HEAT TRANSFER OVER A FLAT PLATE IN POROUS MEDIUM WITH HEAT SOURCE AND VISCOUS DISSIPATION IN SLIP FLOW REGIME

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ABSTRACT

In this analysis, MHD boundary layer flow and heat transfer of a fluid with slip at boundary through a porous medium towards a flat porous plate by taking into the effects of viscous dissipation in presence of heat source/sink is considered. The similarity transformations are used to convert the partial differential equations of the governing equation into the self similar non-linear ordinary differential equation. Numerical solutions of these equations are obtained by Runge-Kutta fourth order with shooting method. Numerical results are obtained for different parameters such as permeability parameter $K$, magnetic parameter $M$, Prandtl Number $Pr$, Eckert Number $Ec$, heat source/sink parameter $\lambda$ and suction/blowing parameter $S$ are drawn graphically and the effects of different flow parameters on velocity and temperature are discussed.

INTRODUCTION

Boundary layer flow and heat transfer problems in the presence of magnetic field through a porous medium with viscous dissipation and heat source have gained tremendous interest amongst researchers for past two decades because of industrial and engineering applications, especially in the enhanced recovery of petroleum resources and packed bed reactors (Pal and Shivkumara, 2006). In engineering science, it finds its application in MHD pumps, MHD bearing, MHD power generators, et al.

The flow and heat transfer over a flat plate has been widely studied from both theoretical and experimental standpoint in the past few decades. Blasius (1908) probably was first who discussed the formation of the velocity boundary layer due to the flow on a flat plate and Pohlhausen (1921) extended Blasius problem for the heat transfer. Howarth (1938) studied the various aspects of the Blausis flat plate flow problem. Abu-sitta (1994) establish the existence of a solution for flow past a flat plate. An approximate solution of the classical Blausis equation using Adomian decomposition method was reported by Wang (2004). Cortell (2005) presented a numerical investigation of the classical Blausis flat plate problem. Recently Mukhopadhyay and Layek (2009) presented the radiation effects on forced convective flow and heat transfer over a porous plate in porous medium. The study through porous medium can benefit several areas like catalytic reactors, filtering devices and heat exchanges etc.

In every the investigations mentioned above, the non slip condition at the boundary was assumed. The assumption of non slip condition does no longer valid and should be replaced by a boundary condition relating to the shear rate at the boundary. Beavers and Joseph (1967) had used the partial slip condition in study of fluid flow past permeable wall. The effects of slip boundary condition on the flow of Newtonian fluid due to a stretching sheet were explained by Andersson (2002) and Wang (2002). Raptis and Kafousian (1982) have investigated the problem of MHD free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Rashad (2008) discussed MHD and thermal radiation effects on heat and mass transfer in steady boundary layer flow over a vertical flat plate embedded in a fluid saturated porous media in the presence of the thermophoresis particle deposition effect.

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Viscous dissipation changes the temperature distributions by playing a role like energy source, which leads to affected heat transfer rates. The merit of the effect of viscous dissipation depends on whether the plate is being cooled or heated. Heat transfer analysis over porous surface is of much practical interest due to its abundant applications. Anjali devi and Ganga(2009) have considered the Viscous dissipation effects on MHD flows past stretching porous surfaces in porous media. A new dimension is added to the above mentioned study of Mukhopadhyay et al.(2005) by considering the effects of porous media. Alam and Sattar(2000) studied unsteady MHD free convection and mass transfer flow in rotating system with hall current viscous dissipation and joule heating. Mhd, free convection and mass transfer with hall current, viscous dissipation, joule heating and thermal diffusion was studied by Singh (2003). MHD effects on heat transfer over stretching sheet embedded in porous medium with variable viscosity, viscous dissipation and heat source/sink was studied by Dessie and Kishan (2014). Jamalabadi and Park (2014) presented thermal radiation, Joule heating and viscous dissipation effects on MHD forced convection flow with uniform surface temperature. Mukhopadhyay et al. (2011) studied the effects of variable viscosity on the boundary layer flow and heat transfer of fluid flow through a porous medium towards a stretching sheet in presence of heat generation or absorption. MHD flow and heat transfer over stretching/shrinking sheets with external magnetic field, viscous dissipation and joule effects was presented by Jafar, et al.(2012). Reddy et.al (2015) studied Internal heat generation and viscous dissipation effects on nanofluids over a moving vertical plate with convective boundary conditions. Reddy (2015) studied MHD flow over a vertical moving porous plate with viscous dissipation by considering double diffusive convection in the presence of chemical reaction. Sreenivasulu et al. (2013) presented the radiation and viscous dissipation effects on steady MHD maragoni convection flow over a permeable flat surface with heat generation or absorption.

