International Journal of Mathematical Archive-8(1), 2017, 65-72
Available online through www.ijma.info ISSN 2229 – 5046

PRE*GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

M. JEYACHITRA*[1], K. BAGEERATHI[2]

[1]Department of Mathematics,
Sri Muthukumaran Arts and Science College, Chennai, Tamil Nadu, India.

[2]Department of Mathematics,
Aditanar College of Arts and Science, Tiruchendur, Tamil Nadu, India.

(Received On: 27-12-16; Revised & Accepted On: 21-01-17)

ABSTRACT

*In this paper a new class of generalized closed sets, namely p*g-closed sets is introduced in topological spaces. We find some basic properties and characterizations of p*g-closed sets.*

Mathematics Subject Classification: *54A05.*

Key Words: *g-closed sets, p*g-closed sets, g*p-closed sets, πgp-closed sets.*

1. INTRODUCTION

In 1970, N. Levine [8] introduced the concept of generalized closed sets (briefly g-closed). In 1982, Dunham [6] introduced the generalized closure (briefly g-closure). In 1996, H. Maki, J. Umehara and T. Noiri [4, 10] introduced the class of pre generalized closed sets and used them to obtain properties of pre-T1/2 spaces. Selvi [20] introduced pre*closed sets using the g-closure operator due to Dunham. Y. Gnanambal [7], H. Maki, R. Devi, K. Balachandran [9], J. Dontchev [4, 5], Veerakumar [23, 24, 25], N. Palaniappan and K. C. Rao [17], N. Nagaveni [14], J. H. Park [18], S. Muthuvel and R. Parimelazhagan, Milby Mathew [12, 13], Sarsak. M. S. and N. Rajesh [19], introduced and investigated gpr-closed, α-closed, αg-closed, gsp-closed, πg-closed, g*-closed, g*p-closed, pre semi closed, rg-closed, wg-closed, rwg-closed, πgp-closed, α^m-closed, b*-closed, πgsp-closed respectively.

In this paper we introduce a new class of sets called p*g-closed sets. We give characterizations of p*g-closed sets also investigate some fundamental properties of p*g-closed set.

2. PRELIMINARIES

Throughout this paper (X, τ) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. (X, τ) will be replaced by X if there is no changes of confusion. We recall the following definitions and results.

Definition 2.1: Let (X, τ) be a topological space. A subset A of X is said to be generalized closed [8] (briefly g-closed) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an open in (X, τ).

Definition 2.2: Let (X, τ) be a topological space and A $\subseteq$ X. The generalized closure of A [6], denoted by cl*(A) and is defined by the intersection of all g-closed sets containing A and generalized interior of A [6], denoted by int*(A) and is defined by union of all g-open sets contained in A.

Corresponding Author: M. Jeyachitra[1], [1]Department of Mathematics,*
Sri Muthukumaran Arts and Science College, Chennai, Tamil Nadu, India.

Definition 2.3: Let(X, τ) be a topological space and $A \subseteq X$. Then
- (i) A is α-open if $A \subseteq int(cl(int(A)))$ and α-closed if $cl(int(cl(A))) \subseteq A$ [15].
- (ii) A is pre open if $A \subseteq int(cl(A))$ and pre closed if $cl(int(A)) \subseteq A$ [11].
- (iii) A is pre*open if $A \subseteq int*(cl(A))$ and pre*closed if $cl*(int(A)) \subseteq A$ [20].
- (iv) A is regular open if $A = int(cl(A))$ and regular closed if $A = cl(int(A))$ [21].
- (v) A is semi pre open if $A \subseteq cl(int(cl(A)))$ and semi pre closed if $int(cl(int(A))) \subseteq A$ [1].
- (vi) π-closed set [26] if A is a finite intersection of regular closed sets. The complement of a π-closed set is called a π-open set.
- (vii) a regular α-open set (briefly rα-open) [22] if there is a regular open set U such that $U \subseteq A \subseteq \alpha cl(U)$.

Definition 2.4: Let (X, τ) be a topological space and $A \subseteq X$. The pre closure of A [11], denoted by pcl(A) and is defined by the intersection of all pre closed sets containing A.

