \\
2 & \text{if } 0 \leq \nu \leq 1, \\
4\nu - 2 & \text{if } \nu \geq 1.
\end{cases}
$$

When $\nu < 0$ or $\nu > 1$, the equality holds if and only if $p(z)$ is $(1+z)/(1-z)$ or one of its rotations. If $0 < \nu < 1$, then the equality holds if and only if $p(z)$ is $(1+z)/(1-z')$ or one of its rotations. If $\nu = 0$, the equality holds if and only if

$$
p(z) = \left( \frac{1 + \frac{1}{2} \lambda}{1-z} \right) \frac{1+z}{1-z} + \left( \frac{1 - \frac{1}{2} \lambda}{1-z} \right) \frac{1+z}{1-z} \hspace{1cm} (0 \leq \lambda \leq 1)
$$

or one of its rotations. If $\nu = 1$, the equality holds if and only if $p(z)$ is the reciprocal of one of the functions such that the equality holds in the case of $\nu = 0$.

Also the above upper bound is sharp, and it can be improved as follows when $0 < \nu < 1$: © 2011, IJMA. All Rights Reserved
Lemma: 1.4 \[5\] If \( p_1(z) = 1 + c_1 z + c_2 z^2 + \cdots \) is a function with positive real part, then
\[
|c_2 - \mu c_1^2| \leq 2 \max \{1, |2\mu - 1|\},
\]
where \( \mu \) is complex and the result is sharp for the functions given by
\[
p(z) = \frac{1 + z^2}{1 - z^2}, \quad p(z) = \frac{1 + z}{1 - z}.
\]

2. MAIN RESULTS

Our main result is contained in the following theorem:

**Theorem: 2.1** If \( f(z) \) given by (1.1) belongs to \( M(\lambda, \phi, t) \), then
\[
\left| a_3 - \mu a_1^3 \right| \leq \begin{cases} 
\frac{1}{(1 + 2\lambda)(2 + t)(1 - t)} \left[ B_2 + B_1 \left( \frac{1 + t}{1 - t} \right) - \frac{\mu B_1 (1 + 2\lambda)(2 + t)}{(1 + \lambda)^2 (1 - t)} \right] & \text{if } \mu \leq \sigma_1 \\
B_1 \left( 1 + 2\lambda)(2 + t)(1 - t) \right)^{-1} & \text{if } \sigma_1 \leq \mu \leq \sigma_2 \\
\frac{1}{(1 + 2\lambda)(2 + t)(1 - t)} \left[ B_2 + B_1 \left( \frac{1 + t}{1 - t} \right) - \frac{\mu B_1 (1 + 2\lambda)(2 + t)}{(1 + \lambda)^2 (1 - t)} \right] & \text{if } \mu \geq \sigma_2
\end{cases}
\]

where
\[
\sigma_1 = \frac{(1 + \lambda)^2 (1 - t)}{B_1 (1 + 2\lambda)(2 + t)} \left\{ -1 + \frac{B_2}{B_1} \left( \frac{1 + t}{1 - t} \right) \right\}
\]
and
\[
\sigma_2 = \frac{(1 + \lambda)^2 (1 - t)}{B_1 (1 + 2\lambda)(2 + t)} \left\{ 1 + \frac{B_2}{B_1} \left( \frac{1 + t}{1 - t} \right) \right\}.
\]

The result is sharp.

**Proof:** Let \( f \in M(\lambda, \phi, t) \). Then there exists a Schwarz function \( w(z) \in A \) such that
\[
\frac{(1 - t)\lambda z^2 \phi'(z) + z \phi'(z)}{(1 - \lambda)(f(z) - f(tz)) + \lambda z(f'(z) - tf'(tz))} = \phi(w(z))
\]
\[
(z \in \Delta; |t| \leq 1, t \neq 0, 1 \leq \lambda \leq 1)
\]
(2.1)

If \( p_1(z) \) is analytic and has positive real part in \( \Delta \) and \( p_1(0) = 1 \), then
\[
p_1(z) = \frac{1 + w(z)}{1 - w(z)} = 1 + c_1 z + c_2 z^2 + \cdots \quad (z \in \Delta).
\]
(2.2)

From (2.2), we obtain
\[
w(z) = \frac{c_1}{2} z + \frac{1}{2} \left( \frac{c_2 - c_1^2}{2} \right) z^2 + \cdots.
\]
(2.3)
which gives

\[
b_1 = (1+\lambda)(1-t)a_2 \quad \text{and} \quad b_2 = (1+\lambda)^2(t^2-1)a_2 + (1+2\lambda)(2-t-t^2)a_3.
\]

(2.5)

