International Journal of Mathematical Archive-8(1), 2017, 156-160
Available online through www.ijma.info ISSN 2229 – 5046

# BIANCHI TYPE V ELECTROMAGNETIC STRING DUST COSMOLOGICAL MODEL

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***(Received On: 22-12-16; Revised & Accepted On: 27-01-17)***

***ABSTRACT***

*We have investigated Bianchi type V cosmological model with the matter string dust coupled with electromagnetic field in general theory of relativity. And will obtain the deterministic solution of Einstein's field equation, Further some physical and geometrical properties of the model are also discussed.*

***Key Words:*** *Bianchi type V, String dust, electromagnetic field, general theory of relativity.*

## I. INTRODUCATION

Bianchi type cosmological models are important in the sense that these models are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view, anisotropic universe has a greater generality than isotropic models. The simplicity of the field equations made Bianchi space time useful in constructing models of spatially homogeneous and anisotropic cosmologies. Hence, these models are to be known as very much suitable models of our universe. Therefore, studies of these models create much more interest.

Adhav, K.S., Nimkar, A.S. [1] have investigated Non-Existence of Bianchi Type-V String Cosmological Model with Bulk Viscous fluid in General Relativity. Adhav, K.S.; Katore, S.D. [2] investigated Higher Dimensional Bianchi Type-V Universe in Creation Field Cosmology. Adhav, K.S.; Dawande, M.V. [3] have studied LRS Bianchi Type-V Universe in Creation Field Cosmology. Baghel, Prashant Singh; Singh, Jagdish Prasad. [4] studied Bianchi Type-V universe with bulk viscous matter and time varying gravitational and cosmological constants. Bali, Raj; Swati Singh [5]have obtained Bianchi Type-V Inflationary universe with decaying vacuum energy (^). Bali [6-10] have studied Bianchi Type-V fluid cosmological models in presence of decaying vacuum energy and also studied Bianchi Type-V Magnetized String Dust Universe With Variable Magnetic Permeability and Conformally Flat Tilted Bianchi Type-V Cosmological Models in General Relativity, Bianchi Type-V Bulk Viscous Barotropic Fluid Cosmological Model with variable G & ^ and also Bianchi Type-V Magnetized string dust cosmological models with petrov-type degenerate. Camci, U.; Yavuz, I.; Baysal, H. [11] studied Generation of Bianchi Type V Universes Filled with A Perfect Fluid. Chauhan, D.S.; Singh, R.P.[12]have investigated Bianchi Type-V Cosmological Models in Scalar Tensor Theory of Gravitation with some observable quantities. Chirde, V.R.; Rahate, P.N. [13] investigated Bianchi Type-V Isotropic cosmological model with strange Quark matter attached to cosmic string. Coley, A.A. [14] Bianchi V Imperfect Fluid Cosmology. Deo, Karan Singh, Rewar [15]pointed Bianchi Type-V string Dust Cosmological model in general relativity. Dubey, R.K.; Tripathi, S.K.[16] have studied On Mathematical Analysis for Bianchi Type-V cosmological Model with time varying Gravitational term G. Dwivedi, Uttam Kumar, [17-19] investigated Bulk Viscous Bianchi Type-V Cosmological models with stiff fluid and time dependent cosmological term ^ and also Bianchi Type-V Models with decaying cosmological term ^ and Bianchi Type-V Cosmological Models with time varying ^. Fransworth, D.L.[20]have pointed Generation of Bianchi Type-V Bulk Viscous Cosmological Model. Kandalkar, S.P.; Wasnik, A.P.; Gawande, S.P. [21] investigated Bianchi Type-V String dust Cosmological Model with Bulk Viscous

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Magnetic Field. Katore, S.D., Shaikh, A.Y. [22] studied Bianchi Type-V Magnetized Anisotropic Dark Energy Models with constant Deceleration Parameter. Mishra, B.; Mohanty, G. [23] have investigated Bianchi Type-V Cosmological Model in Scale Invarient Theory. Mohd. Zeyauddin; Saha, Bijan [24] have investigated Bianchi type-V bulk viscous cosmological models with particle creation in general relativity. Deo S. D. and Roughe A K [25] have studied Non Existence of Bianchi type I, V and $VI_0$ cosmic string in Bimetric relativity.

