PRE*GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper a new class of generalized closed sets, namely p*g-closed sets is introduced in topological spaces. We find some basic properties and characterizations of p*g-closed sets.

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Key Words: g-closed sets, p*g-closed sets, g*p-closed sets, πgp-closed sets.

1. INTRODUCTION


In this paper we introduce a new class of sets called p*g-closed sets. We give characterizations of p*g-closed sets also investigate some fundamental properties of p*g-closed set.

2. PRELIMINARIES

Throughout this paper (X, τ) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. (X, τ) will be replaced by X if there is no changes of confusion. We recall the following definitions and results.

Definition 2.1: Let (X, τ) be a topological space. A subset A of X is said to be generalized closed [8] (briefly g-closed) if cl(A) ⊆ U whenever A ⊆ U and U is an open in (X, τ).

Definition 2.2: Let (X, τ) be a topological space and A ⊆ X. The generalized closure of A [6], denoted by clp(A) and is defined by the intersection of all g-closed sets containing A and generalized interior of A [6], denoted by intp(A) and is defined by union of all g-open sets contained in A.

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**Definition 2.3:** Let \((X, \tau)\) be a topological space and \(A \subseteq X\). Then

1. \(A\) is \(\alpha\)-open if \(A \subseteq \text{int}(\text{cl}(A))\) and \(\alpha\)-closed if \(\text{cl}(\text{int}(\text{cl}(A))) \subseteq A\) [15].
2. \(A\) is pre open if \(A \subseteq \text{int}(\text{cl}(A))\) and pre closed if \(\text{cl}(\text{int}(\text{cl}(A))) \subseteq A\) [11].
3. \(A\) is \(\alpha\)-pre open if \(A \subseteq \text{int}(\text{cl}(A))\) and \(\alpha\)-pre closed if \(\text{cl}(\text{int}(\text{cl}(A))) \subseteq A\) [20].
4. \(A\) is regular open if \(A = \text{int}(\text{cl}(A))\) and regular closed if \(A = \text{cl}(\text{int}(\text{cl}(A)))\) [21].
5. \(A\) is semi pre open if \(A \subseteq \text{int}(\text{cl}(A))\) and semi pre closed if \(\text{int}(\text{cl}(A)) \subseteq A\) [1].
6. \(\pi\)-closed set [26] if \(A\) is a finite intersection of regular closed sets. The complement of a \(\pi\)-closed set is called a \(\alpha\)-open set.

**Remark 2.6:**

\[
\text{regular closed} \rightarrow \pi\text{-closed} \rightarrow \text{closed} \rightarrow \text{g*-closed} \rightarrow \text{g-closed} \\
\alpha\text{-closed} \rightarrow \text{pre closed} \rightarrow \text{pre*closed} \\
\text{semi pre closed}
\]

**Definition 2.4:** Let \((X, \tau)\) be a topological space and \(A \subseteq X\). The pre closure of \(A\) [11], denoted by \(\text{pcl}(A)\), is defined by the intersection of all pre closed sets containing \(A\).

**Definition 2.5:** Let \((X, \tau)\) be a topological space. A subset \(A\) of \(X\) is said to be

1. \(\alpha\)-generalized closed set (briefly \(\alpha g\)-closed) [9] if \(\text{acl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).
2. generalized pre closed set (briefly \(gp\)-closed) [10] if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).
3. strongly generalized closed set (briefly \(g*\)-closed) [23] if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g\)-open in \((X, \tau)\).
4. generalized \(\ast\) pre closed set (briefly \(g*\)-closed) [24] if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g\)-open in \((X, \tau)\).
5. regular generalized closed set (briefly \(rg\)-closed) [17] if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular open in \((X, \tau)\).
6. weakly generalized closed set (briefly \(wg\)-closed) [14] if \(\text{cl}(\text{int}(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).
7. generalized pre regular closed set (briefly \(gpr\)-closed) [7] if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular open in \((X, \tau)\).
8. generalized semi preclosed set (briefly \(gsp\)-closed) [5] if \(\text{spcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).
9. pre semi closed set [25] if \(\text{spcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g\)-open in \((X, \tau)\).
10. \(\pi\)gp-closed set [18] if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\pi\)-open in \((X, \tau)\).
11. regular weakly generalized closed set (briefly \(rwg\)-closed) [14] if \(\text{cl}(\text{int}(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular open in \((X, \tau)\).
12. \(b\)-closed set [13] if \(\text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)) \subseteq A\).
13. \(b\*)\)-closed set [13] if \(\text{int}(\text{cl}(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(b\)-open in \((X, \tau)\).
14. \(\alpha\*)\)-closed set [12] if \(\text{int}(\text{cl}(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\alpha\)-open in \((X, \tau)\).
15. \(\pi\)-generalized semi pre closed set [19] (briefly \(\pi\)gp-closed) if \(\text{spcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\pi\)-open in \((X, \tau)\).

