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MULTIGRID METHOD FOR SOLVING THE GENERALIZED EQUAL WIDTH WAVE EQUATION

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ABSTRACT

Numerical solution of the generalized equal width (GEW) equation is obtained by using the multigrid method based on finite difference method. The motion of a single solitary wave, interaction of two solitary waves and the Maxwellian initial condition pulse are studied using the proposed method. The numerical solutions are compared with the known analytical solutions. Using L_2, L_∞ error norms and conservative properties of mass, momentum and energy, accuracy and efficiency of the mentioned method will be established through comparison with other methods.

Keywords: *Multigrid Method, Finite Difference Method, GEW Equation Mathematics Subject Classification: 34K28, 65D05, 65D25.*

1. INTRODUCTION

The GEW equation is given by the following form:

$$u_t + \epsilon u^p u_x - \mu u_{xxt} = 0, \tag{1}$$

with the physical boundary condition s$u \to 0$ as $x \to \pm\infty$, where t is time and x is the space coordinate, p is a positive integer, ε and μ are a positive parameter. For this study boundary conditions are chosen

$$\begin{aligned} &u(a,t) = 0, \quad u(b,t) = 0, \\ &u_x(a,t) = 0, \; u_x(b,t) = 0, \\ &u_{xxt}(a,t) = 0, \; u_{xxt}(b,t) = 0, \end{aligned} \tag{2}$$

and the initial condition as

$$u(x,0) = f(x), a \le x \le b,$$

where f is a localized disturbance inside the considered interval and it will be determined later. In the fluid problems,$u$is related to the wave amplitude of the water surface or similar physical quantity. In the plasma applications,$u$is negative of the electrostatic potential. Therefore, the solitary wave solution of equation (1) has an important role in the motion of non-linear dispersive waves.

In the literature, the GEW equation has been many studies. Hamdi *et al.* [1] got the exact solitary wave solutions of the generalized EW and the generalized EW-Burges equation. The collocation method based on quadratic was presented by Evans and Raslan [2], Raslan [3] introduced cubic B-splines to get the numerical solution of the GEW equation. Petrov-Galerkin finite element method using a quadratic B-spline function as the trial function was investigated for solving the GEW equation by Roshan [4]. The GEW equation was solved numerically using the meshless method based on a global collocation with standard types of radial basis functions (RBFs) by Panahipour [5]. Taghizadeh *et al.* [6] have constructed the homogeneous balance method to obtain the exact travelling wave solutions of the GEW equation. Battal Gazi Karakoc and Geyikli [7] solved the GEW equation by a cubic B-splines Galerkin appoach.

In this paper, we study the GEW equation by using the multigrid method. We derive a numerical method based on the multigrid technique based on finite difference method for obtainning the numerical solution of GEW equation in Section 2. In Section 3, we introduce the numerical results for solving the GEW equation through some well known standard problems.

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2. NUMERICAL METHOD

We apply the full multigrid algorithm for the GEW equation as shown in [8-11]. Assuming the initial condition $u(x,0)=f(x)$ and the solution $u(x,t)$, $a\le x\le b, 0\le t\le T$ has the usual partition with a space step size Δx and a time step size Δt $(t_{K+1}=t_K+\Delta t, K=0,1,2,...)$. For convenience, the GEW equation (1) is rewritten in the form:

$$u_t+\frac{\varepsilon}{p+1}(u^{p+1})_x-\mu u_{xxt}=0, \tag{3}$$

The back-time and centre-space difference for equation (3) is

$$\frac{u_{i,n}^k-u_{i,n-1}^k}{\Delta t}+\frac{\varepsilon}{p+1}\frac{\left(u_{i+1,n}^k\right)^{p+1}-\left(u_{i-1,n}^k\right)^{p+1}}{2\Delta x} -\mu\frac{\left(u_{i+1,n}^k-u_{i+1,n-1}^k\right)-2\left(u_{i,n}^k-u_{i,n-1}^k\right)+\left(u_{i-1,n}^k-u_{i-1,n-1}^k\right)}{(\Delta x)^2(\Delta t)}=0, \tag{4}$$

where $i=1,\dots,2^k-1, n=1,\dots,2^k, k=1,\dots,M$ for a set grids $G^1,G^2,\dots,G^M$.

Step-1: $K=0, u(x,0)=f(x)$.