Definition 2.5: Let (X, τ) be a topological space. A subset A of X is said to be
1. α-generalized closed set (briefly αg-closed) [9] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
2. generalized pre closed set (briefly gp-closed) [10] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
3. strongly generalized closed set (briefly g$*$closed) [23] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ).
4. generalized$*$pre closed set (briefly g$*$p-closed) [24] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ).
5. regular generalized closed set (briefly rg-closed) [17] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ).
6. weakly generalized closed set (briefly wg-closed) [14] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
7. generalized pre regular closed set (briefly gpr-closed) [7] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ).
8. generalized semi preclosed set (briefly gsp-closed) [5] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
9. pre semi closed set [25] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ).
10. πgp-closed set [18] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in (X, τ).
11. regular weakly generalized closed set (briefly rwg-closed) [14] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ).
12. b-closed set [13] if $cl(int(A)) \cap int(cl(A)) \subseteq A$.
13. b*-closed set [13] if $int(cl(A)) \subseteq U$ whenever $A \subseteq U$ and U is b-open in (X, τ).
14. α^m-closed set [12] if $int(cl(A)) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ).
15. π-generalized semi pre closed set [19] (briefly πgsp-closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in (X, τ).

The complements of the above mentioned closed sets are their respective open sets.

Remark 2.6:

$$\begin{tikzcd}
\text{regular closed} \arrow[r] & \pi\text{-closed} \arrow[r] & \text{closed} \arrow[r] \arrow[d] & g^*\text{-closed} \arrow[r] & g\text{-closed} \arrow[d] \\
 & & \alpha\text{-closed} \arrow[r] \arrow[d] & \text{pre closed} \arrow[r] \arrow[dl] & \text{pre*closed} \\
 & & \text{semi pre closed} & &
\end{tikzcd}$$

Remark 2.7:

$$\begin{tikzcd}
\text{regular open} \arrow[r] & \pi\text{-open} \arrow[r] & \text{open} \arrow[r] \arrow[d] & g^*\text{-open} \arrow[r] & g\text{-open} \arrow[d] \\
 & & \alpha\text{-open} \arrow[r] \arrow[d] & \text{pre open} \arrow[r] \arrow[dl] & \text{pre*open} \\
 & & \text{semi pre open} & &
\end{tikzcd}$$

Theorem 2.8: **[3]** Let (X, τ) be a topological space. Then $pcl(A \cap B) \subseteq pcl(A) \cap pcl(B)$.

Lemma 2.9: [1] For any subset A of X, pcl(A) = A ∪ cl(int(A)).

Lemma 2.10: [2] If A is semi closed in X, then pcl(A∪B) = pcl(A) ∪ pcl(B).

Theorem 2.11: [20] Arbitrary union of pre*open sets is pre*open.

3. PRE * GENERALIZED CLOSED SETS

Definition 3.1: A subset A of a topological space (X, τ) is called pre∗generalized closed (briefly p∗g-closed) if pcl(A) ⊆ U whenever A ⊆ U and U is pre∗open in (X, τ).

Theorem 3.2: Let (X, τ) be a topological space. Then every closed set is p∗g-closed.

Proof: Let A be a closed set. Let A ⊆ U, U is pre*open. Since A is closed , cl(A) = A ⊆ U. But pcl(A) ⊆ cl(A). Thus we have pcl(A) ⊆ U whenever A ⊆ U and U is pre∗open. Therefore, A is p∗g-closed.

Remark 3.3: The converse of the above theorem need not be true, as seen from the following example.

Example 3.4: Consider the space (X, τ) where X = {a, b, c} and τ = {ϕ, {c}, X}. Then {a} and {b} are p∗g-closed but not closed.

Theorem 3.5: Let (X, τ) be a topological space. Then every regular closed set is p∗g-closed.

Proof: Let A be a regular closed set. Let A ⊆ U, U is pre*open. By Remark 2.6, pcl(A) ⊆ rcl(A) = A ⊆ U. Thus we have pcl(A) ⊆ U whenever A ⊆ U and U is pre∗open. Therefore, A is p∗g-closed.