In this paper, we have investigated Bianchi type V string dust coupled with electromagnetic field in general theory of relativity and obtained the solution of field equations. Further we have discussed the physical and geometrical behaviour of bianchi type V cosmological model.

## II. THE METRIC AND FIELD EQUATIONS

Here we consider the spatially homogeneous Bianchi type-V metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + e^{2\alpha x}\left(B^2dy^2 + C^2dz^2\right) \tag{2.1}$$

Where $\alpha$ is non -zero constant and A, B and C are functions of t only.

The energy momentum tensor of the source string dust coupled with electromagnetic field is denoted by

$$T_i^{\ j} = \varepsilon v_i v^j - \lambda x_i x^j + E_i^{\ j} \tag{2.2}$$

Here, $\varepsilon$ is the rest energy density of the system of string, $\lambda$ the string tension density, The four velocity vector of the fluid satisfies $v_i v^i = -1 = -x_i x^i$.

Electromagnetic field is defined as

$$E_i^{\ j} = -F_{ir}F^{jr} + \frac{1}{4}F_{ab}F^{ab}g_i^{\ j} \tag{2.3}$$

Where, $E_i^{\ j}$ is electromagnetic energy tensor and $F_i^{\ j}$ is the electromagnetic field tensor.

We assume that $F_{13}$ is the only non-vanishing component of $F_{ij}$ which corresponds to the presence of magnetic field along y-direction.

The Einstein field equation in the general theory of relativity is given by

$$R_i^{\ j} - \frac{1}{2}Rg_i^{\ j} = -8\pi G T_i^{\ j} \tag{2.4}$$

Where, $R_i^{\ j}$ is known as Ricci tensor and $R = g^{ij}R_{ij}$ is the Ricci scalar and $T_i^{\ j}$ is the energy momentum tensor for matter.

The field equations (2.4) together with the line element (2.1) with equations (2.2) and (2.3) we get

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = 8\pi G\left[\lambda - \frac{\left(F_{23}\right)^2}{2B^2C^2e^{4\alpha x}}\right] \tag{2.5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = 8\pi G\left[\frac{\left(F_{23}\right)^2}{2B^2C^2e^{4\alpha x}}\right] \tag{2.6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = 8\pi G\left[\frac{\left(F_{23}\right)^2}{2B^2C^2e^{4\alpha x}}\right] \tag{2.7}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3\alpha^2}{A^2} = 8\pi G\left[\varepsilon - \frac{\left(F_{23}\right)^2}{2B^2C^2e^{4\alpha x}}\right] \tag{2.8}$$

$$\left(\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0 \tag{2.9}$$

 

After integration, we have

$$A^2 = k^2(BC) \tag{2.10}$$

Where $k$ is a constant of integration.

## III. SOLUTION OF THE FIELD EQUATIONS

From equation (2.6) and (2.7) with (2.9) we obtain

$$\frac{C\ddot{B}-\ddot{C}B}{BC} = -\frac{1}{2}\left[\frac{C\dot{B}-\dot{C}B}{BC}\right] \tag{3.1}$$

After integration we have

$$C^2\frac{d}{dt}\left(\frac{B}{C}\right) = \frac{k_1}{\sqrt{BC}} \tag{3.2}$$

where $k_1$ is constant of integration.