The complements of the above mentioned closed sets are their respective open sets.

**Remark 2.7:**

\[
\text{regular open} \rightarrow \pi\text{-open} \rightarrow \text{open} \rightarrow \text{g*-open} \rightarrow \text{g-open} \\
\alpha\text{-open} \rightarrow \text{pre open} \rightarrow \text{pre*open} \\
\text{semi pre open}
\]

**Theorem 2.8:** [3] Let \((X, \tau)\) be a topological space. Then \(\text{pcl}(A \cap B) \subseteq \text{pcl}(A) \cap \text{pcl}(B)\).
Lemma 2.9: [1] For any subset $A$ of $X$, $\text{pcl}(A) = A \cup \text{cl}(\text{int}(A))$.

Lemma 2.10: [2] If $A$ is semi closed in $X$, then $\text{pcl}(A \cup B) = \text{pcl}(A) \cup \text{pcl}(B)$.

Theorem 2.11: [20] Arbitrary union of pre-open sets is pre-open.

3. PRE-* GENERALIZED CLOSED SETS

Definition 3.1: A subset $A$ of a topological space $(X, \tau)$ is called pre-*generalized closed (briefly $p^*g$-closed) if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is pre-open in $(X, \tau)$.

Theorem 3.2: Let $(X, \tau)$ be a topological space. Then every closed set is $p^*g$-closed.

Proof: Let $A$ be a closed set. Let $A \subseteq U$, $U$ is pre-open. Since $A$ is closed, $\text{cl}(A) = A \subseteq U$. But $\text{pcl}(A) \subseteq \text{cl}(A)$. Thus we have $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is pre-open. Therefore, $A$ is $p^*g$-closed.

Remark 3.3: The converse of the above theorem need not be true, as seen from the following example.

Example 3.4: Consider the space $(X, \tau)$ where $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{c\}, X\}$. Then $\{a\}$ and $\{b\}$ are $p^*g$-closed but not closed.

Theorem 3.5: Let $(X, \tau)$ be a topological space. Then every regular closed set is $p^*g$-closed.

Proof: Let $A$ be a regular closed set. Let $A \subseteq U$, $U$ is pre-open. By Remark 2.6, $\text{pcl}(A) \subseteq \text{rcl}(A) = A \subseteq U$. Thus we have $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is pre-open. Therefore, $A$ is $p^*g$-closed.

Remark 3.6: The converse of the above theorem need not be true, as seen from the following example.

Example 3.7: Consider the space $(X, \tau)$ where $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a, b\}, \{c\}, X\}$. Then $\{b, c\}$ and $\{a\}$ are $p^*g$-closed but not regular closed.

Theorem 3.8: Let $(X, \tau)$ be a topological space. Then every $\alpha$-closed set is $p^*g$-closed.

Proof: Let $A$ be an $\alpha$-closed set. Let $A \subseteq U$, $U$ is pre-open. By Remark 2.6, $\text{pcl}(A) \subseteq \text{cl}(A) = A \subseteq U$. Thus we have $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is pre-open. Therefore, $A$ is $p^*g$-closed.

Remark 3.9: The converse of the above theorem need not be true, as seen from the following example.

Example 3.10: Consider the space $(X, \tau)$ where $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a, b\}, X\}$. Then $\{a\}$, $\{b\}$ are $p^*g$-closed but not $\alpha$-closed.

Theorem 3.11: Let $(X, \tau)$ be a topological space. Then every $p^*g$-closed set is $gp$-closed.

Proof: Let $A$ be a $p^*g$-closed set. Let $A \subseteq U$, $U$ is open. Then by Remark 2.7, $U$ is pre-open. Since $A$ is $p^*g$-closed, $\text{pcl}(A) \subseteq U$. Thus we have $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open. Therefore, $A$ is $gp$-closed.