Remark 3.6: The converse of the above theorem need not be true, as seen from the following example.

Example 3.7: Consider the space (X, τ) where X = {a, b, c} and τ = {ϕ, {a, b}, {c}, X}. Then {b, c} and {a} are p∗g-closed but not regular closed.

Theorem 3.8: Let (X, τ) be a topological space. Then every α-closed set is p∗g-closed.

Proof: Let A be a α-closed set. Let A ⊆ U, U is pre*open. By Remark 2.6, pcl(A) ⊆ αcl(A) = A ⊆ U. Thus we have pcl(A) ⊆ U whenever A ⊆ U and U is pre∗open. Therefore, A is p∗g-closed.

Remark 3.9: The converse of the above theorem need not be true, as seen from the following example.

Example 3.10: Consider the space (X, τ) where X = {a, b, c} and τ = {ϕ, {a, b}, X}. Then {a}, {b} are p∗g-closed but not α-closed.

Theorem 3.11: Let (X, τ) be a topological space. Then every p*g-closed set is gp-closed.

Proof: Let A be a p*g-closed set. Let A ⊆ U, U is open. Then by Remark 2.7, U is pre∗open. Since A is p*g-closed, pcl(A) ⊆ U. Thus we have pcl(A) ⊆ U whenever A ⊆ U and U is open. Therefore, A is gp-closed.

Remark 3.12: The converse of the above theorem need not be true, as seen from the following example.

Example 3.13: Consider the space (X, τ) where X = {a, b, c} and τ = {ϕ, {c}, X}. Then {a, c} and {b, c} are gp-closed but not p*g-closed.

Theorem 3.14: Let (X, τ) be a topological space. Then every p*g-closed set is gpr-closed.

Proof: Let A be a p*g-closed set. Let A ⊆ U, U is regular open. Then by Remark 2.7, U is pre∗open. Since A is p*g-closed, pcl(A) ⊆ U. Thus we have pcl(A) ⊆ U whenever A ⊆ U and U is regular open. Therefore, A is gpr-closed.

Remark 3.15: The converse of the above theorem need not be true, as seen from the following example.

Example 3.16: Consider the space (X, τ) where X = {a, b, c} and τ = {ϕ, {a, b}, X}. Then {a, b} is gpr-closed but not p*g-closed.

Theorem 3.17: Let (X, τ) be a topological space. Then every p*g-closed set is wg-closed.

Proof: Let A be a p*g-closed set. Let $A \subseteq U$, U is open. Then by Remark 2.7, U is pre*open. Since A is p*g-closed, $pcl(A) \subseteq U$. By Lemma 2.9, $A \cup cl(int(A)) \subseteq U$. Thus we have $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open. Therefore, A is wg-closed.

Remark 3.18: The converse of the above theorem need not be true, as seen from the following example.

Example 3.19: Consider the space (X, τ) where $X = \{a, b, c\}$ and $\tau = \{\phi, \{c\}, X\}$. Then $\{a, c\}$ and $\{b, c\}$ are wg-closed but not p*g-closed.

Theorem 3.20: Let (X, τ) be a topological space. Then every p*g-closed set is rwg-closed.

Proof: Let A be a p*g-closed set. Let $A \subseteq U$, U is regular open. Then by Remark 2.7, U is pre∗open. Since A is p*g-closed, $pcl(A) \subseteq U$. By Lemma 2.9, $A \cup cl(int(A)) \subseteq U$. Thus we have $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open. Therefore, A is rwg-closed.

Remark 3.21: The converse of the above theorem need not be true, as seen from the following example.

Example 3.22: Consider the space (X, τ) where $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then $\{a, b\}$ is rwg-closed but not p*g-closed.

Theorem 3.23: Let (X, τ) be a topological space. Then every p*g-closed set is πgp-closed.

Proof: Let A be a p*g-closed set. Let $A \subseteq U$, U is π-open. Then by Remark 2.7, U is pre∗open. Since A is p*g-closed, $pcl(A) \subseteq U$. Thus we have $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open. Therefore, A is πgp-closed.

