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MHD BOUNDARY LAYER SLIP FLOW AND HEAT TRANSFER OVER A POROUS FLAT PLATE EMBEDDED IN A POROUS MEDIUM

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ABSTRACT

A mathematical model is presented for analyzing the steady magneto- hydrodynamic (MHD) boundary layer forced convective flow and heat transfer of an electrically conducting fluid past a porous plate embedded in a porous medium with slip conditions at the boundary. The governing system of partial differential equations is first transformed into a system of ordinary differential equations by introducing an appropriate similarity transformation, which is then solved numerically using a finite difference method. Velocity and temperature profiles are shown graphically for different values of parameters involved and discussed in detail. In the presence of magnetic field, both fluid velocity and temperature increase with increasing suction and decrease for increasing blowing. Results indicate that temperature decrease with thermal slip parameter. On the other hand for increasing Prandtl Number, thermal boundary layer thickness decreases rapidly.

Keywords: *Boundary Layer Slip Flow, MHD, Heat Transfer, Porous Medium, Suction/Blowing, Flat Porous Plate.*

INTRODUCTION

In recent years boundary layer flow behaviour and heat transfer problems in the presence of magnetic field through a porous medium have attracted the attention of a number of scholars because of their possible application in many branches of science and technology, especially in the enhanced recovery of petroleum resources and packed bed reactors (Pal and Shivkumara, 2006).

The flow and heat transfer over a flat plate has been widely studied from both theoretical and experimental standpoint in the past few decades. Blasius (1908) probably was first who discussed the formation of the velocity boundary layer due to the flow on a flat plate and Pohlhausen (1921) extended Blasius problem for the heat transfer. Howarth (1938) studied the various aspects of the Blausis flat plate flow problem. Abusitta (1994) establish the existence of a solution for flow past a flat plate. An approximate solution of the classical Blassius equation using Adomian decomposition method was reported by Wang (2004). Cortell (2005) presented a numerical investigation of the classical Blausis flat plate problem. Recently Mukhopadhyay and Layek (2009) presented the radiation effects on forced convective flow and heat transfer over a porous plate in porous medium. The study through porous medium can benefit several areas like catalytic reactors, filtering devices and heat exchanges etc.

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In every the investigations mentioned above, the non slip condition at the boundary was assumed. The assumption of non slip condition does no longer valid and should be replaced by a boundary condition relating to the shear rate at the boundary. Beavers and Joseph(1967) had used the partial slip condition in study of fluid flow past permeable wall. The effects of slip boundary condition on the flow of Newtonian fluid due to a stretching sheet were explained by Andersson(2002) and Wang(2002). Raptis and Kafousian (1982) have investigated the problem of MHD free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Rashad (2008) discussed MHD and thermal radiation effects on heat and mass transfer in steady boundary layer flow over a vertical flat plate embedded in a fluid saturated porous media in the presence of the thermophoresis particle deposition effect. Pal and Talukdar (2010) presented an analytical solution of unsteady MHD convective heat and mass transfer past a vertical permeable plate with thermal radiation and chemical reaction in the presence of slip at the boundary . Mukhopadhyay and Bhattacharyya (2011) explained an analysis for boundary layer forced convective flow and heat transfer past a moving porous plate parallel to a moving stream. Bhattacharyya (2012) explained slip effects on boundary layer flow and mass transfer with chemical reaction over a permeable flat plate in porous medium. Ahmed and Das (2013) investigated effects of thermal radiation and chemical reaction on MHD unsteady mass transfer flow past a semi infinite vertical porous plate embedded in a porous medium in a slip flow regime with variable suction. Sharma and Konwar (2014) presented effects of MHD flow heat and mass transfer about a horizontal cylinder in porous medium. Dessie and Kishan (2014) discussed an heat transfer over stretching sheet embedded in porous medium with variable viscosity, viscous dissipation and heat source.

Despite the large number of previous work done by different researchers dealing with MHD fluid flow, heat and mass transfer, boundary layer flow in porous media, there is a still considerable need for more comprehensive and reliable methods of accurately predicting the fluid flow, heat and mass transfer characteristics in many problems. The aim of this paper is to study the heat transfer in steady MHD boundary layer flow with partial slip conditions. Thermal slip is also considered which gives interesting features regarding such flow. The slip model of Andersson (2002) is taken here in a modified form. Using similarity solution technique, the governing partial differential equations of flow and heat transfer are transformed into a set of self similar non- linear differential equation. These equations with boundary condition are solved numerically using shooting method. Computed numerical results are plotted and the characteristics of the flow and heat transfer are thoroughly analyzed.