The present work is concerned with the effects of viscous dissipation and heat transfer on MHD boundary layer flow on a flat porous plate immersed in a porous media in the presence of heat source/sink in slip flow regime. Thermal slip is also considered which gives interesting features regarding such flow. The slip model of Andersson (2002) is taken here in a modified form. An efficient Numerical shooting technique with a fourth order Runge- Kutta scheme is used to solve the normalized boundary layer equations and the effects of various physical parameters on the flow field and heat transfer characteristics is discussed in detail.

MATHEMATICAL FORMULATION

Consider a steady two-dimensional flow of an incompressible viscous, thermally and electrically conducting fluid past over a flat porous plate immersed in a porous medium which is subject to slip boundary conditions at the interface of porous and fluid layers. A uniform transverse magnetic field of magnitude \( B_0 \) is applied. It is assumed that there is no applied voltage which implies the absence of an electric field. The governing equations for this investigation are based on the balance of mass, linear momentum and energy. Taking in to consideration these assumptions the equation that describe the physical situation can be written in Cartesian frame of references, as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k} (u - U_\infty) + \frac{\sigma B_0^2}{\rho} (u - U_\infty), \quad (2)
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty) + \frac{\mu}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^2, \quad (3)
\]

Subject to the appropriate boundary conditions with partial slip:

\[
y = 0: \quad u = L_i \left( \frac{\partial u}{\partial y} \right), \quad v = v_w \quad T = T_w + D_i \left( \frac{\partial T}{\partial y} \right), \quad (4)
\]

\[
y \to \infty: \quad u \to U_\infty, \quad T \to T_\infty. \quad (5)
\]
where \( u \) and \( v \) are the velocity components in x- and y- directions respectively, \( \nu(=\mu/\rho) \) is the kinematic viscosity, \( \rho \) is the density, \( \mu \) is the dynamic viscosity, \( k \) is the permeability of the porous medium, \( U_\infty \) is the free stream velocity, \( T \) is the temperature, \( \kappa \) is the thermal conductivity of the fluid, \( \sigma \) is the electrical conductivity, \( C_p \) is the specific heat, \( Q_0(Js^{-1}m^{-3}k^{-1}) \) is the dimensional heat generation \((Q_0>0)\) or absorption \((Q_0<0)\) coefficient, \( L_1 = L\sqrt{\frac{U_\infty x}{\nu}} \) and \( D_1 = D\sqrt{\frac{U_\infty x}{\nu}} \) are the velocity and thermal slip factor with \( L \) and \( D \) being initial values of the slip factors, \( T_w \) is the temperature of the plate and \( T_\infty \) is the free stream temperature, both assumed to be constants. Here \( \nu \) is prescribed suction or blowing through the porous plate and is given by \( \nu = \frac{V_0}{\sqrt{X}} \), \( V_0 \) being constant with \( V_0 < 0 \) for suction and \( V_0 > 0 \) for blowing.