Since \( \phi(z) \) is univalent and \( p < \phi \), therefore using (2.3), we obtain

\[
p(z) = \phi(w(z)) = 1 + B_1 c_1 z + \left( \frac{1}{2} - \frac{c_2}{2} \right) B_1 + \frac{1}{4} c_1^2 B_2 \right) z^2 + \cdots \quad (z \in \Delta),
\]

(2.6)

Now from (2.4), (2.5) and (2.6), we have

\[
(1+\lambda)(1-t)a_2 = \frac{B_1 c_1}{2},
\]

\[
(1+\lambda)^2(t^2-1)a_2 + (1+2\lambda)(2-t-t^2)a_3 = \frac{1}{2} \left( c_2 - \frac{c_1^2}{2} \right) B_1 + \frac{1}{4} c_1^2 B_2,
\]

\[
|t| \leq 1, \ t \neq 1, \ 0 \leq \lambda \leq 1.
\]

Therefore we have

\[
a_3 - \mu a_2^2 = \frac{B_1}{2(1+2\lambda)(2+t)(1-t)} \left[ c_2 - \nu c_1^2 \right]
\]

(2.7)

where

\[
\nu = \frac{1}{2} \left[ 1 - \frac{B_2}{B_1} - B_1 \left( \frac{1+t}{1-t} \right) + \mu B_1 \left( \frac{1+2\lambda}{1-t} \right) \right].
\]

Our result now follows by an application of Lemma 1.3. To shows that these bounds are sharp, we define the functions \( K_{\phi_n} \) \((n = 2, 3 \ldots)\) by

\[
(1-t)[\lambda z^2 K_{\phi_n}(z) + zK_{\phi_n}'(z)]
\]

\[
(1-\lambda)[K_{\phi_n}(z) - K_{\phi_n}(t z)] + \lambda z[K_{\phi_n}'(z) - tK_{\phi_n}'(t z)]
\]

\[
= \phi(z^{-1}),
\]

\[
K_{\phi_n}(0) = 0 = [K_{\phi_n}]'(0) - 1
\]

and the function \( F_\eta \) and \( G_\eta \) \((0 \leq \eta \leq 1)\) by

\[
(1-t)[\lambda z^2 F_\eta(z) + zF_\eta'(z)]
\]

\[
(1-\lambda)[F_\eta(z) - F_\eta(t z)] + \lambda z[F_\eta'(z) - tF_\eta'(t z)]
\]

\[
= \phi \left( \frac{z(\eta + z)}{(1+\eta z)} \right),
\]

\[
F_\eta(0) = 0 = [F_\eta]'(0) - 1
\]

and

\[
(1-t)[\lambda z^2 G_\eta(z) + zG_\eta'(z)]
\]

\[
(1-\lambda)[G_\eta(z) - G_\eta(t z)] + \lambda z[G_\eta'(z) - tG_\eta'(t z)]
\]

\[
= \phi \left( \frac{-z(\eta + z)}{(1+\eta z)} \right),
\]

\[
G_\eta(0) = 0 = [G_\eta]'(0) - 1.
\]

Obviously the functions \( K_{\phi_n}, F_\eta, G_\eta \in M(\lambda, \alpha, t) \). Also we write \( K_\phi := K_{\phi_1}. \) If \( \mu < \sigma_1 \) or \( \mu > \sigma_2 \), then equality holds if and only if \( f \) is \( K_\phi \) or one of its rotations. When \( \sigma_1 < \mu < \sigma_2 \), the equality holds if and only if \( f \) is \( K_{\phi_n} \) or one of its...
rotations. \( \mu = \sigma_1 \) then equality holds if and only if \( f \) is \( F_n \) or one of its rotations. \( \mu = \sigma_2 \) then equality holds if and only if \( f \) is \( G_n \) or one of its rotations.

If \( \sigma_1 \leq \mu \leq \sigma_2 \), in view of Lemma 1.3, Theorem 2.1 can be improved.

**Theorem: 2.2** Let \( f(z) \) given by (1.1) belongs to \( \mathcal{M}(\lambda, \alpha, t) \) and \( \sigma_3 \) be given by

\[
\sigma_3 = \frac{1}{B_1} \left[ \frac{(1 + \lambda)^2(1 - t)}{(1 + 2\lambda)(2 + t)} \right] + \frac{B_2}{B_1} \left( \frac{1 + t}{1 - t} \right)
\]

If \( \sigma_1 < \mu \leq \sigma_3 \), then

\[
\left| a_3 - \mu a_2^2 \right| \leq \frac{B_1}{(1 + 2\lambda)(2 + t)(1 - t)}.
\]

If \( \sigma_3 < \mu \leq \sigma_2 \), then

\[
\left| a_3 - \mu a_2^2 \right| \leq \frac{B_1}{(1 + 2\lambda)(2 + t)(1 - t)}.
\]

For \( \lambda = 0 \) in Theorem 2.1 we get Fekete-Szegö inequality for functions to be in the class \( \mathcal{S}^*(\phi, t) \) which was given by Goyal and Goswami [3].