Remark 3.12: The converse of the above theorem need not be true, as seen from the following example.

Example 3.13: Consider the space $(X, \tau)$ where $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a, b\}, X\}$. Then $\{a, c\}$ and $\{b, c\}$ are $gp$-closed but not $p^*g$-closed.

Theorem 3.14: Let $(X, \tau)$ be a topological space. Then every $p^*g$-closed set is $gpr$-closed.

Proof: Let $A$ be a $p^*g$-closed set. Let $A \subseteq U$, $U$ is regular open. Then by Remark 2.7, $U$ is pre-open. Since $A$ is $p^*g$-closed, $\text{pcl}(A) \subseteq U$. Thus we have $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open. Therefore, $A$ is $gpr$-closed.

Remark 3.15: The converse of the above theorem need not be true, as seen from the following example.

Example 3.16: Consider the space $(X, \tau)$ where $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a, b\}, X\}$. Then $\{a, b\}$ is $gpr$-closed but not $p^*g$-closed.
Theorem 3.17: Let $(X, \tau)$ be a topological space. Then every $p*g$-closed set is $wg$-closed.

Proof: Let $A$ be a $p*g$-closed set. Let $A \subseteq U$, $U$ is open. Then by Remark 2.7, $U$ is $pre*open$. Since $A$ is $p*g$-closed, $pcl(A) \subseteq U$. By Lemma 2.9, $A \cup cl(int(A)) \subseteq U$. Thus we have $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is open. Therefore, $A$ is $wg$-closed.

Remark 3.18: The converse of the above theorem need not be true, as seen from the following example.

Example 3.19: Consider the space $(X, \tau)$ where $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{c\}, X\}$. Then $\{a, c\}$ and $\{b, c\}$ are $wg$-closed but not $p*g$-closed.

Theorem 3.20: Let $(X, \tau)$ be a topological space. Then every $p*g$-closed set is $rwg$-closed.

Proof: Let $A$ be a $p*g$-closed set. Let $A \subseteq U$, $U$ is regular open. Then by Remark 2.7, $U$ is $pre*open$. Since $A$ is $p*g$-closed, $pcl(A) \subseteq U$. By Lemma 2.9, $A \cup cl(int(A)) \subseteq U$. Thus we have $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open. Therefore, $A$ is $rwg$-closed.

Remark 3.21: The converse of the above theorem need not be true, as seen from the following example.

Example 3.22: Consider the space $(X, \tau)$ where $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Then $\{a, b\}$ is $rwg$-closed but not $p*g$-closed.

Theorem 3.23: Let $(X, \tau)$ be a topological space. Then every $p*g$-closed set is $\pi gp$-closed.

Proof: Let $A$ be a $p*g$-closed set. Let $A \subseteq U$, $U$ is $\pi$-open. Then by Remark 2.7, $U$ is $pre*open$. Since $A$ is $p*g$-closed, $pcl(A) \subseteq U$. Thus we have $pcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\pi$-open. Therefore, $A$ is $\pi gp$-closed.

Remark 3.24: The converse of the above theorem need not be true, as seen from the following example.

Example 3.25: Consider the space $(X, \tau)$ where $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$. Then $\{a, b\}$ is $\pi gp$-closed but not $p*g$-closed.

Theorem 3.26: Let $(X, \tau)$ be a topological space. Then every $p*g$-closed set is $gsp$-closed.

Proof: Let $A$ be a $p*g$-closed set. Let $A \subseteq U$, $U$ is open. Then by Remark 2.7, $U$ is $pre*open$. Since $A$ is $p*g$-closed, $pcl(A) \subseteq U$. But $spcl(A) \subseteq pcl(A) \subseteq U$. Thus we have $spcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open. Therefore, $A$ is $gsp$-closed.

Remark 3.27: The converse of the above theorem need not be true, as seen from the following example.

Example 3.28: Consider the space $(X, \tau)$ where $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$. Then $\{a, b\}$ is $gsp$-closed but not $p*g$-closed.

Theorem 3.29: Let $(X, \tau)$ be a topological space. Then every $p*g$-closed set is $\pi gsp$-closed.