We look for a similarity solution of the Equations (1)-(3) subject to the boundary conditions (4)-(5) of the following form

\[
\psi(x,y) = \sqrt{U_\infty \nu x} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{(T_w - T_\infty)}, \quad \eta = \sqrt{\frac{U_\infty}{\nu x}}
\]

Where \( \psi \) is the stream function defined as \( u = \frac{\partial \psi}{\partial y} \) and \( v = -\frac{\partial \psi}{\partial x} \) which identically satisfied Equation (1). Substituting (6) into Equations (2) and (3) we obtain the following ordinary differential equations

\[
f'''' + \frac{1}{2} ff''' - K (f' - 1) + M (f' - 1) = 0 \quad (7)
\]

\[
\theta'' + \frac{1}{2} Pr f \theta' + \lambda Pr \theta + Ec Pr f'^2 = 0 \quad (8)
\]

Subject to the boundary conditions (4) and (5), which become

\[
\eta = 0; \quad f(\eta) = S, \quad f'(0) = \delta f'(0), \quad \theta(\eta) = 1 + \beta \theta'(\eta) \quad (9)
\]

\[
\eta \to \infty; \quad f'(\eta) \to 1, \quad \theta(\eta) \to 0 \quad (10)
\]

Where prime denotes differentiation with respect to \( \eta \), \( S = -2\sqrt{U_\infty \nu} \), where \( S > 0 \) (i.e. \( \nu < 0 \)) suction or \( S < 0 \) (i.e. \( \nu > 0 \)) blowing effect, \( \delta = LU_\infty / \nu \) is the velocity slip parameter and \( \beta = DU_\infty / \nu \) is the thermal slip parameter, \( K \) is permeability number, \( M \) is magnetic interaction number, \( Pr \) is the Prandtl number, \( Ec \) is the Eckert number, \( \lambda \) is heat source parameter, which are defined as;

\[
K = \frac{1}{(Da_x Re_x)}, \quad Da_x = k_0 x, \quad Re_x = \frac{U_\infty x}{\nu}, \quad M = \frac{\sigma B^2}{\rho U_\infty}, \quad Pr = \frac{\mu C_p}{\kappa}, \quad Ec = \frac{U_\infty^2}{C_p(T_w - T_\infty)}, \quad \lambda = \frac{Q_0 x}{\rho C_p U_\infty} \quad (11)
\]

Where \( Da_x \) is local Darcy number and \( Re_x \) is local Reynolds number.

**NUMERICAL METHOD FOR SOLUTION**

Equations (7) and (8) constitute a highly non-linear coupled boundary value problem of third order and second order respectively. Exact solution does not seem to be feasible for complete set of equations (7) and (8) and therefore we have developed a most effective numerical shooting technique with fourth order Runge-Kutta technique by converting these equations into initial value problem. Boundary conditions are also simultaneously reduced into first order ordinary differential equations.

We set

\[
f' = p, \quad p' = q, \quad q' = -\frac{1}{2} fq + K(p - 1) - M(p - 1) \quad (12)
\]

\[
\theta' = z, \quad z' = -\left[ \frac{1}{2} Pr fz + \lambda Pr \theta + Ec Pr q^2 \right] \quad (13)
\]

With the boundary conditions

\[
f(0) = 0, \quad p(0) = \delta q(0), \quad \theta(0) = 1 + \beta z(0) \quad (14)
\]
To solve equations (12) and (13) with (14) as an IVP, we must need values for $q(0)$ i.e. $f''(0)$ and $z(0)$ i.e. $\theta'(0)$ but no such values are given. The initial values for $f''(0)$ and $\theta'(0)$ are chosen and by applying fourth order Runge-Kutta Method, solution are obtained. We compare the calculated values for $f'(\eta)$ and $\theta(\eta)$ at $\eta_c(-20)$ with the given boundary condition $f'(\eta_0) = 1$ and $\theta(\eta_0) = 0$ and adjust estimated values of $f''(0)$ and $\theta'(0)$ to give a better approximation for the solution. Different values of $\eta$ are taken in our numerical computations so that obtained numerical values are independent of $\eta$ chosen. The step size $\Delta \eta = 0.001$ is used to obtain the numerical solution with sixth decimal ($1 \times 10^{-6}$) accuracy as criteria of convergence.