FORMULATION OF THE PROBLEM

Consider a two- dimensional steady flow of a viscous incompressible, electrically conducting fluid past a porous plate embedded in a porous medium in the presence of applied transverse magnetic field of a constant strength B_0 . The induced magnetic field is assumed to be small as compared to applied magnetic field and therefore it is neglected. All the fluid properties are assumed to be constant throughout the motion. Under the usual boundary layer approximation the basic governing equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\upsilon}{k}(u-U_\infty)+\frac{\sigma_c B_0^2}{\rho}(u-U_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_P}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u and v are the velocity components in x- and y- directions respectively, $\upsilon(=\mu/\rho)$ is the kinematic viscosity, ρ is the density, μ is the coefficient of viscosity, k is the permeability of the porous medium, U_∞ is the free stream velocity, T is the temperature, κ is the thermal conductivity of the fluid, σ_c is the electrical conductivity of the fluid.

The appropriate boundary conditions with partial slip for velocity and temperature are:

$$y=0: \quad u = L_1(\partial u/\partial y), \quad \mathrm{v} = \mathrm{v}_w, \quad T = T_w + D_1(\partial T/\partial y) \tag{4}$$

$$y \to \infty: \quad u \to U_\infty, \quad \mathrm{T} \to T_\infty \tag{5}$$

where $L_1 = L\sqrt{\frac{U_\infty x}{\upsilon}}$ and $\mathrm{D}_1 = D\sqrt{\frac{U_\infty x}{\upsilon}}$ are the velocity and thermal slip factors respectively with L and D being initial values of the slip factors having the same dimension of length, T_w is the temperature of the plate and T_∞ is the free stream temperature, both assumed to be constants. Here v_w is prescribed suction or blowing through the porous plate and is given by $\mathrm{v}_w = \frac{v_0}{\sqrt{x}}$, v_0 being constant with $v_0 < 0$ for suction and $v_0 > 0$ for blowing.

ANALYSIS

The continuity equation (1) is identically satisfied by a stream function $\psi(x, y)$, defined as:

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \tag{6}$$

For the solution of momentum and energy equations (2) and (3), the following similarity transformation are defined

$$\psi(x, y) = \sqrt{U_\infty \upsilon x}\, f(\eta), \tag{7}$$

$$\theta(\eta) = \frac{\mathrm{T} - \mathrm{T}_\infty}{(\mathrm{T}_w - \mathrm{T}_\infty)}, \tag{8}$$

Where $$\eta = y\sqrt{\frac{U_\infty}{\upsilon x}} \tag{9}$$

Thus the momentum and energy equation (2) and (3) after some simplifications reduce to

$$f''' + \frac{1}{2} ff'' - K(f'-1) + M(f'-1) = 0 \tag{10}$$

$$\theta'' + \frac{1}{2}\Pr f\theta' = 0 \tag{11}$$

The corresponding boundary condition is:

$$\left.\begin{aligned} &\eta = 0; \quad f(\eta) = \mathrm{S}, \quad f'(\eta) = \delta f''(\eta), \quad \theta(\eta) = 1 + \beta\theta'(\eta) \\ &\eta \to \infty; \quad f'(\eta) \to 1, \quad \theta(\eta) \to 0 \end{aligned}\right\} \tag{12}$$

The physical quantities of interest are the local skin friction coefficient c_f and heat transfer rate i.e. the Nussult number Nu_x are:

$$c_f = \frac{2\tau_w}{\rho U_\infty^2} \tag{13}$$

where the surface shear stress τ_w is defined as

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{14}$$

and the heat transfer between the surface and the fluid conventionally expressed is dimensionless as a local Nussult number is given by

$$Nu_x = \frac{x}{\kappa(T_w - T_\infty)} q_w \tag{15}$$

Where the surface heat flux q_w is defined as

$$q_w = -\kappa \left(\frac{\partial T}{\partial y} \right)_{y=0} \tag{16}$$

Using the similarity variable equations (7), (8) and (9), we obtain

$$C_f = \frac{f''(0)}{\mathrm{Re}_x}, \qquad Nu_x = -\theta'(0)\,\mathrm{Re}_x^{1/2} \tag{17}$$

Where the non-dimensional parameters

$K = \dfrac{1}{(Da_x \mathrm{Re}_x)}$ (Permeability parameter)

$Da_x = \dfrac{k_0}{x}$ (Local Darcy parameter)

$\mathrm{Re}_x = \dfrac{U_\infty x}{\upsilon}$ (Reynolds number)

$M = \dfrac{\sigma_c B_0^2 x}{\rho U_\infty}$ (Magnetic interaction parameter)

$\mathrm{Pr} = \dfrac{\mu c_p}{\kappa}$ (Prandtl number)

$\mathrm{S} = -2 v_0 \big/ \sqrt{(U_\infty \upsilon)}$, (Suction / injection parameter)

$\delta = LU_\infty / \upsilon$ (Velocity slip parameter)

$\beta = DU_\infty / \upsilon$ (Thermal slip parameter)