Step-2: Starting from$k=1$ in the coarse grid, we can calculate the approximate value $u_{i,n}$ at two points using equation (4) leading to:

$$u_{i,n}^1=u_{i,n-1}^1+\frac{1}{(2(\Delta x)^2+4\mu)}\Big[2\mu\left(u_{i+1,n}^1+u_{i-1,n}^1-u_{i+1,n-1}^1-u_{i-1,n-1}^1\right) -\frac{\varepsilon}{p+1}(\Delta x)(\Delta t)\left(\left(u_{i+1,n}^1\right)^{p+1}-\left(u_{i-1,n}^1\right)^{p+1}\right)\Big];i=1,n=1,2. \tag{5}$$

The right hand side for equation (5) can be computed using the initial and boundary conditions.

Step-3: Interpolating the grid functions from the coarse grid to fine grid using linear interpolation I_k^{k+1}, in which

$$u^{k+1}=I_k^{k+1}u^k, \tag{6}$$

Step-4: Doing relaxation sweep on G^{k+1} using the point relaxation

$$u_{i,n}^{k+1}=u_{i,n-1}^k+\frac{1}{(2(\Delta x)^2+4\mu)}\Big[2\mu\left(u_{i+1,n}^k+u_{i-1,n}^k-u_{i+1,n-1}^k-u_{i-1,n-1}^k\right) -\frac{\varepsilon}{p+1}(\Delta x)(\Delta t)\left(\left(u_{i+1,n}^k\right)^{p+1}-\left(u_{i-1,n}^k\right)^{p+1}\right)\Big]; \quad i=1,\dots,2^{k+1}-1, n=1,\dots,2^{k+1}. \tag{7}$$

Step-5: Computing the residuals r^{k+1} on $G^{k+1}$and inject them into G^k using full weighting restriction I_{k+1}^k to get r^k as:

$$r^k=I_{k+1}^k r^{k+1}, \tag{8}$$

Step-6: Computing an approximate solution of error e^k.

Step-7: Interpolating the solution of errore^konto G^{k+1},

$$e^{k+1}=I_k^{k+1}e^k, \tag{9}$$

and adding it to u^{k+1} which is the approximate value of$u$ on the fine grid with $k=2$.

By taking this solution on coarse grid and repeating steps 3-7, we obtain the approximate values of$u$on the grid with $k=3$ and so $k=4,5,\dots,M$ the final value is the solution at the time level $K+1$.

Step-8: $K=K+1$, go to step 2 (lead to the solution at higher time level as needed).

3. NUMERICAL EXAMPLES AND RESULTS

In this section, numerical solutions of GEW equation are obtained for standard problems as: the motion of single solitary wave, interaction of two solitary waves and the development of Maxwellian initial condition into solitary waves.

The analytical solution of the GEW equation (1) can be obtained by using the transformation $u(x,t)=f(x-ct)$, where$c$ represents the constant velocity of the wave travelling in the positive direction of the$x$ -axis and the analytic solution can be expressed in the form [2, 3, 4, 7]

$$u(x,t)=\sqrt[p]{\frac{c(p+1)(p+2)}{2\varepsilon}sech^2\left[\frac{p}{2\sqrt{\mu}}(x-ct-x_0)\right]}, \tag{10}$$

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Where x_0 is an arbitrary constant. For the GEW equation, it is important to discuss the following three invariant conditions given in [2, 3, 4, 7] which respectively correspond to conversation of mass, momentum and energy:

$$I_1 = \int_a^b u\,dx = \Delta x \sum_{i=1}^{N} u_{i,n},$$
$$I_2 = \int_a^b (u^2 + \mu(u_x)^2)dx = \Delta x \sum_{i=1}^{N} \left(\left(u_{i,n}\right)^2 + \mu\left((u_x)_{i,n}\right)^2\right), \quad (11)$$
$$I_3 = \int_a^b u^{p+1}\,dx = \Delta x \sum_{i=1}^{N} (u_{i,n})^{p+2}.$$

The accuracy of the method is measured by both the L_2 error norm

$$L_2 = \|u^{exact} - u_N\|_2 = \sqrt{\Delta x \sum_{i=0}^{N} \left|u_i^{exact} - (u_N)_i\right|^2}\,, \quad (12)$$

and the L_∞ error norm

$$L_\infty = \|u^{exact} - u_N\|_\infty = \max_i |u_i^{exact} - (u_N)_i|, \quad (13)$$

to show how good the numerical results in comparison with the exact results.