RESULTS AND DISCUSSION

In order to analyze the effects of the various physical parameters, numerical computations have been performed. The effects of various physical parameters on velocity profile and temperature profile have been discussed and are shown graphically. Velocity profile $f'(\eta)$ and the skin friction coefficient $f''(0)$ do not vary with either Eckert number (Ec) or Prandtl number (Pr). This is expected since Eckert number (Ec) or Prandtl number (Pr) only appears in the temperature equation that is equation (13). Velocity profile does not effect by heat source/sink parameter $(\lambda)$. Fig.1(a)-(c) illustrate the effects of permeability parameter $K$ on the velocity and temperature profiles respectively. It is observed that the velocity increases with increasing value of $K$. hence thickness boundary layer decreases in velocity profile. The temperature profile increases with increase in $K$ in both of case presence/absence of heat source/sink parameter. Therefore thermal boundary layer thickness increases with increasing $K$.

Fig. 2(a)-(c) depict the velocity and temperature profile to the effects of magnetic parameter $M$. Here different values of $M$ leads to decrease velocity and increase the boundary layer thickness as shown in figure (a). 2(b) and (c) show that with increasing value of $M$, firstly temperature increase but as far we move it decreases in both presence/absence of heat source/sink parameter.

Fig. 3 shows the effects of Prandtl number on heat transfer. It can be noticed that an increase in Prandtl number reduces the thermal boundary layer thickness. In heat transfer problems, the Prandtl number controls the relative thickening of momentum and thermal boundary layers. Hence Prandtl number can be used to increase the rate of cooling in conducting flow.

Fig. 4 is the graphical representation of the temperature profiles for different values of heat source/sink parameter $(\lambda)$. From figure it is observed that the temperature profiles decreases for increasing of the heat sink $(\lambda < 0)$ and due to increase in heat source $(\lambda > 0)$ temperature increases so that the thickness of thermal boundary layer reduces for the increase in heat sink parameter but it decreases with heat source parameter. This result is very much significant for the flow where heat transfer is given prime important.

In fig. 5(a) and (b) the effects of viscous dissipation i.e. Eckert number Ec on the temperature profile exhibited. The Eckert number expresses the relation between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. It can be seen from figures the effects of Ec leads to increase temperature profile in case of presence/absence of heat source/sink parameter.

Fig.6(a)-(c) presents influence of suction $S$ ($S>0$) and blowing $S(S<0)$ parameter on the velocity and temperature profile in presence of magnetic field at the boundary for flat plate embedding in porous medium. With the increasing $S(S>0)$, the fluid velocity increases as shown in fig.6(a). Suction causes to increase of fluid velocity and reduces the thickness of fluid boundary layer due to sucking fluid particles through porous wall. Hence the velocity gradient increases. Opposite behaviour is noted for blowing $S(S<0)$. Fig.6(b) and (c) shows that the temperature $\theta$ increases with increasing suction parameter $S$ ($S>0$) but far away it decreases with increasing suction parameter $S$ ($S>0$). The temperature decreases with increasing blowing parameter $S(S<0)$ but far away it increase with increasing blowing parameter $S(S<0)$. However the behaviour is found to be not uniform throughout the analysis.