We assume that

$$BC = \xi, \frac{B}{C} = \tau \tag{3.3}$$

Using equation (3.3) in equation (3.2) we have

$$\frac{\dot{\xi}}{\xi} = \frac{k_1}{\tau^{3/2}} \tag{3.4}$$

To get deterministic model of the universe, we apply string dust condition $\varepsilon = \lambda$. Thus equation (2.5) and (2.8) with (2.10) we have

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}-\frac{1}{2}\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]^2+\frac{2}{k^2(BC)} = 0 \tag{3.5}$$

Using equation (3.3) and (3.4) in equation (3.5) we have

$$2\ddot{\xi}-2\frac{\dot{\xi}^2}{\xi} = \frac{-k_1^2}{\xi^2}-\frac{4}{k^2} \tag{3.6}$$

Now assuming $\dot{\xi} = f(\xi)$, equation (3.6) takes the from

$$\frac{d}{d\xi}\left(f^2\right)-\frac{2}{\xi}f^2 = \frac{-k_1^2}{\xi^2}-\frac{4}{k^2} \tag{3.7}$$

Equation (3.7) has the general solution

$$f^2 = \left(\frac{d\xi}{dt}\right)^2 = \frac{a^2+b^2\xi^2+\xi^3k_2}{\xi} \tag{3.8}$$

Where $a^2 = \frac{k_1^2}{3}, b^2 = \frac{4}{k^2}$ and $k_2$ is the constant of integration.

The Bianchi type –V model in this case reduces to the form, substituting, $\xi = T$, $k^2x = X$ , y=Y, z=Z in (2.1) we get

$$ds^2 = -\left[\frac{TdT^2}{a^2+b^2T^2+k_2T^3}\right]+TdX^2+Te^{2\alpha X}\left[\tau dY^2+\tau^{-1}dZ^2\right] \tag{3.9}$$

 

**PHYSICAL AND GEOMETRICAL BEHAVIOUR OF THE MODEL**

It is clear that, the expansion $\theta$, The shear $\sigma$, The spatial volume $R^3$, The deceleration parameter $q$ for the model (3.9) are given by

$$\theta = \frac{3}{2T}\left[\sqrt{\frac{a^2}{T} + b^2T + k_2T^2}\right] \tag{3.10}$$

$$\sigma = \frac{k_1}{2T^{3/2}} \tag{3.11}$$

$$R^3 = \sqrt{k}T^{3/2} \tag{3.12}$$

and

$$q = 2 - \frac{2b^2T^2 + 3T^3}{a^2 + b^2T^2 + k_2T^3} \tag{3.13}$$

**IV. CONCLUSION**

In the present study, we have investigated the effect of string dust with electromagnetic field in bianchi type V Universe. Einstein's field equations have been solved exactly with suitable physical and kinematical parameters. However for large value of T, it isotropizes. The deceleration parameter q > 0, hence the model decelerates.