Proof: Let $A$ be a $p*g$-closed set. Let $A \subseteq U$, $U$ is $\pi$-open. Then by Remark 2.7, $U$ is $pre*open$. Since $A$ is $p*g$-closed, $pcl(A) \subseteq U$. But $spcl(A) \subseteq pcl(A) \subseteq U$. Thus we have $spcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\pi$-open. Therefore, $A$ is $\pi gsp$-closed.

Remark 3.30: The converse of the above theorem need not be true, as seen from the following example.

Example 3.31: Consider the space $(X, \tau)$ where $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a, b\}, X\}$. Then $\{a, b\}$ is $\pi gsp$-closed but not $p*g$-closed.

Theorem 3.32: Let $(X, \tau)$ be a topological space. Then every $p*g$-closed set is pre semi closed.

Proof: Let $A$ be a $p*g$-closed set. Let $A \subseteq U$, $U$ is $g$-open. Then by Remark 2.7, $U$ is $pre*open$. Since $A$ is $p*g$-closed, $pcl(A) \subseteq U$. But $spcl(A) \subseteq pcl(A) \subseteq U$. Thus we have $spcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g$-open. Therefore, $A$ is pre semi closed.

Remark 3.33: The converse of the above theorem need not be true, as seen from the following example.
Example 3.34: Consider the space \((X, \tau)\) where \(X = \{a, b, c\}\) and \(\tau = \{\emptyset, \{c\}, \{b, c\}, X\}\). Then \(\{a, c\}\) is pre semi closed but not \(p^*g\)-closed.

Theorem 3.35: Let \((X, \tau)\) be a topological space. Then every \(p^*g\)-closed set is \(g^*p\)-closed.

Proof: Let \(A\) be a \(p^*g\)-closed set. Let \(A \subseteq U\), \(U\) is \(g\)-open. Then by Remark 2.7, \(U\) is \(\pre^*\)open. Since \(A\) is \(p^*g\)-closed, \(\pcl(A) \subseteq U\). Thus we have \(\pcl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g\)-open. Therefore, \(A\) is \(g^*p\)-closed.

Remark 3.36: The converse of the above theorem need not be true, as seen from the following example.

Example 3.37: Consider the space \((X, \tau)\) where \(X = \{a, b, c\}\) and \(\tau = \{\emptyset, \{c\}, \{b, c\}, X\}\). Then \(\{a, c\}\) is \(g^*p\)-closed but not \(p^*g\)-closed.

Theorem 3.38: Let \((X, \tau)\) be a topological space. If \(A\) and \(B\) are two \(p^*g\)-closed in \(X\), then \(A \cap B\) is \(p^*g\)-closed.

Proof: Let \(U\) be \(\pre^*\)open such that \(A \cap B \subseteq U\). Then by Theorem 2.11, \(U \cup (X-B)\) is \(\pre^*\)open containing \(A\). Since \(A\) is \(p^*g\)-closed, \(\pcl(A) \subseteq U \cup (X-B)\).

Now \(\pcl(A \cap B) \subseteq \pcl(A) \cap \pcl(B) \subseteq \pcl(A) \cap \cl(B) = \pcl(A) \cap B \subseteq (U \cup (X-B)) \cap B = U \cap B \subseteq U\). Thus we have \(\pcl(A \cap B) \subseteq U\), \(U\) is \(\pre^*\)open and \(A \cap B \subseteq U\). Therefore \(A \cap B\) is \(p^*g\)-closed.

Remark 3.39: In general, union of any two \(p^*g\)-closed sets in \((X, \tau)\) need not be a \(p^*g\)-closed set, as seen from the following example.

Example 3.40: Consider the space \((X, \tau)\) where \(X = \{a, b, c\}\) and \(\tau = \{\emptyset, \{a, b\}, X\}\). Here, \(\{a\}\) and \(\{b\}\) are \(p^*g\)-closed. But their union \(\{a, b\}\) is not \(p^*g\)-closed.

Remark 3.41: The above discussions are summarized in the following implications.