3.1The Motion of Single Solitary Wave

Consider equation (1) with boundary conditions (2) and the initial condition

$$u(x,0) = \sqrt[p]{\frac{c(p+1)(p+2)}{2\varepsilon}} sech^2\left[\frac{p}{2\sqrt{\mu}}(x - x_0)\right], \quad (14)$$

different values of p, c and amplitude $\sqrt[p]{\frac{c(p+1)(p+2)}{2\varepsilon}}$ and the same values of $\Delta x = 0.1, \Delta t = 0.2, \varepsilon = 3, \mu = 1$ and $x_0 = 30$ over the interval [0, 80] is considered to coincide with papers[4, 7].

Case-1: we choose the quantities $p = 2, c = \frac{1}{32}$ and $\frac{1}{2}$. Hence, the solitarywave has amplitude 0.25 and 1, respectively.

The calculated quantities of the invariants are presented in Tables 1, 2. As can be seen in Table 1, 2, three invariants are almost constant as the time increases and we have found out that the quantity of the error norms L_2 and L_∞ are reasonably small.

Table-1: Invariants and error norms for single solitary wave with $p = 2$, amplitude $= 0.25, \Delta x = 0.1$, $\Delta t = 0.2, \varepsilon = 3, \mu = 1$ and $0 \le x \le 80$.

t	I_1	I_2	I_3	$L_2 \times 10^5$	$L_\infty \times 10^5$
0	0.7853966199	0.1666662968	0.005208333331	0.000000000	0.00000
5	0.7853966129	0.1666662138	0.005208328142	0.036593427	0.03523
10	0.7853966027	0.1666661274	0.005208322719	0.073126604	0.07055
15	0.7853965918	0.1666660402	0.005208317288	0.109768130	0.10597
20	0.7853965730	0.1666659518	0.005208311746	0.146341818	0.14142

Table-2: Invariants and error norms for single solitary wave with$p = 2$, amplitude $= 1, \Delta x = 0.1$, $\Delta t = 0.2, \varepsilon = 3, \mu = 1$ and $0 \le x \le 80$.

t	I_1	I_2	I_3	$L_2 \times 10^3$	$L_\infty \times 10^3$
0	3.141586518	2.666660747	1.333333333	0.000000000	0.0000000
5	3.141586479	2.666501571	1.333013988	0.099074261	0.1031408
10	3.141586472	2.666022600	1.332694855	0.198280452	0.2065635
15	3.141586474	2.665703797	1.332375906	0.297634570	0.3097387
20	3.141586466	2.665385179	1.332057146	0.397150640	0.4121295

Case-2: if $p = 3, c =$0.001 and 0.3, the solitary wave has amplitude 0.15 and 1. The obtained resultsare given in Tables 3,4. It is observed from Table 3, 4 that three invariants are nearly unchanged as the time processes and the values of the error norms L_2 and $L_\infty$are adequately small.

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Table-3: Invariants and error norms for single solitary wave with $p = 3$, amplitude $= 0.15$, $\Delta x = 0.1$, $\Delta t = 0.2, \varepsilon = 3, \mu = 1$ and $0 \le x \le 80$.

t	I_1	I_2	I_3	$L_2 \times 10^7$	$L_\infty \times 10^7$
0	0.4189155504	0.05497501959	0.00007330778	0.0000000000	0.000
5	0.4189155480	0.05497501753	0.00007330777	0.1727595028	0.178
10	0.4189155417	0.05497501594	0.00007330777	0.3343260913	0.340
15	0.4189155392	0.05497501447	0.00007330776	0.5053116238	0.506
20	0.4189155318	0.05497501238	0.00007330775	0.6821458112	0.691

Table-4: Invariants and error norms for single solitary wave with$p = 3$, amplitude $= 1, \Delta x = 0.1$, $\Delta t = 0.2, \varepsilon = 3, \mu = 1$ and $0 \le x \le 80$.

t	I_1	I_2	I_3	$L_2 \times 10^3$	$L_\infty \times 10^3$
0	2.804364205	2.463133856	0.9855655469	0.0000000000	0.0000000
5	2.804364169	2.462970930	0.9854023227	0.0490037002	0.0611103
10	2.804364161	2.462808123	0.9852392180	0.0981543350	0.1220840
15	2.804364127	2.462645334	0.9850761297	0.1474675836	0.1827248
20	2.804364080	2.462482607	0.9849131086	0.1969502601	0.2432723

Case-3: we take the parameters$p = 4, c =$0.02. This leads to amplitude1. The obtained results are listed in Table 5 which clearly shows that the change of the invariants from their initial count are small. Also, we observed that the quantity of the error normsL_2 and L_∞ is sensibly small.