The non-linear coupled differential equations (10) and (11) together with boundary conditions (12) forms a two point boundary value problem (BVP). First we convert above two point BVP into initial value problem by applying the shooting technique and then solved by Runge-Kutta fourth order method . For this we set the following first order system

$$f' = p. \quad p' = q. \quad q' = -\frac{1}{2} fq + K(\mathrm{p}-1) - \mathrm{M}(\mathrm{p}-1) \tag{18}$$

$$\theta' = z. \quad z' = -\frac{1}{2} \mathrm{Pr}\ fz \tag{19}$$

With the boundary conditions

$$f(0) = 0. \quad \mathrm{p}(0) = \delta q(0) \quad \theta(0) = 1 + \beta\, \mathrm{z}(0) \tag{20}$$

In order to integrate Equations (18) and (19) as IVP one requires a value for $q(0)$ i.e. $f''(0)$ and $\mathrm{z}(0)$ i.e. $\theta'(0)$ but no such values are given in the boundary conditions. The suitable guess values for $f''(0)$ and $\theta'(0)$ are chosen by the shooting technique and then solved the system by using fourth order Runge- Kutta method. Taking a suitable finite value of $\eta\ (\to \infty)$, say η_∞, we compare the calculated values of $f'(\eta)$ and $\theta(\eta)$ at $\eta_\infty(20)$ with the given boundary conditions $f'(\eta_\infty) = 1$ and $\theta(\eta_\infty) = 0$ and adjust estimated values of $f''(0)$ and $\theta'(0)$ to give a better approximation for the solution using Secant method. The step size is taken as $\Delta\eta = 0.01$. The above procedure is repeated until we get the result upto desired degree of accuracy 10^{-6}. All the computations are done in the Matlab software.

RESULTS AND DISCUSSION

The numerical computations have been carried out for several values of parameters involved in these equations, namely, the permeability parameter K, the magnetic field parameter M, velocity slip parameter δ, thermal slip parameter β, and the Prandtl number Pr.

The effects of permeability parameter K on velocity profile are shown in figure1 for both slip and non slip conditions. It is observed that the velocity f' increases with increasing value of K or in other words the momentum boundary layer thickness decreases with increasing value of K for both slip and non-slip conditions. Now we shall pay our attention to figure2 which is plotted to the velocity profile f' for different values of magnetic parameter M. The effect of M leads to decrease the velocity profile f' and and consequently the thickness of momentum boundary layer increases with increasing M for both slip and non-slip conditions .

Figure3 presents the influence of suction S (S>0) and blowing S(S<0) parameter on the velocity and temperature profile in presence of magnetic field at the boundary for flat plate embedding in porous medium. With the increasing S(S>0), the fluid velocity increases. Suction causes to increase of fluid velocity and reduces the thickness of fluid boundary layer due to sucking fluid particles through porous wall. Hence the velocity gradient increases. Opposite behaviour is noted for blowing S(S<0).

Figure4 indicates that as the magnitude of $\delta(a)$ increases, the fluid velocity increases monotonically. Increasing in slip parameter, causes to decrease the momentum boundary layer thickness.

Figure5 presents that the temperature θ increases with K for slip as well as non slip conditions. Increase in permeability parameter K cause to increase in thermal boundary layer thickness. Figure6 shows that temperature profile θ decreases with M and thermal boundary layer thickness decreases as M as increases. Figure7 shows that the temperature θ increases with increasing suction parameter S (S>0). The thermal boundary layer thickness increases with S (S>0) which cause a decrease in the rate of heat transfer. The temperature decreases with increasing blowing parameter S(S<0).

We have studied effects of the velocity slip parameter on the temperature profile figure. Figure8 shows that, temperature profile $\theta(\eta)$ decreases with increasing in velocity slip parameter (δ). The rate of heat transfer is enhanced with velocity due to slip near the plate since when it increases, then thermal boundary layer decreases.

From figure9 we see that increasing value of Prandtl Number Pr has the rapid effects of decreasing the temperature profile $\theta(\eta)$ and thermal boundary layer thickness for both slip and non slip conditions. Figure10 presents the effect of thermal slip parameter on temperature profile. The temperature as well as the heat transfer rate decrease with increasing value of thermal slip parameter.

Figure11 is graphical presentation of skin friction coefficient $f''(0)$ with magnetic parameter M. It is observed that the skin friction coefficient decreases rapidly and approaches zero as the slip starts to increase and magnetic parameter affects conversely, i.e. it increases with increasing M. The skin friction coefficient is maximum at the non-slip conditions which are similar to the observation of Cao and Baker (2009).