Remark 3.24: The converse of the above theorem need not be true, as seen from the following example.

Example 3.25: Consider the space (X, τ) where $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then $\{a, b\}$ is πgp-closed but not p*g-closed.

Theorem 3.26: Let (X, τ) be a topological space. Then every p*g-closed set is gsp-closed.

Proof: Let A be a p*g-closed set. Let $A \subseteq U$, U is open. Then by Remark 2.7, U is pre∗open. Since A is p*g-closed, $pcl(A) \subseteq U$. But $spcl(A) \subseteq pcl(A) \subseteq U$. Thus we have $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open. Therefore, A is gsp-closed.

Remark 3.27: The converse of the above theorem need not be true, as seen from the following example.

Example 3.28: Consider the space (X, τ) where $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{a, b\}, X\}$. Then $\{a, c\}$ is gsp-closed but not p*g-closed.

Theorem 3.29: Let (X, τ) be a topological space. Then every p*g-closed set is πgsp-closed.

Proof: Let A be a p*g-closed set. Let $A \subseteq U$, U is π-open. Then by Remark 2.7, U is pre∗open. Since A is p*g-closed, $pcl(A) \subseteq U$. But $spcl(A) \subseteq pcl(A) \subseteq U$. Thus we have $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open. Therefore, A is πgsp-closed.

Remark 3.30: The converse of the above theorem need not be true, as seen from the following example.

Example 3.31: Consider the space (X, τ) where $X = \{a, b, c\}$ and $\tau = \{\phi, \{a, b\}, X\}$. Then $\{a, b\}$ is πgsp-closed but not p*g-closed.

Theorem 3.32: Let (X, τ) be a topological space. Then every p*g-closed set is pre semi closed.

Proof: Let A be a p*g-closed set. Let $A \subseteq U$, U is g-open. Then by Remark 2.7, U is pre∗open. Since A is p*g-closed, $pcl(A) \subseteq U$. But $spcl(A) \subseteq pcl(A) \subseteq U$. Thus we have $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open. Therefore, A is pre semi closed.

Remark 3.33: The converse of the above theorem need not be true, as seen from the following example.

Example 3.34: Consider the space (X, τ) where X = {a, b, c} and τ = {ϕ, {c}, {b, c}, X}. Then {a, c} is pre semi closed but not p*g-closed.

Theorem 3.35: Let (X, τ) be a topological space. Then every p*g-closed set is g*p-closed.

Proof: Let A be a p*g-closed set. Let A ⊆ U, U is g-open. Then by Remark 2.7, U is pre∗open. Since A is p*g-closed, pcl(A) ⊆ U. Thus we have pcl(A) ⊆ U whenever A ⊆ U and U is g-open. Therefore, A is g*p-closed.

Remark 3.36: The converse of the above theorem need not be true, as seen from the following example.

Example 3.37: Consider the space (X, τ) where X = {a, b, c} and τ = {ϕ, {c}, {b, c}, X}. Then {a, c} is g*p-closed but not p*g-closed.

Theorem 3.38: Let (X, τ) be a topological space. If A and B are two p∗g-closed in X, then A∩B is p∗g-closed.