Table-5: Invariants and error norms for single solitary wave with$p = 4$, amplitude $= 1, \Delta x = 0.1$, $\Delta t = 0.2, \varepsilon = 3, \mu = 1$ and $0 \le x \le 80$.

t	I_1	I_2	I_3	$L_2 \times 10^3$	$L_\infty \times 10^3$
0	2.622052433	2.355649252	0.7853981633	0.000000000	0.0000000
5	2.622052366	2.355555685	0.7853043758	0.030041836	0.0430082
10	2.622052330	2.355462162	0.7852106141	0.060207252	0.0857518
15	2.622052332	2.355368741	0.7851169641	0.090498118	0.1281052
20	2.622052336	2.355275363	0.7850233414	0.120915385	0.1713846

The comparison of our results with the ones obtained by Petrov-Galerkin method [4] and cubic B-splines Galerkin [7] at$t = 20$ is given in Table 6. From this table, we can conclude that the values of three invariants are to be close to each other. The magnitude of our error norms is smaller than the ones given by [4, 7] for p = 2, 3 and 4.

Table-6: For$p = 2, 3$ and 4, Comparisons of result for the single solitary wave with $\Delta x = 0.1$, $\Delta t = 0.2$, $\varepsilon = 3, \mu = 1$ and $0 \le x \le 80$.

	p	2	3	4
I_1	Petrov-Galerkin (quadratic)[4]	0.7853980	0.4189160	2.6220600
	Galerkin (cubic) [7]	0.7853968	0.4189154	2.6327833
	Present method	0.7853965	0.4189155	2.6220523
I_2	Petrov-Galerkin (quadratic)[4]	0.1666690	0.0549783	2.3561500
	Galerkin (cubic) [7]	0.1666663	0.0549805	2.3730032
	Present method	0.1666659	0.0549750	2.3552753
I_3	Petrov-Galerkin (quadratic)[4]	0.0052082	0.0000733	0.7853440
	Galerkin (cubic) [7]	0.0052082	0.0000733	0.8023383
	Present method	0.0052083	0.0000733	0.7850233
$L_2 \times 10^3$	Petrov-Galerkin (quadratic)[4]	0.0025017	0.0000640	2.3049900
	Galerkin (cubic) [7]	0.0783378	0.0028248	8.9061700
	Present method	0.0014634	0.0000682	0.1209153
$L_\infty \times 10^3$	Petrov-Galerkin (quadratic)[4]	0.0027516	0.0000820	1.8828500
	Galerkin (cubic) [7]	0.0444850	0.0018329	8.2199100
	Present method	0.0014142	0.0000691	0.1713846

3.2 Interaction of Two Solitary Waves

We use the initial condition

$$u(x,0) = \sum_{i=1}^{2} \sqrt[p]{\frac{c_i(p+1)(p+2)}{2\varepsilon} sech^2\left[\frac{p}{2\sqrt{\mu}}(x - x_i)\right]}, \tag{15}$$

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which produces two positive solitary waves having different amplitudes of magnitudes 1 and 0.5 at the samedirection, where c_i and x_i, $i = 1, 2$ are arbitrary constants.

Three sets of parameters have been constructed by taking the values of first parameters $p = 2, c_1 = 0.5$ and$c_2 = 0.125$, second parameters$p = 3, c_1 = 0.3$ and $c_2 = 0.0375$ and third parameters $p = 4, c_1 = 0.2$ and $c_2 = \frac{1}{80}$. The computer program is run until time$t = 60, 100$ and 120, respectively. The other parameters are considered as $\Delta x = 0.1$, $\Delta t = 0.25, \varepsilon = 3, \mu = 1, x_1 = 15, x_2 = 30$ and $0 \le x \le 80$ to coincide with papers [4, 7]. To prove the conserved quantities of the invariantsI_1, I_2 andI_3, the calculated values are given in Table 7, 8, 9 which show that the invariant quantities are compatible with [7].