CONCLUSION

The present study gives the numerical solution for MHD effects on the boundary layer flow and heat transfer over a flat porous plate embedded in a porous medium with slip at the boundary. The self similar equation obtained for this flow and are solved by the finite differences method using shooting technique. Our study reveals that the thickness of velocity boundary layer decreases with increase in permeability parameter, velocity slip parameter and applied suction. Whereas it increases with applied blowing and magnetic parameter. The thermal boundary layer thickness increases with increase in permeability parameter and applied suction. Whereas it decreases with velocity slip parameter, magnetic parameter and applied blowing.

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REFERENCES

1. Abussita, A.M.M., 1994. A note on a certain boundary-layer equation. Appl.Math. Comput. 64, 73–77.
2. Ahmed, N., Das, K.K., 2013. MHD mass transfer flow past a vertical porous plate embedded in a porous medium in a slip flow regime with thermal radiation and chemical reaction. 3, 230-239.
3. Andersson, H.I., 2002. Slip flow past a stretching surface. Acta Mech. 158, 121–125.

4. Beavers, G.S., Joseph, D.D., 1967. Boundary conditions at a naturally permeable wall. J. Fluid Mech. 30, 197–207.
5. Bhattacharya, K., 2012. Slip effects on boundary layer flow and amss transfer wuth chemical reaction over a permeable flat plate in a porous medium. Frontiers in Heat and Mass Transfer. 3, 043006, 1-6.
6. Blasius, H., 1908. Grenzschichten in Flüssigkeiten mit kleiner Reibung. Z. Math. U. Phys. 56, 1–37.
7. Cao, K., Baker, J., 2009. Slip effects on mixed convective flow and heat transfer from a vertical plate. Int. J. Heat Mass Trans. 52, 3829–3841.
8. Cortell, R., 2005. Numerical solutions of the classical Blasius flat-plate problem. Appl. Math. Comp. 170, 706–710.
9. Cortell, R., 2008. A numerical tackling on Sakiadis flow with thermal radiation. Chin. Phys. Lett. 25, 1340–1342.
10. Dessie, H., Kishan, N., 2014. MHD effects on heat transfer over stretching sheet embedded in porous medium with variable viscosity, viscous dissipation and heat transfer. Ain Shams Eng. Journal. 5, 967-977.
11. Howarth, L., 1938. On the solution of the laminar boundary layer equations. Proc. Roy. Soc. Lond. A 164, 547–579.
12. Mukhopadhyay, S., Layek, G.C., 2009. Radiation effect on forced convective flow and heat transfer over a porous plate in a porous medium. Meccanica 44, 587–597.
13. Mukhopadhyay, S., Bhattacharya, K., 2011. Steady boundary layer flow and heat transfer over a porous moving plate in presence of thermal radiation. Int. Journal of Heat and Mass transfer. 54, 2751-2757.
14. Pal, D., Shivakumara, I.S., 2006. Mixed convection heat transfer from a vertical heated plate embedded in a sparsely packed porous medium. Int. J. Appl. Mech. Eng. 11, 929–939.
15. Pal, D., Talukdar, B., 2010. Perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Commun. Nonlinear Sci. Numer. Simul. 15, 1813–1830.
16. Pohlhausen, E., 1921. Der Wärmeaustausch zwischen festen Körpern und Flüssigkeiten mit kleiner Reibung und kleiner Wärmeleitung. Z. Angew. Math. Mech. 1, 151-121.
17. Raptis, A., Kafousian, N., 1982. MHD free convection flow and mass treansfer through a porous medium bounded by an infite vertical porous plate with constant heat flux. Canadian Journal of Physics, 12, 1725-1729.
18. Rashad, A.M., 2008. Influence of radiation on MHD free convection from a vertical flat plate embedded in porous media with thermophoretic deposition of particles. Commun. Nonlinear Sci. Numer. Simul. 13, 2213–2222.
19. Sharma, B.R., Konwar, H., 2014. MHD flow, heat and mass transfer about a horizontal cylinder in porous medium. Int. Journal of Innovative Research in Sci., Eng. And Tech. 3, 16459-16469.
20. Wang, C.Y., 2002. Flow due to a stretching boundary with partial slip—an exact solution of the Navier–Stokes equations. Chem. Eng. Sci. 57, 3745–3747.
21. Wang, L., 2004. A new algorithm for solving classical Blasius equation. Appl. Math. Comp. 157, 1–9.