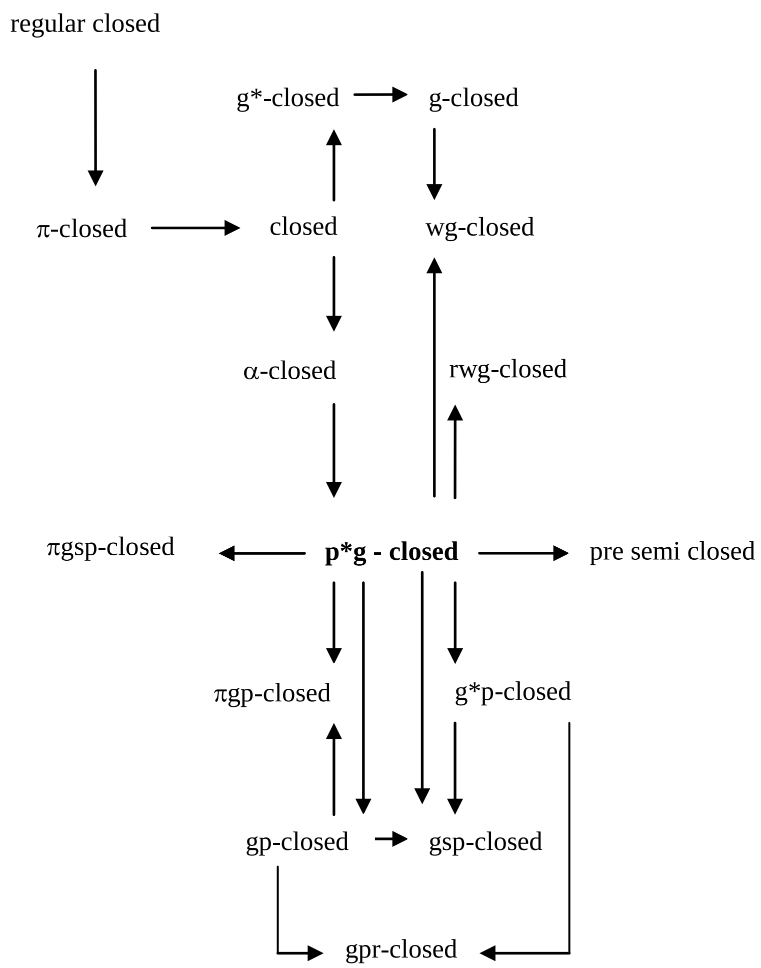
Proof: Let U be pre*open such that A∩B ⊆ U. Then by Theorem 2.11, U ∪ (X-B) is pre*open containing A. Since A is p*g-closed, pcl(A) ⊆ U ∪ (X-B).

Now pcl(A ∩ B) ⊆ pcl(A) ∩ pcl(B) ⊆ pcl(A) ∩ cl(B) = pcl(A) ∩ B ⊆ (U ∪ (X-B)) ∩ B = U ∩ B ⊆ U. Thus we have pcl(A∩B) ⊆ U, U is pre*open and A∩B ⊆ U. Therefore A∩B is p*g-closed.

Remark 3.39: In general, union of any two p*g-closed sets in (X, τ) need not be a p*g-closed set, as seen from the following example.

Example 3.40: Consider the space (X, τ) where X = {a, b, c} and τ = {ϕ, {a, b}, X}. Here, {a} and {b} are p∗g-closed. But their union {a, b} is not p∗g-closed.

Remark 3.41: The above discussions are summarized in the following implications.



Remark 3.42: p*g-closedness and rg-closedness are independent concepts as we illustrate by means of the following example.

Example 3.43: Let X = {a, b, c, d} and τ = {ϕ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, X}. Then the set {c} is p*g-closed but not rg-closed and also {a, b} is rg-closed but not p*g-closed.

Remark 3.44: p*g-closedness and g-closedness are independent concepts as we illustrate by means of the following example.

Example 3.45: Let X = {a, b, c} and τ = {ϕ, {a}, {a, b}, X}. Then the set {b} is p*g-closed but not g-closed and also {a, c} is g-closed but not p*g-closed.

Remark 3.46: p*g-closedness and g*-closedness are independent concepts as we illustrate by means of the following example.

Example 3.47: Let X = {a, b, c} and τ = {ϕ, {a}, {a, b}, X}. Then the set {b} is p*g-closed but not g*-closed and also {a, c} is g*-closed but not p*g-closed.

Remark 3.48: p*g-closedness and αg-closedness are independent concepts as we illustrate by means of the following examples.

Example 3.49:

i. Let X = {a, b, c} and τ = {ϕ, {a, b}, X}. Then the set {a} and {b} are p*g-closed but not αg-closed.
ii. Let X = {a, b, c} and τ = {ϕ, {c}, X}. Then the set {a, c} and {b, c} are αg-closed but not p*g-closed.

Remark 3.50: p*g-closedness and regular α-closedness are independent concepts as we illustrate by means of the following example.