CONCLUSION

In this paper the effects of viscous dissipation and an external uniform magnetic field on the flow and temperature distribution of an electrically conducting fluid over a porous plate in the presence of porous medium and heat source/sink with partial slip conditions at boundary. The governing equations were developed and transformed using appropriate similarity transformation and then solved numerically using Runge-Kutta scheme with shooting method. The following conclusion can be drawn from computed results:

- The effect of permeability parameter $K$ is to increase both velocity and temperature.
• Prandtl Number (Pr) and Eckert Number (Ec) do not affect velocity. Prandtl Number Pr leads to decrease temperature while Eckert number Ec leads to increase temperature in both case presence/absence of heat source/sink parameter.
• The temperature profile is lower throughout the boundary layer for heat sink parameter and higher for heat source parameter.
• The effects of magnetic parameter M leads to increase boundary layer thickness in starting but decreases later.
• The thermal and momentum boundary layer thickness increases with applied suction and applied blowing respectively whether decrease with applied blowing and applied suction respectively.

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REFERENCES

Figure-1(a): Velocity profile $f'(\eta)$ for various values of $K$ with $M=0.2$, $Pr=0.3$, $Ec=0.5$, $\delta=0.1$, $\beta=0.1$ and $S=0.2$.

Figure-1(b): Temperature profile $\theta(\eta)$ for various values of $K$ with $M=0.2$, $Pr=0.3$, $Ec=0.5$, $\delta=0.1$, $\beta=0.1$, $S=0.2$ and in absence of heat source/sink.
Figure-1(c): Temperature profile $\theta(\eta)$ for various values of $K$ with $M=0.2$, $Pr=0.3$, $Ec=0.5$, $\delta=0.1$, $\beta=0.1$, $S=0.2$ and in presence of heat source/sink.

Figure-2(a): Velocity profile $f'(\eta)$ for various values of $M$ with $K=0.1$, $Pr=0.3$, $Ec=0.5$, $\delta=0.1$, $\beta=0.1$ and $S=0.2$.

Figure-2(b): Temperature profile $\theta(\eta)$ for various values of $M$ with $K=0.1$, $Pr=0.3$, $Ec=0.5$, $\delta=0.1$, $\beta=0.1$, $S=0.2$ and in absence of heat source/sink.
Figure-2(c): Temperature profile $\theta(\eta)$ for various values of $M$ with $K=0.1$, $Pr=0.3$, $Ec=0.5$, $\delta=0.1$, $\beta=0.1$, $S=0.2$ and in presence of heat source/sink.

Figure-3: Temperature profile $\theta(\eta)$ for various values of $Pr$ with $K=0.1$, $M=0.2$, $Ec=0.5$, $\delta=0.1$, $\beta=0.1$, and $S=0.2$.

Figure-4: Temperature profile $\theta(\eta)$ for various values of $\lambda$ with $K=0.1$, $M=0.2$, $Ec=0.5$, $Pr=0.3$, $\delta=0.1$, $\beta=0.1$, and $S=0.2$. 

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Figure-5(a): Temperature profile $\theta(\eta)$ for various values of Ec with $K=0.1$, $M=0.2$, $Pr=0.3$, $\delta=0.1$, $\beta=0.1$, $S=0.2$ and in absence of heat source/sink.

Figure-5(b): Temperature profile $\theta(\eta)$ for various values of Ec with $K=0.1$, $M=0.2$, $Pr=0.3$, $\delta=0.1$, $\beta=0.1$, $S=0.2$ and in presence of heat source/sink.

Figure-6(a): Velocity profile $f'(\eta)$ for various values of S with $K=0.1$, $Pr=0.3$, $Ec=0.5$, $\delta=0.1$, $\beta=0.1$ and $M=0.2$. 
Figure-6(b): Temperature profile $\theta(\eta)$ for various values of $S$ with $K=0.1$, $M=0.2$, $Pr=0.3$, $\delta=0.1$, $\beta=0.1$, $Ec=0.5$ and in absence of heat source/sink.

Figure-6(c): Temperature profile $\theta(\eta)$ for various values of $S$ with $K=0.1$, $M=0.2$, $Pr=0.3$, $\delta=0.1$, $\beta=0.1$, $Ec=0.5$ and in presence of heat source/sink.

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