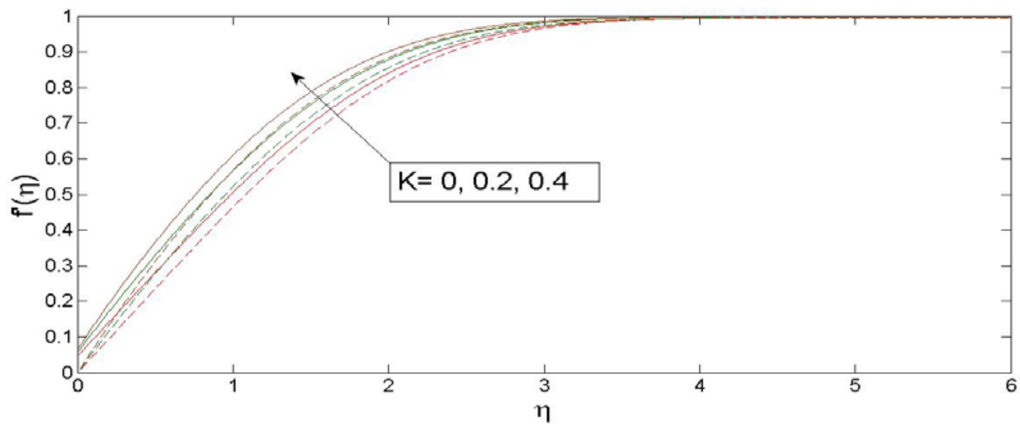

Figure-1: Velocity profile $f'(\eta)$ for various values of K with Slip and non-slip condition and M=0.2, Pr=0.3, S=0.2

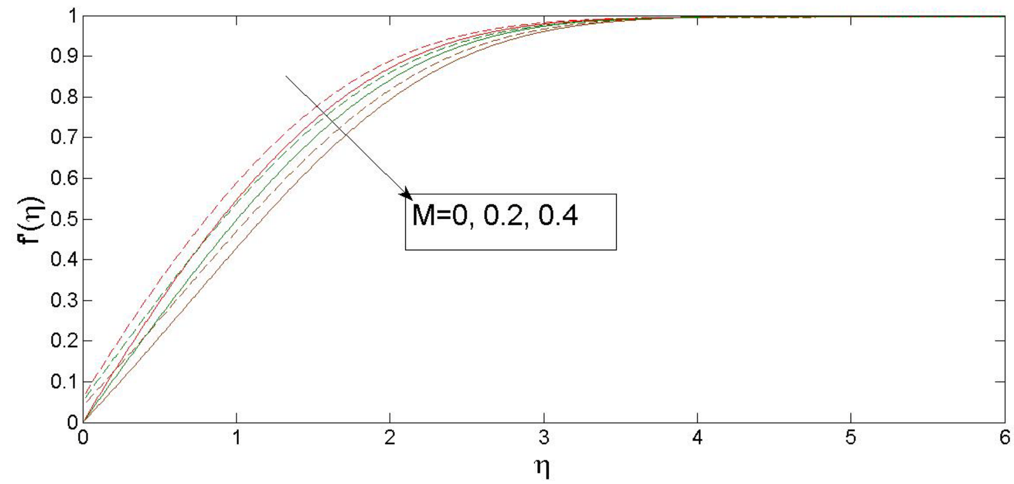

Figure-2: Velocity profile $f'(\eta)$ for various values of M with Slip and non-slip condition and K=0.1, Pr=0.3, S=0.2

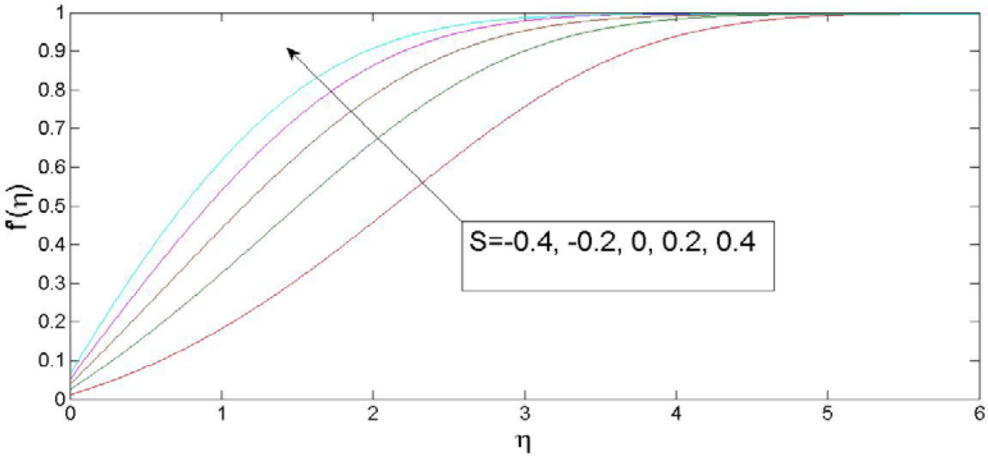

Figure-3: Velocity profile $f'(\eta)$ for various values of S with M=0.2, Pr=0.3, K=0.1, δ =0.1, β =0.1