**V. REFERENCES**

1. Adhav, K.S.; Nimkar, A.S.; Dawande, M.V.; Ugale, M.R., Non-Existence of Bianchi Type- V String Cosmological Model with Bulk Viscous fluid in General Relativity, Rom. Journ. Phys. Vol.54, Nos.1-2, (2007), 207-212.
2. Adhav, K.S.; Katore, S.D.; Bansod, A.S.; Gadodia, P.S., Higher Dimensional Bianchi Type-V Universe in Creation Field Cosmology, Natural Science, Vol. 2, No. 5, (2010), 484-488.
3. Adhav, K.S.; Dawande, M.V.; Raut, R.B., LRS Bianchi Type-V Universe in Creation Field Cosmology, Bulg. J. Phys. 38, (2011), 364-370.
4. Baghel, Prashant singh; Singh, Jagdish Prasad., Bianchi Type-V universe with bulk viscous matter and time varying gravitational and cosmological constants, Research in Astronomy and Astrophysics, Vol. 12, No. 11, (2012), 1457-1466.
5. Bali, Raj; Swati Singh, Bianchi Type-V Inflationary universe with decaying vacuum energy (^), Middle-East Journal of Scientific Research, 15(8), (2013), 1087-1091.
6. Bali, Raj; Singh, Pratibha; Singh, J.P., Bianchi Type-V fluid cosmological models in presence of decaying vacuum energy,. arXiv: 1202. 305Ov1 [gr-qc], (2012), 1-10.
7. Bali, Raj, Bianchi Type-V Magnetized String Dust Universe With Variable Magnetic Permeability, EJTP 5, No. 19, (2008), 105-114.
8. Bali, Raj; Meena, B.L. ,Conformally Flat Tilted Bianchi Type-V Cosmological Models in General Relativity, Pramana – journal of physics, Vol. 62, No. 5, (2004), 1007-1014.
9. Bali, Raj; Tinker, Seema.,Bianchi Type-V Bulk Viscous Barotropic Fluid Cosmological Model with variable G & ^ , Chin. Phys. Lett. Vol. 25, No. 8, (2008), 3090-3093.
10. Bali, Raj ; Pareek, Umesh, K. ,Bianchi Type-V Magnetized string dust cosmological models with petrov-type degenerate, Pramana – journal of physics, Vol. 72, No. 5, (2009), 787-796.
11. Camci, U.; Yavuz, I.; Baysal, H.; Tahran, I.; Yilmaz, I., Generation of Bianchi Type V Universes Filled with A Perfect Fluid, Astrophysics and Space Science, 275 (4), (2001), 391-400.
12. Chauhan, D.S.; Singh, R.P., Bianchi Type-V Cosmological Models in Scalar Tensor Theory of Gravitation with some observable quantities, AJMP, Vol. 1, No. 1, (2012), 23-45.
13. Chirde, V.R.; Rahate, P.N. ,Bianchi Type-V Isotropic cosmological model with strange Quark matter attached to cosmic string, .International Journal of Mathematical Archieve, Vol. 3, No. 2, (2012), 426-431.
14. Coley, A.A., Bianchi V Imperfect Fluid Cosmology, General Relativity and Gravitation, 22, (1990), 3.
15. Deo, Karan Singh, Rewar, Bianchi Type-V string Dust Cosmological model in general relativity, Journal of Rajasthan Academy of Physical Sciences. Vol. 12, No. 3, (2013), 337-344.
16. Dubey, R.K.; Tripathi, S.K., On Mathematical Analysis for Bianchi Type-V cosmological Model with time varying Gravitational term G, International Journal of Physics and Mathematical Sciences, Vol. 1, No. 1, (2011), 19-27.

 

17. Dwivedi, Uttam Kumar, Bulk Viscous Bianchi Type-V Cosmological models with stiff fluid and time dependent cosmological term ^ , International Journal of Physics and Mathematical Sciences, Vol. 2, No. 2, (2012), 17-20.
18. Dwivedi, Uttam Kumar, Bianchi Type-V Models with decaying cosmological term ^ , International Journal of Management, IT and Engineering, Vol. 2, Issue 7, (2012), 568-573.
19. Dwivedi, U.K.; Tiwari, R.K. Bianchi Type-V Cosmological Models with time varying ^ , International Journal of Physics and Mathematical Sciences, Vol. 2, No. 2, (2012), 1-7.
20. Fransworth, D.L., Generation of Bianchi Type-V Bulk Viscous Cosmological Model, J. Math. Phys. 8, (1967), 2315.
21. Kandalkar, S.P.; Wasnik, A.P., Gawande, S.P. Bianchi Type-V String dust Cosmological Model with Bulk Viscous Magnetic Field, IJSER, Vol. 4, Issue 3, (2013),1-4.
22. Katore, S.D., Shaikh, A.Y., Bianchi Type-V Magnetized Anisotropic Dark Energy Models with constant Deceleration Parameter, The African Review of Physics, 9:0035, (2014), 269-276.
23. Mishra, B.; Mohanty, G., Bianchi Type-V Cosmological Model in Scale Invarient Theory, Bulg. J. Phys. 34, (2007), 252-259
24. Mohd. Zeyauddin; Saha, Bijan, Bianchi type-V bulk viscous cosmological models with particle creation in general relativity, The European Physical Journal Plus 129, 177, (2014), (23 Pages) DOI: 10.1140/epjp/i2014-14177-0, ArXiv: 1306.2168 [gr-qc].
25. S. D. Deo and Roughe, A K Non Existence of Bianchi type I, V and $VI_0$ cosmic string in Bimetric relativity, Ultra Scientist of physical science, vol.22 no.2, (2010), 535- 538.

**Source of support: Nil, Conflict of interest: None Declared.**