# CORDIAL AND PRODUCT CORDIAL LABELING FOR THE EXTENDED DUPLICATE GRAPH OF SPLITTING GRAPH OF PATH

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## ABSTRACT

*In this paper, we prove that the extended duplicate graph of splitting graph of path admits cordial, total cordial, product cordial and total product cordial labeling.*

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*Keywords: Graph labeling, Duplicate graph, Splitting graph, Cordial labeling, Product cordial labeling.*

## 1. INTRODUCTION

The origin of graph labeling can be attributed to Rosa. E.Sampthkumar [1, 2] introduced the concept of splitting graph and duplicate graph. Gallian [3] provide the literature on survey of different types of graph labeling. The idea of cordial labeling was introduced by Cahit [4]. P.Lawrence Rozario Raj and S.Koilraj [5] have proved the cordial labeling for the splitting graph of some standard graphs. The concept of product cordial labeling and total product labeling was introduced by Sundaram and Somasundaram [6, 7]. K.Thirusangu, B.Selvam and P.P. Ulaganathan have proved that the extended duplicate graph of twig graphs is cordial and total cordial [8].

## 2. PRELIMINARIES

In this section, we give some basic definitions which are relevant to this paper. Let $G(V, E)$ be a finite, simple and undirected graph with p vertices and q edges.

**Definition 2.1 Splitting graph:** For each vertex v of a graph G, take a new vertex $v'$. Join $v'$ to all the vertices of G adjacent to v. The graph Spl(G) thus obtained is called splitting graph of G.

**Illustration 1:** The Splitting graph of Path Spl ($P_6$)



*Spl($P_6$)*
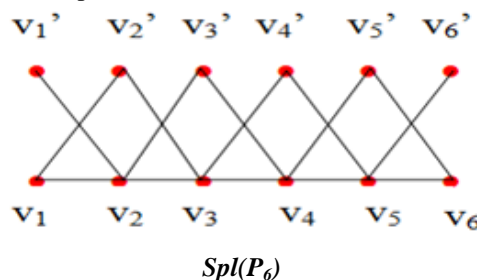
**Definition 2.2 Duplicate Graph:** Let $G (V, E)$ be a simple graph and the duplicate graph of G is $DG = (V_1, E_1)$, where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f: V \to V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set $E_1$ of DG is defined as the edge $ab$ is in E if and only if both $ab'$ and $a'b$ are edges in $E_1$.

**Definition 2.3 Extended duplicate graph of Splitting graph of Path:** Let DG = $(V_1, E_1)$ be a duplicate graph of splitting graph of path G(V,E). Extended duplicate graph of splitting graph of path is obtained by adding the edge $v_2v_2'$ to the duplicate graph. It is denoted by EDG Spl($P_m$). Clearly it has 4m vertices and 6m-5 edges, $m \geq 2$.

**Illustration 2:** Extended duplicate graph of Splitting graph of Path.



$$EDG(Spl(P_3))$$
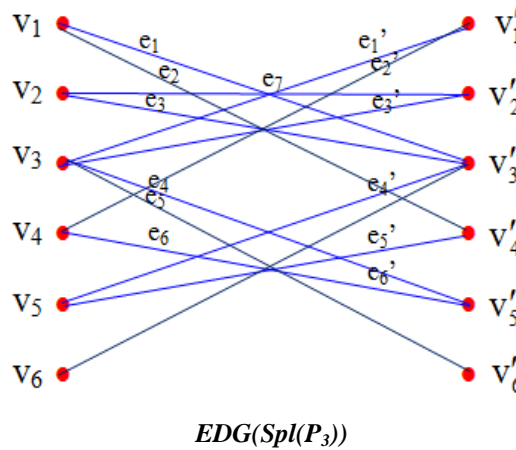
**Definition 2.4 Cordial labeling:** A function $f$: V → {0, 1} such that each edge $uv$ receives the label $|f(u) - f(v)|$ is said to be cordial labeling if the number of vertices labeled '0' and the number of vertices labeled '1' differ by at most one, and the number of edges labeled '0' and the number of edges labeled '1' differ by at most one.