**Theorem: 2.3** If \( f(z) \) is given by (1.1) belongs to \( \mathcal{M}(\lambda, \phi, t) \) then

\[
\left| a_3 - \mu a_2^2 \right| \leq \frac{B_1}{(1 + 2\lambda)(2 + t)(1 - t)} \max \left\{ \left| \frac{B_2}{B_1} + \frac{B_1(1 + t)}{(1 - t)} - \frac{(1 + 2\lambda)(2 + t)(1 - t)}{(1 + \lambda)^2(1 - t)} \mu B_1 \right| \right\}.
\]

The result is sharp.

**Proof:** By applying the Lemma 1.4 in (2.7) we get Theorem 2.3. The result is sharp for the functions defined by

\[
\frac{(1 - t)[\lambda z^2 f'(z) + zf(z)]}{(1 - \lambda)[f(z) - f(tz)] + \lambda z[f'(z) - tf'(tz)]} = \phi(z^2)
\]

and

\[
\frac{(1 - t)[\lambda z^2 f'(z) + zf(z)]}{(1 - \lambda)[f(z) - f(tz)] + \lambda z[f'(z) - tf'(tz)]} = \phi(z).
\]

### 3. APPLICATIONS TO FUNCTIONS DEFINED BY FRACTIONAL DERIVATIVES

For two analytic functions \( f(z) = z + \sum_{n=0}^{\infty} a_n z^n \) and \( g(z) = z + \sum_{n=0}^{\infty} g_n z^n \), their convolution (or Hadamard product) is defined to be the function \( (f * g)(z) = z + \sum_{n=0}^{\infty} a_n g_n z^n \). For a fixed \( g \in \mathcal{A} \), let \( \mathcal{M}^f(\lambda, \phi, t) \) be the class of functions \( f \in \mathcal{A} \) for which \( (f * g) \in \mathcal{M}(\lambda, \phi, t) \).

**Definition: 3.1** Let \( f(z) \) be analytic in a simply connected region of the \( z \)-plane containing origin. The fractional derivative of \( f \) of order \( \delta \) is defined by

\[
\frac{d^\delta}{dz^\delta} f(z) = \frac{1}{\Gamma(1 - \delta)} \int_0^z (z - w)^{1-\delta} f'(w) \, dw.
\]
where the multiplicity of \((z - \zeta)^\delta\) is removed by requiring that \(\log(z - \zeta)\) is real for \((z - \zeta) > 0\).

Using Definition 3.1, Owa and Srivastava (see [7, 8]; see also [13, 14]) introduced a fractional derivative operator \(\Omega^\delta : \mathcal{A} \to \mathcal{A}\) defined by

\[
(\Omega^\delta f)(z) = (2 - \delta)z^\delta D^\delta_z f(z), \quad (\delta \neq 2, 3, 4, \ldots).
\]

The class \(M^\delta(\lambda, \phi, t)\) consists of the functions \(f \in \mathcal{A}\) for which \(\Omega^\delta f \in M(\lambda, \phi, t)\). The class \(M^\delta(\lambda, \phi, t)\) is a special case of the class \(M^\delta(\lambda, \phi, t)\) when \((z, z_1)\) \((n \geq 1) \Rightarrow \sum_{n=2}^{\infty} \frac{\Gamma(n+1)}{\Gamma(n+1-\delta)} z^n = 1\).

Now applying Theorem 2.1 for the function \((f \ast g)(z) = z + g_2 z^2 + g_3 z^3 + \cdots\), we get following theorem after an obvious change of the parameter \(\mu\):

**Theorem: 3.2** Let \(g(z) = z + \sum_{n=2}^{\infty} g_n z^n\) \((g_n > 0)\). If \(f(z)\) is given by (1.1) belongs to \(M^\delta(\lambda, \phi, t)\) then

\[
|a_3 - \mu a_2^2| \leq \frac{1}{g_3(1+2\lambda)(2+t)(1-t)} \left[ B_2 + B_1 \frac{1+t}{1-t} - \mu \frac{g_3}{g_2} B_1^2 \frac{(1+2\lambda)(2+t)}{(1+\lambda)^2(1-t)} \right] \quad \text{if } \mu \leq \eta_1
\]

\[
- \frac{1}{g_3(1+2\lambda)(2+t)(1-t)} \left[ B_2 + B_1 \frac{1+t}{1-t} - \mu \frac{g_3}{g_2} B_1^2 \frac{(1+2\lambda)(2+t)}{(1+\lambda)^2(1-t)} \right] \quad \text{if } \eta_1 \leq \mu \leq \eta_2
\]