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 regular closed  
    ↓            ↓                ↓            ↓                ↓            ↓                ↓            ↓                ↓            ↓
 g*-closed     g-closed     g-closed     g-closed     g-closed     g-closed     g-closed     g-closed     g-closed
    ↓            ↓                ↓            ↓                ↓            ↓                ↓            ↓                ↓            ↓
 π-closed      closed        closed        closed        closed        closed        closed        closed        closed
    ↓            ↓                ↓            ↓                ↓            ↓                ↓            ↓                ↓            ↓
 α-closed      rgw-closed   rgw-closed   rgw-closed   rgw-closed   rgw-closed   rgw-closed   rgw-closed   rgw-closed
    ↓            ↓                ↓            ↓                ↓            ↓                ↓            ↓                ↓            ↓
 πgsp-closed   p*g-closed   p*g-closed   p*g-closed   p*g-closed   p*g-closed   p*g-closed   p*g-closed   p*g-closed
    ↓            ↓                ↓            ↓                ↓            ↓                ↓            ↓                ↓            ↓
 πgp-closed   g*p-closed   g*p-closed   g*p-closed   g*p-closed   g*p-closed   g*p-closed   g*p-closed   g*p-closed
    ↓            ↓                ↓            ↓                ↓            ↓                ↓            ↓                ↓            ↓
 gp-closed    gsp-closed   gsp-closed   gsp-closed   gsp-closed   gsp-closed   gsp-closed   gsp-closed   gsp-closed
    ↓            ↓                ↓            ↓                ↓            ↓                ↓            ↓                ↓            ↓
 gpr-closed
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Remark 3.42: p*g-closedness and rg-closedness are independent concepts as we illustrate by means of the following example.

Example 3.43: Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$. Then the set $\{c\}$ is p*g-closed but not rg-closed and also $\{a, b\}$ is rg-closed but not p*g-closed.

Remark 3.44: p*g-closedness and g-closedness are independent concepts as we illustrate by means of the following example.

Example 3.45: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$. Then the set $\{c\}$ is p*g-closed but not g-closed and also $\{a, c\}$ is g-closed but not p*g-closed.

Remark 3.46: p*g-closedness and g*-closedness are independent concepts as we illustrate by means of the following example.

Example 3.47: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$. Then the set $\{b\}$ is p*g-closed but not g*-closed and also $\{a, c\}$ is g*-closed but not p*g-closed.

Remark 3.48: p*g-closedness and $\alpha g$-closedness are independent concepts as we illustrate by means of the following examples.

Example 3.49: i. Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a, b\}, X\}$. Then the set $\{a\}$ and $\{b\}$ are p*g-closed but not $\alpha g$-closed.

Example 3.50: p*g-closedness and regular $\alpha$-closedness are independent concepts as we illustrate by means of the following example.

Example 3.51: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$. Then the set $\{c\}$ is p*g-closed but not regular $\alpha$-closed and also $\{a\}$ and $\{b\}$ are regular $\alpha$-closed but not p*g-closed.

Remark 3.52: p*g-closedness and b*-closedness are independent concepts as we illustrate by means of the following examples.

Example 3.53: i. Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a, b\}, X\}$. Then the set $\{a\}$ and $\{b\}$ are p*g-closed but not b*-closed.

Example 3.54: p*g-closedness and $\alpha m$-closedness are independent concepts as we illustrate by means of the following examples.

Example 3.55: i. Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a, b\}, X\}$. Then the set $\{a\}$ and $\{b\}$ are p*g-closed but not $\alpha m$-closed.

Example 3.56: p*g-closedness and $\alpha m$-closedness are independent concepts as we illustrate by means of the following examples.

Example 3.57: i. Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Then $\{a\}$ and $\{b\}$ are $\alpha m$-closed but not p*g-closed.

Remark 3.58: In this section, we investigate some basic characterization of p*g-closed set in topological spaces.

4. CHARACTERIZATION

In this section, we investigate some basic characterization of p*g-closed set in topological spaces.
Theorem 4.1: If A is g-closed and p*g-closed, then A is wg-closed.

Proof: Suppose A is g-closed and p*g-closed. By Remark 2.6, pcl(A) ⊆ cl(A) which implies pcl(A) ⊆ cl(A) ⊆ U. By Lemma 2.9, A ⊆ cl(int(A)) ⊆ U. Thus we have cl(int(A)) ⊆ U whenever A ⊆ U and U is open. Therefore, A is wg-closed.

Remark 4.2: The converse of the above theorem need not be true, as seen from the following example.

Example 4.3: Consider the space (X, τ) where X = {a, b, c} and τ = {∅, {c}, X}. Here, {a, c} and {b, c} are both g-closed and wg-closed but not p*g-closed.

Theorem 4.4: If A is g-closed and p*g-closed, then A is g*p-closed.