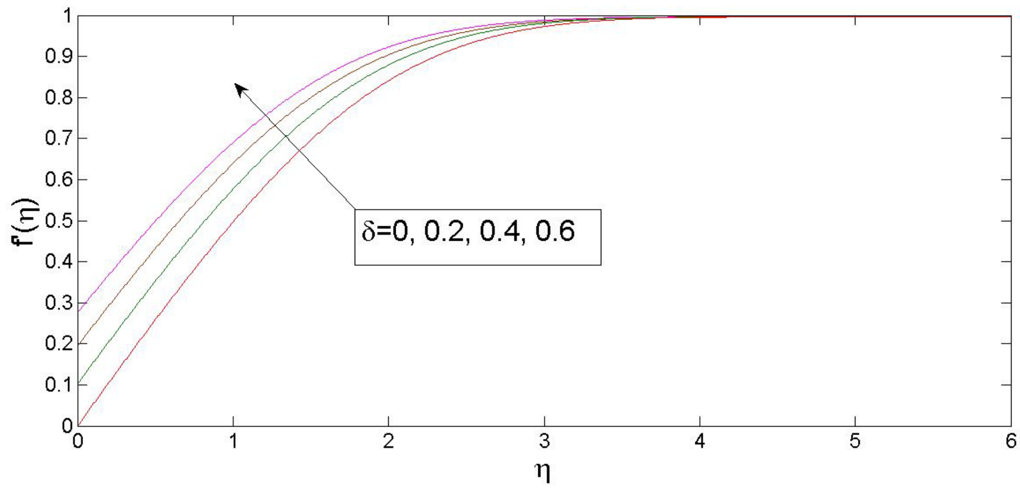

Figure-4: Velocity profile $f'(\eta)$ for various values of δ with K=0.1, M=0.2, Pr=0.3, S=0.2, β =0.1

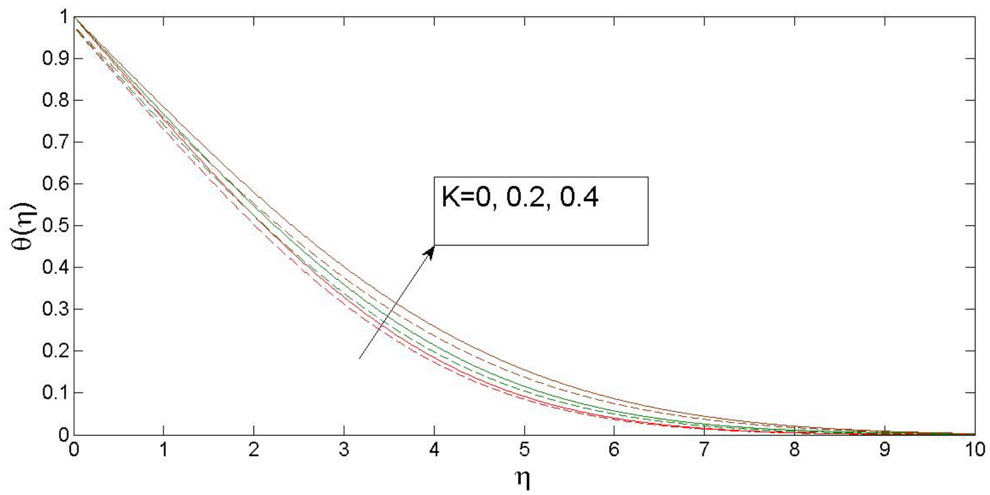

Figure-5: temperature profile $\theta(\eta)$ for various values of K with Slip and non-slip condition and M=0.2, Pr=0.3, S=0.2

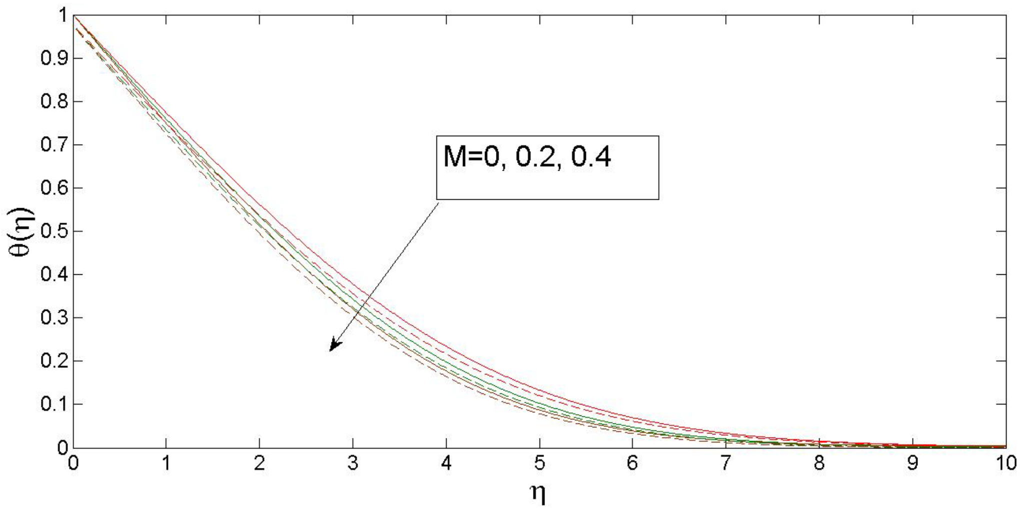

Figure-6: temperature profile $\theta(\eta)$ for various values of M with Slip and non-slip conditions and K=0.1, Pr=0.3, S=0.2