Example 3.51: Let X = {a, b, c} and τ = {ϕ, {a}, {b}, {a, b}, X}. Then {c} is p*g-closed but not regular α-closed and also {a} and {b} are regular α-closed but not p*g-closed.

Remark 3.52: p*g-closedness and b*-closedness are independent concepts as we illustrate by means of the following examples.

Example 3.53:

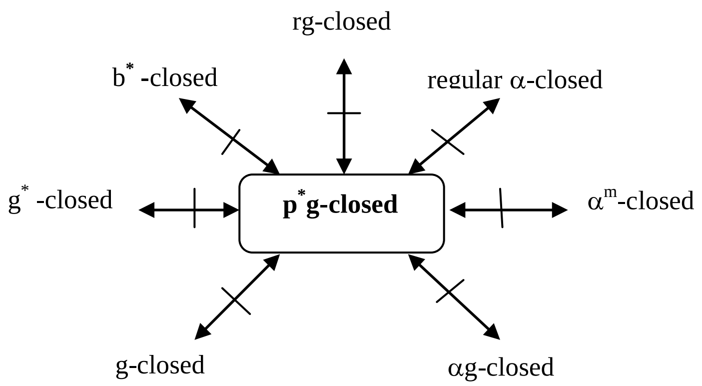
i. Let X = {a, b, c} and τ = {ϕ, {a, b}, X}. Then the set {a} and {b} are p*g-closed but not b*-closed.
ii. Let X = {a, b, c} and τ = {ϕ, {a}, {b}, {a, b}, X}. Then {a} and {b} are b*-closed but not p*g-closed.

Remark 3.54: p*g-closedness and α^m-closedness are independent concepts as we illustrate by means of the following examples**.**

Example 3.55:

i. Let X = {a, b, c} and τ = {ϕ, {a, b}, X}. Then the set {a} and {b} are p*g-closed but not α^m-closed.
ii. Let X = {a, b, c} and τ = {ϕ, {a}, {b}, {a, b}, X}. Then {a} and {b} are α^m-closed but not p*g-closed.

Remark 3.56:



4. CHARACTERIZATION

In this section, we investigate some basic characterization of p*g-closed set in topological spaces.

Theorem 4.1: If A is g-closed and p*g-closed, then A is wg-closed.

Proof: Suppose A is g-closed and p*g-closed. By Remark 2.6, $pcl(A) \subseteq cl(A)$ which implies $pcl(A) \subseteq cl(A) \subseteq U$. By Lemma 2.9, $A \cup cl(int(A)) \subseteq U$. Thus we have $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open. Therfore, A is wg-closed.

Remark 4.2: The converse of the above theorem need not be true, as seen from the following example.

Example 4.3: Consider the space (X, τ) where $X = \{a, b, c\}$ and $\tau = \{\phi, \{c\}, X\}$. Here, $\{a, c\}$ and $\{b, c\}$ are both g-closed and wg-closed but not p*g-closed.

Theorem 4.4: If A is g-closed and p*g-closed, then A is g*p-closed.

Proof: Suppose A is g-closed and p*g-closed. By Remark 2.6, $pcl(A) \subseteq cl(A)$ which implies $pcl(A) \subseteq cl(A) \subseteq U$ and by Remark 2.7, U is g-open. Thus we have $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open. Therefore, A is g*p-closed.

Remark 4.5: The converse of the above theorem need not be true, as seen from the following example.

Example 4.6: Consider the space (X, τ) where $X = \{a, b, c\}$ and $\tau = \{\phi, \{c\}, \{b, c\}, X\}$. Here, $\{a, c\}$ is both g-closed and g*p-closed but not p*g-closed.

Theorem 4.7: Let A be any p*g-closed set in (X, τ). If $A \subseteq B \subseteq pcl(A)$, then B is also a p*g-closed set.

Proof: Let $B \subseteq U$ where U is pre*open in (X, τ). Then $A \subseteq U$. Also since A is p*g-closed, $pcl(A) \subseteq U$. Since $B \subseteq pcl(A)$, $pcl(B) \subseteq pcl(pcl(A)) = pcl(A) \subseteq U$. This implies, $pcl(B) \subseteq U$. Thus B is a p*g-closed set.