Proof: Suppose A is g-closed and p*g-closed. By Remark 2.6, pcl(A) ⊆ cl(A) which implies pcl(A) ⊆ cl(A) ⊆ U and by Remark 2.7, U is g-open. Thus we have pcl(A) ⊆ U whenever A ⊆ U and U is g-open. Therefore, A is g*p-closed.

Remark 4.5: The converse of the above theorem need not be true, as seen from the following example.

Example 4.6: Consider the space (X, τ) where X = {a, b, c} and τ = {∅, {c}, {b, c}, X}. Here, {a, c} is both g-closed and g*p-closed but not p*g-closed.

Theorem 4.7: Let A be any p*g-closed set in (X, τ). If A ⊆ B ⊆ pcl(A), then B is also a p*g-closed set.

Proof: Let B ⊆ U where U is pre*open in (X, τ). Then A ⊆ U. Also since A is p*g-closed, pcl(A) ⊆ U. Since B ⊆ pcl(A), pcl(B) ⊆ pcl(pcl(A)) = pcl(A) ⊆ U. This implies, pcl(B) ⊆ U. Thus B is a p*g-closed set.

Theorem 4.8: If a set A is p*g-closed in X, then pcl(A) - A contains no non empty pre*open set in X.

Proof: Suppose X - {x} is not pre*open. Then X is the only pre*open set containing X - {x}. This implies pcl(X - {x}) ⊆ U whenever A ⊆ U and U is pre*open. Therefore, A is closed.

Remark 4.9: The converse of the above theorem need not be true, as seen from the following example.

Example 4.10: If pcl(A) - A contains no non empty pre*open set in X, then A is not a p*g-closed set. Consider X = {a, b, c} with the topology τ = {∅, {a}, {b}, {a, b}, X} and A = {a, b}. Then pcl(A) - A = X - {a, b} = {c} contains no non empty pre*open set in X, but A is not a p*g-closed set in X.

Theorem 4.11: For every element x in a space X, the set X - {x} is p*g-closed or pre*open.

Proof: Suppose X - {x} is not pre*open. Then X is the only pre*open set containing X - {x}. This implies pcl(X - {x}) ⊆ U. Hence X - {x} is p*g-closed.

Theorem 4.12: Let A and B be p*g-closed sets in (X, τ) such that cl(A) = pcl(A) and cl(B) = pcl(B). Then A∪B is p*g-closed.

Proof: Let A∪B ⊆ U, where U is pre*open. Then A ⊆ U and B ⊆ U. Since A and B are p*g-closed, pcl(A) ⊆ U and pcl(B) ⊆ U. Now cl(A∪B) = cl(A) ∪ cl(B) = pcl(A) ∪ pcl(B) ⊆ U. But pcl(A∪B) ⊆ cl(A∪B). So, pcl(A∪B) ⊆ cl(A∪B) ⊆ U whenever A∪B ⊆ U, U is pre*open. Hence A∪B is p*g-closed.

Theorem 4.13: The union of two p*g-closed sets is p*g-closed if at least one of them is semi closed.

Proof: Let A and B be two p*g-closed sets in X. Suppose A is semi closed. To prove that A∪B is p*g-closed. Let A∪B ⊆ U and U is pre*open. Then A ⊆ U and B ⊆ U. Since A and B are p*g-closed, pcl(A) ⊆ U and pcl(B) ⊆ U. Therefore, pcl(A) ∪ pcl(B) ⊆ U. Since by Lemma 2.10, pcl(A∪B) ⊆ U. Thus we have pcl(A∪B) ⊆ U whenever A∪B ⊆ U and U is pre*open. Therefore A∪B is p*g-closed.

Theorem 4.14: If A ⊆ Y ⊆ X and A is p*g-closed in X then A is p*g-closed relative to Y.

Proof: Given that A ⊆ Y ⊆ X and A is p*g-closed set in X. To prove that A is p*g-closed set relative to Y. Let us assume that A ⊆ Y∩U, where U is pre*open in X. Since A is p*g-closed, A ⊆ U. This implies that pcl(A) ⊆ U. It follows that Y ∩ pcl(A) ⊆ Y ∩ U. That is, A is p*g-closed relative to Y.
5. CONCLUSION

The present paper has introduced a new concept of p*g-closed set in topological spaces. It also analyzed some of the properties. The implication shows the relationship between p*g-closed sets and the other existing sets.

6. REFERENCES


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