Table-7: The invariants for interaction of two solitary wave with $p = 2, c_1 = 0.5, c_2 = 0.125, x_1 = 15, x_2 = 30$, $\Delta x = 0.1, \Delta t = 0.2, \varepsilon = 3, \mu = 1$ and $0 \le x \le 80$.

	I_1		I_2		I_3	
t	Present method	[7]	Present method	[7]	Present method	[7]
0	4.712379141	4-71237	3.333328363	3.33332	1.416669724	1.41666
10	4.712378516	4.71236	3.333075144	3.33331	1.416419291	1.41665
20	4.712378540	4.71235	3.332822114	3.33332	1.416169063	1.41666
30	4.712378509	4.71260	3.332569134	3.33416	1.415918898	1.41758
40	4.712378494	4.71234	3.332316258	3.33345	1.415668856	1.41699
50	4.712378491	4.71210	3.332063495	3.33290	1.415418902	1.41652
60	4.712378530	4.71213	3.331810916	3.33296	1.415169155	1.41651

Table-8: The invariants for interaction of two solitary wave with$p = 3, c_1 = 0.3, c_2 = 0.0375,\ x_1 = 15, x_2 = 30$, $\Delta x = 0.1, \Delta t = 0.2, \varepsilon = 3, \mu = 1$ and $0 \le x \le 80$.

	I_1		I_2		I_3	
t	Present method	[7]	Present method	[7]	Present method	[7]
0	4.206537586	4.20653	3.079567950	3.07987	1.016366206	1.01634
10	4.206537036	4.20653	3.079440079	3.07989	1.016238629	1.01634
20	4.206537063	4.20652	3.079312391	3.07988	1.016111204	1.01634
30	4.206537080	4.20653	3.079184721	3.07991	1.015983819	1.01634
40	4.206537125	4.20677	3.079057141	3.07050	1.015856513	1.01634
50	4.206537123	4.20793	3.078929536	3.07362	1.015729168	1.01633
60	4.206537137	4.20616	3.078802008	3.07947	1.015601922	1.01633
70	4.206537185	4.20558	3.078674537	3.07863	1.015474709	1.01634
80	4.206537175	4.20509	3.078547045	3.07800	1.015347499	1.01633
90	4.206537170	4.20490	3.078419573	3.07777	1.015220316	1.01633
100	4.206537204	4.20503	3.078292202	3.07797	1.015093215	1.01634

Table-9: Theinvariants for interaction of two solitary wave with$p = 4, c_1 = 0.2, c_2 = \frac{1}{80}, x_1 = 15, x_2 = 30$, $\Delta x = 0.1, \Delta t = 0.2, \varepsilon = 3, \mu = 1$ and $0 \le x \le 80$.

	I_1		I_2		I_3	
t	Present method	[7]	Present method	[7]	Present method	[7]
0	3.933078206	3.93307	2.944562781	2.94521	0.7976713094	0.79766
10	3.933077741	3.93310	2.944489638	2.94529	0.7975980766	0.79773
20	3.933077729	3.93309	2.944416562	2.94527	0.7975248961	0.79771
30	3.933077759	3.93309	2.944343551	2.94527	0.7974517703	0.79770
40	3.933077788	3.93310	2.944270522	2.94529	0.7973786281	0.79773
50	3.93307770	3.93320	2.944197494	2.94553	0.7973054617	0.79795
60	3.933077768	3.93388	2.944124470	2.94703	0.7972323156	0.79942
70	3.933077796	3.93601	2.944051552	2.94212	0.7971592672	0.79505
80	3.933077795	3.93285	2.943978608	2.94529	0.7970862078	0.79862
90	3.933077870	3.93222	2.943905767	2.94436	0.7970132516	0.79812
100	3.933077870	3.93161	2.943832843	2.94366	0.7969402008	0.79805
110	3.933077872	3.93095	2.943805678	2.94291	0.7968864548	0.79799
120	3.933077871	3.93026	2.943798537	2.94212	0.7967767656	0.79794

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3.3 The Maxwellian Initial Condition

Evolution of a train of solitary waves of the GEW equation has been studied using the Maxwellian initial condition

$$u(x,0) = e^{-(x-20)^2}, \tag{16}$$

and the boundary conditions $u(0,t) = u_x(0,t) = u(40,t) = u_x(40,t) = 0$.