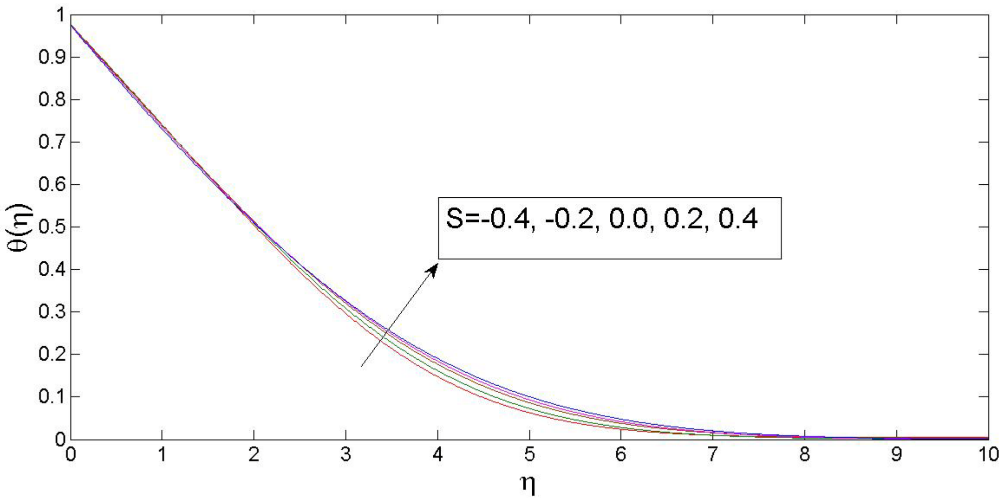

Figure-7: temperature profile $\theta(\eta)$ for various values of S with M=0.2, Pr=0.3, K=0.1, δ =0.1, β =0.1

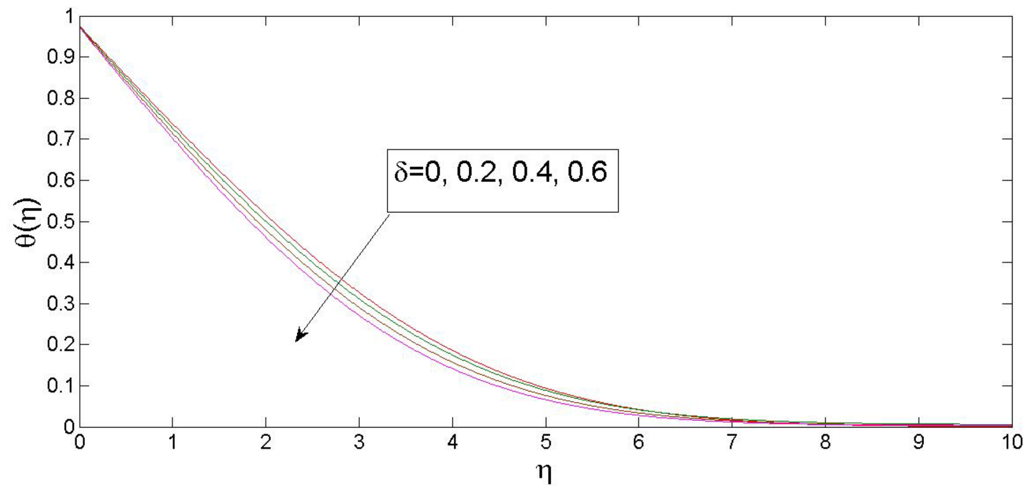

Figure-8: temperature profile $\theta(\eta)$ for various values of δ with K=0.1, M=0.2, Pr=0.3, S=0.2, β =0.1