Theorem 4.8: If a set A is p*g-closed in X, then pcl(A) - A contains no non empty pre*open set in X.

Proof: Let $U \subseteq pcl(A) - A$ be a non empty pre*open set. Then $U \subseteq pcl(A)$ and $A \subseteq X - U$, we have $pcl(A) \subseteq X-U$. So $U \subseteq X - pcl(A)$. Therefore $U \subseteq pcl(A) \cap (X - pcl(A)) = \{\phi\}$. Hence pcl(A) - A contains no non empty pre*open set in X.

Remark 4.9: The converse of the above theorem need not be true, as seen from the following example.

Example 4.10: If pcl(A) - A contains no non empty pre*open set in X, then A is not a p*g-closed set. Consider $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $A = \{a, b\}$. Then $pcl(A) – A = X - \{a, b\} = \{c\}$ contains no non empty pre*open set in X, but A is not a p*g-closed set in X.

Theorem 4.11: For every element x in a space X, the set $X - \{x\}$ is p*g-closed or pre*open.

Proof: Suppose $X - \{x\}$ is not pre*open. Then X is the only pre*open set containing $X - \{x\}$. This implies $pcl(X - \{x\}) \subseteq X$. Hence $X - \{x\}$ is p*g-closed.

Theorem 4.12: Let A and B be p*g-closed sets in (X, τ) such that $cl(A) = pcl(A)$ and $cl(B) = pcl(B)$. Then $A \cup B$ is p*g-closed.

Proof: Let $A \cup B \subseteq U$, where U is pre*open. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are p*g-closed, $pcl(A) \subseteq U$ and $pcl(B) \subseteq U$. Now $cl(A \cup B) = cl(A) \cup cl(B) = pcl(A) \cup pcl(B) \subseteq U$. But $pcl(A \cup B) \subseteq cl(A \cup B)$. So, $pcl(A \cup B) \subseteq cl(A \cup B) \subseteq U$. Therefore $pcl(A \cup B) \subseteq U$ whenever $A \cup B \subseteq U$, U is pre*open. Hence $A \cup B$ is p*g-closed.

Theorem 4.13: The union of two p*g-closed sets is p*g-closed if at least one of them is semi closed.

Proof: Let A and B be two p*g-closed sets in X. Suppose A is semi closed. To prove that $A \cup B$ is p*g-closed. Let $A \cup B \subseteq U$ and U is pre*open. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are p*g-closed, $pcl(A) \subseteq U$ and $pcl(B) \subseteq U$. Therefore, $pcl(A) \cup pcl(B) \subseteq U$. Since by Lemma 2.10, $pcl(A \cup B) \subseteq U$. Thus we have $pcl(A \cup B) \subseteq U$ whenever $A \cup B \subseteq U$ and U is pre*open. Therefore $A \cup B$ is p*g-closed.

Theorem 4.14: If $A \subseteq Y \subseteq X$ and A is p*g-closed in X then A is p*g-closed relative to Y.

Proof: Given that $A \subseteq Y \subseteq X$ and A is a p*g-closed set in X. To prove that A is p*g-closed set relative to Y. Let us assume that $A \subseteq Y \cap U$, where U is pre*open in X. Since A is p*g-closed, $A \subseteq U$. This implies that $pcl(A) \subseteq U$. It follows that $Y \cap pcl(A) \subseteq Y \cap U$. That is, A is p*g-closed relative to Y.

5. CONCLUSION

The present paper has introduced a new concept of p*g-closed set in topological spaces. It also analyzed some of the properties. The implication shows the relationship between p*g-closed sets and the other existing sets.