It is known that the behaviour of the solution with the Maxwellian condition (16) depends on the values of μ. So we have considered various values for μ. The computations are carried out for the cases $\mu = 0.1, 0.05, 0.025$ and 0.01 which are used in the earlier papers [3, 4]. The numerical conserved quantities with $\mu = 0.1, 0.05, 0.025$ and 0.01 are given in Table 10, 11, 12. It is observed that the obtained values of the invariants remain almost constant during the computer run.

Table-10: Invariants of GEW equation using the Maxwellian condition, $p = 2, \mu = 0.1, 0.05, 0.025, 0.01$.

t	μ	I_1	I_2	I_3	μ	I_1	I_2	I_3
0	0.1	1.7724503	1.3786858	0.88622692	0.025	1.7724503	1.2846570	0.88622692
4		1.7724503	1.3770924	0.88460963		1.7724503	1.2835450	0.88287632
8		1.7724503	1.3755023	0.88300024		1.7724503	1.2824399	0.87957135
12		1.7724503	1.3739073	0.88139732		1.7724504	1.2813364	0.87629839
0	0.05	1.7724503	1.3159999	0.88622692	0.01	1.7724503	1.2658513	0.88622692
4		1.7724503	1.3150250	0.88373231		1.7724503	1.2646271	0.88203137
8		1.7724503	1.3140541	0.88125940		1.7724503	1.2634136	0.87792085
12		1.7724503	1.3130844	0.87880282		1.7724503	1.2622013	0.87386607

Table-11: Invariants of GEW equation using the Maxwellian condition, $p = 3, \mu = 0.1, 0.05, 0.025, 0.01$.

t	μ	I_1	I_2	I_3	μ	I_1	I_2	I_3
0	0.1	1.7724503	1.3786858	0.79266545	0.025	1.7724503	1.2846570	0.79266545
4		1.7724503	1.3782203	0.79135853		1.7724503	1.2839697	0.78959818
8		1.7724503	1.3777554	0.79005758		1.7724503	1.2832853	0.78657424
12		1.7724503	1.3772901	0.78876131		1.7724503	1.2826000	0.78357877
0	0.05	1.7724503	1.3159999	0.79266545	0.01	1.7724503	1.2658513	0.79266545
4		1.7724503	1.3154128	0.79050965		1.7724503	1.2650769	0.78861198
8		1.7724503	1.3148271	0.78837224		1.7724503	1.2643080	0.78464973
12		1.7724503	1.3142410	0.78624809		1.7724503	1.2635366	0.78074216

Table-12: Invariants of GEW equation using the Maxwellian condition, $p = 4, \mu = 0.1, 0.05, 0.025, 0.01$.

t	μ	I_1	I_2	I_3	μ	I_1	I_2	I_3
0	0.1	1.7724503	1.3786858	0.72360125	0.025	1.7724503	1.2846570	0.72360125
4		1.7724503	1.3783822	0.72249261		1.7724503	1.2841870	0.72072800
8		1.7724503	1.3780785	0.72138878		1.7724503	1.2837184	0.71789629
12		1.7724503	1.3777744	0.72028866		1.7724503	1.2832480	0.71509097
0	0.05	1.7724503	1.3159999	0.72360125	0.01	1.7724503	1.2658513	0.72360125
4		1.7724503	1.3156072	0.72167451		1.7724503	1.2653109	0.71962366
8		1.7724503	1.3152148	0.71976393		1.7724503	1.2647734	0.71574262
12		1.7724503	1.3148216	0.71786476		1.7724503	1.2642325	0.71191641

4. CONCLUSION

In this paper, we adapted the use of multigrid technique to study the GEW problem. We investigated our scheme through single solitary wave in which the analytic solution is known. The interaction of two solitary waves and Maxwellian initial condition where the analytic solutions are unknown during the interaction were studied by extending our scheme. By calculating the error norms L_2, L_∞ and conservative properties of mass, momentum and energy the performance and accuracy of the method were illustrated. The computed results showed that our scheme is a successful numerical technique for solving the GEW problem and can be also efficiently applied for solving a large number of physically important non-linear problems.

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