\[
- \frac{1}{g_3(1+2\lambda)(2+t)(1-t)} \left[ B_2 + B_1 \frac{1+t}{1-t} - \mu \frac{g_3}{g_2} B_1^2 \frac{(1+2\lambda)(2+t)}{(1+\lambda)^2(1-t)} \right] \quad \text{if } \mu \geq \eta_2
\]

where

\[
\eta_1 = \frac{g_2^2(1+\lambda)^2(1-t)}{B_1 g_3(1+2\lambda)(2+t)} \left[ -1 + \frac{B_2}{B_1} \frac{1+t}{1-t} \right],
\]

\[
\eta_2 = \frac{g_2^2(1+\lambda)^2(1-t)}{B_1 g_3(1+2\lambda)(2+t)} \left[ 1 + \frac{B_2}{B_1} \frac{1+t}{1-t} \right].
\]

The result is sharp.

Since

\[
\Omega^\delta f(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(n+1)}{\Gamma(n+1-\delta)} z^n.
\]

We have

\[
g_2 := \frac{\Gamma(3)\Gamma(2-\delta)}{\Gamma(3-\delta)} = \frac{2}{2-\delta},
\]

and

\[
g_3 := \frac{\Gamma(4)\Gamma(2-\delta)}{\Gamma(4-\delta)} = \frac{6}{(2-\delta)(3-\delta)}.
\]

For \(g_2, g_3\) given by (3.2) and (3.3) respectively, Theorem 3.2 reduces to the following:

**Theorem: 3.3** Let \(\delta < 2\). If \(f(z)\) is given by (1.1) belongs to \(M^\delta(\lambda, \phi, t)\) then
The result is sharp.

For the case $\lambda = 0$ in Theorem 3.2, we get Fekete-Szegö inequality for functions to be in the class $S(\phi, t)$ which are given by Goyal and Goswami [3].

Theorem: 3.4 Let $g(z) = z + \sum_{n=2}^{\infty} g_n z^n$ ($g_n > 0$). If $f(z)$ is given by (1.1) belongs to $M_{2}(\lambda, \phi, t)$ then

$$|a_3 - \mu a_2|^2 \leq \frac{(2-\delta)(3-\delta)}{6(1+2\lambda)(2+t)(1-t)} \left[ B_2 + B_1^2 \frac{1+t}{1-t} - \frac{3}{2} \mu \left( \frac{2-\delta}{3-\delta} \right) \frac{(1+2\lambda)(2+t)}{(1+\lambda)^2(1-t)B_1^2} \right] \text{ if } \mu \leq \eta_1^*$$

$$-\frac{(2-\delta)(3-\delta)}{6(1+2\lambda)(2+t)(1-t)} \left[ B_2 + B_1^2 \frac{1+t}{1-t} - \frac{3}{2} \mu \left( \frac{2-\delta}{3-\delta} \right) \frac{(1+2\lambda)(2+t)}{(1+\lambda)^2(1-t)B_1^2} \right] \text{ if } \eta_1^* \leq \mu \leq \eta_2^*$$

$$-\frac{(2-\delta)(3-\delta)}{6(1+2\lambda)(2+t)(1-t)} \left[ B_2 + B_1^2 \frac{1+t}{1-t} - \frac{3}{2} \mu \left( \frac{2-\delta}{3-\delta} \right) \frac{(1+2\lambda)(2+t)}{(1+\lambda)^2(1-t)B_1^2} \right] \text{ if } \mu \geq \eta_2^*$$

where

$$\eta_1^* = \frac{2}{3B_1} \left( \frac{3}{2} - \frac{\delta}{2} \right) \left( \frac{1}{(1+\lambda)^2(1-t)} + \frac{1}{(1+2\lambda)(2+t)} \right) \left[ -1 + \frac{B_2}{B_1} + B_1 \left( \frac{1+t}{1-t} \right) \right],$$

$$\eta_2^* = \frac{2}{3B_1} \left( \frac{3}{2} - \frac{\delta}{2} \right) \left( \frac{1}{(1+\lambda)^2(1-t)} + \frac{1}{(1+2\lambda)(2+t)} \right) \left[ 1 + \frac{B_2}{B_1} + B_1 \left( \frac{1+t}{1-t} \right) \right].$$

The result is sharp.

Theorem: 3.5 If $f(z)$ is given by (1.1) belongs to $M_{2}(\lambda, \phi, t)$ then

$$|a_3 - \mu a_2|^2 \leq \frac{B_1}{(1+2\lambda)(2+t)(1-t)g_3} \max \left\{ 1, \frac{B_2}{B_1} + B_1 \left( \frac{1+t}{1-t} \right) - \frac{6(1+2\lambda)(2+t)(2-\delta)}{4(1+\lambda)^2(1-t)(3-\delta)} B_1 \right\}.$$ 

The result is sharp.

Theorems 3.4, Theorem 3.5 were obtained by applying Lemma 1.4.

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