6. REFERENCES

1. Andrijevic, D., Semi preopen sets, Mat. Vesnik, 38(1986), 24-32.
2. Anitha, M., and Thangavelu, P., On Pre Generalized Pre regular Closed sets, Acta Ciencia Indica 31 M (4) (2005), 1035 - 1040, ISSN 0970-0455.
3. El- Deeb, N., Hasanein, I. A., Mashhour, A. S., and Noiri, T., On p-regular spaces, Bull. Math. Soc. Sci. Math. R. S. Roumanie, 22(75)(1983), 311-315.
4. Dontchev, J., and Noiri, T., Quasi Normal Spaces and πg-closed sets, Acta Math. Hungar, 89, 3, (2000), 211-219.
5. Dontchev, J., On generalizing semi pre open sets, Mem Fac. Sci. Kochi. Univ. Ser. A. Maths 16, 35-48, 1995.
6. Dunham, W., A New Closure Operator for non-T1 topologies, Kyungpook Math. J., 22(1982), 55-60.
7. Gnanambal, Y., On generalized pre regular closed sets in topological spaces, Indian J.pure appl. Math.28 (3) (1997), 351-360.
8. Levine, N., Generalized closed sets in topology, Rend. Circ. Mat. Palermo 19(2) (1970), 89-96.
9. Maki, H., Devi, R., and Balachandran, K., Associated topologies of generalized α-closed sets and α-generalised closed sets, Mem . Fac. Sci. Kochi Univ. Ser. A Math., 15(1994), 51-63.
10. Maki, H., Umehara, J., and Noiri, T., Every topological space in Pre- T1/2, Mem. Fac. Sci, Kochi Univ Ser. A. Maths, (17), 33-42, 1996.
11. Mashhour, A. S., Abd El- Monsef, M. E., and El- Deeb, S. N., On Precontinuous and Weak pre continuous mappings, Proc. Math. Phys. Soc. Egypt, 53(1982), 47-53.
12. Milby Mathew and Parimelazhagan, R., α^m-closed sets in Topological Spaces, International Journal of Mathematical Analysis Vol. 8, 2014, no. 47,2325-2329.
13. Muthuvel, S., and Parimelazhgan, R., b*-closed sets in topological spaces, Int. Journal of Math. Analysis. Vol. 6. 2012. No 47, 2317-2323.
14. Nagaveni, N., Studies on Generalization of Homeomorphisms in Topological Spaces, Ph.D Thesis, Bharathiyar Univ. Coimbatore, 1999.
15. Njastad, O., Some classes of nearly open sets, Pacific J. Math., 15(1965), 961-970.
16. Noiri, T., Maki, H., and Umehara, J., " Generalized Preclosed Functions ". Mem. Fac.Sci. Kochi. Univ. (Math) 19, (1998), 13-20.
17. Palaniappan, N., and Rao, K. C., Regular generalized closed sets, Kyungpook Math. J., 33(1993), 211-219.
18. Park, J. H., On πgp-closed sets in topological spaces, Indian J. Pure Appl. Math (to appear)
19. Sarsak, M. S., and Rajesh, N., (2010), π-Generalized semi pre closed sets.International Mathematical Forum, 5(12), 573-578.
20. Selvi, T., and Punitha Dharani, A., Some new class of nearly closed and open sets, Asian Journal of Current Engineering and Maths 1:5 Sep – Oct (2012) 305-307.
21. Stone, M., Application of the theory of Boolean rings to general topology, Trans. Amer.Math. Soc. 41 (1937), 374-481.
22. Vadivel, A., and Vairamanickam, K., rgα-closed sets and rgα-open sets in topological spaces, Int. Journal of Math. Analysis. Vol. 3, 2009, no. 37, 1803-1819.
23. Veerakumar. M. K. R. S., g*-closed sets in topological spaces, Mem. Fac. Sci. Kochi Univ. Math, 21(2000), 1-19.
24. Veera kumar, M. K. R. S., g*-preclosed sets, Acta ciencia Indica (Maths), Meerut, XXVIII (M) (1) 2002, 51-60.
25. Veerakumar, M. K. R. S., Pre semi closed, Indian J.Math, 44(2), 165-181, 2002.
26. Zaitsev, V., On certain class of Topological spaces and their bicompactification, Dokl Akad Nauk SSSR 178(1968). pp 778-779.

Source of support: Nil, Conflict of interest: None Declared.