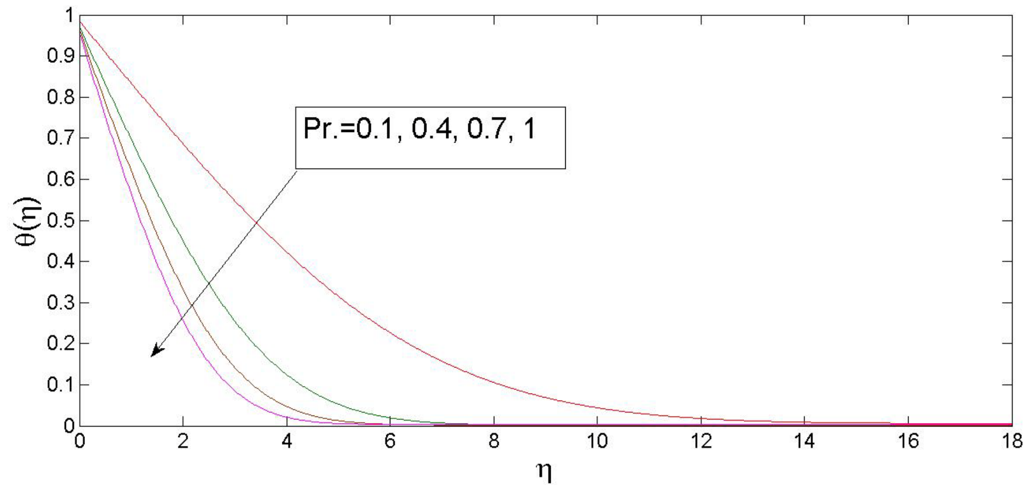

Figure-9: temperature profile $\theta(\eta)$ for various values of Pr with K=0.1, M=0.2, S=0.2, δ =0.1, β =0.1

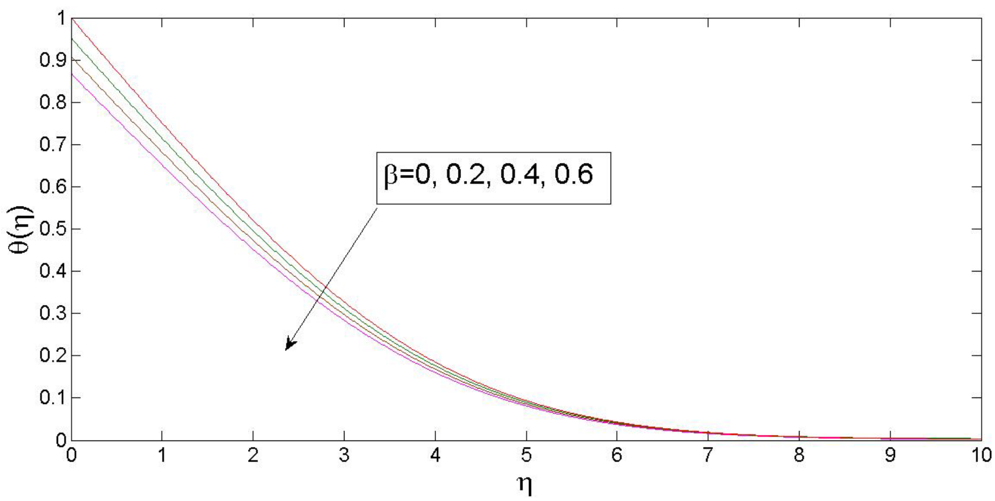

Figure-10: temperature profile $\theta(\eta)$ for various values of β with K=0.1, M=0.2, S=0.2, Pr=0.3, δ =0.1

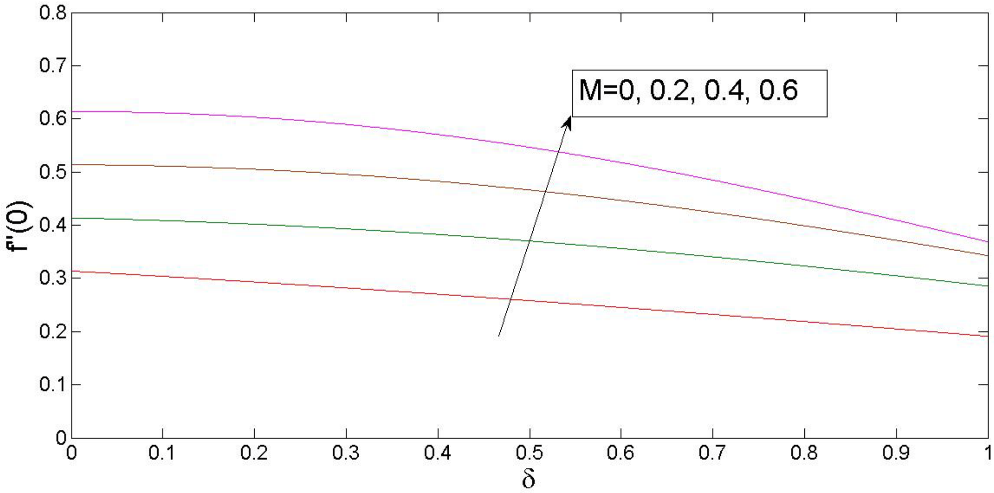

Figure-11: skin friction coefficient $f''(0)$ against δ for various values of M with K=0.1, S=0.2, Pr=0.3, δ =1, β =0.1.

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