**Definition 2.5  Total cordial labeling:** A function $f : V \to \{0,1\}$ such that each edge uv receives the label $|f(u) - f(v)|$ is said to be  total cordial labeling if the number of vertices  and edges labeled '0' and the number of vertices and edges labeled '1' differ by at most one.

**Definition 2.6 Product cordial labeling:** A function $f$: V →{0, 1} such that each edge uv receive the label $f(u) \times f(v)$ is said to be product cordial labeling if the number of vertices labeled '0' and the number of vertices labeled '1' differ by at most one, and the number of edges  labeled '0' and the number of edges labeled '1' differ by at most one.

**Definition 2.7 Total product cordial  labeling:** A function $f : V \to \{0,1\}$ such that each edge uv receives the label $f(u) \times f(v)$ is said to be  total product cordial labeling if the number of vertices and edges labeled '0' and the number of vertices and edges  labeled '1' differ by at most one.

## 3. MAIN RESULTS

### 3.1 Cordial labeling

In  this section, we  present an algorithm and  prove the existence of  cordial  labeling for  the extended duplicate graph of splitting  graph of  path  $P_m$, m ≥ 2.

**Algorithm: 3.1:** Procedure [Cordial labeling for EDG Spl($P_m$), m≥ 2]
V ← {$v_1, v_2, \dots ,v_{2m}, v'_1, v'_2, \dots ,v'_{2m}$}
E ← {$e_1, e_2, \dots.,e_{3m-3}, e_{3m-2}, e'_1,e'_2,\dots..e'_{3m-3}$}
$\quad\quad v_1 \leftarrow 1, v_2 \leftarrow 0, v'_1 \leftarrow 0, v'_2 \leftarrow 1$
for  i = 0 to (m-2) do
$\quad\quad v_{3+2i} \leftarrow 1$
$\quad\quad v_{4+2i} \leftarrow 0$
end for
for i = 0 to [(m-2)/2] do
$\quad\quad v'_{3+4i} \leftarrow 0$
$\quad\quad v'_{4+4i} \leftarrow 1$
end for
for i = 0 to [(m-3)/2] do
$\quad\quad v'_{5+4i} \leftarrow 1$
$\quad\quad v'_{6+4i} \leftarrow 0$
end for
end procedure

**Theorem 3.1:** The extended duplicate graph of splitting graph of path Spl$(P_m)$,  m $\geq$ 2  admits  cordial labeling.

**Proof:** Let Spl$(P_m)$,  m $\geq$ 2  be a  splitting  graph of  path.  Let EDG Spl$(P_m)$,  m $\geq$ 2  be  the extended duplicate graph of splitting  graph of  path.

To label the vertices, define a function $f : V \rightarrow \{0,1\}$ as given in algorithm 3.1.

The  vertices  $v_1$, $v_2$, $v'_1$ and  $v'_2$ receive label  '1', '0', '0'  and ' 1' respectively**;**

for  $0 \leq$ i $\leq$ (m – 2),  the vertices  v$_{3+2i}$ receive label '1' and  the vertices  v$_{4+2i}$ receive label '0'**;**

for  $0 \leq$ i $\leq$ [(m-2)/2],  the vertices  v'$_{3+4i}$ receive label  '0'  and  the vertices  v'$_{4+4i}$ receive label '1'**;**

for  $0 \leq$ i $\leq$ [(m-3)/2],  the vertices  v'$_{5+4i}$ receive label  '1'  and  the  vertices  v'$_{6+4i}$ receive label '0'**.**

Hence the  entire  4m vertices  are  labeled  such that  the  number of  vertices  labeled  '0'  is 2m  which is  same  as  the  number of  vertices  labeled  '1'  and  satisfies  the  required condition.

To obtain the label for edges, we define the induced function   f *: E $\rightarrow$ {0, 1} such that
$$f * (v_i \, v_j) =  = | f(v_i) - f(v_j) |; \; v_i, \, v_j \in V$$

The  induced  function  yields  the  label  '0' for the  edges  e'$_2$, e'$_3$ and  the  label  '1' for  the  edges  e'$_1$  and  e$_{3m-2}$;

for $0 \leq$ i $\leq$ [(m-2)/2] ,  the  edges e$_{1+6i}$  receive label '1';

for $0 \leq$ i $\leq$ [(m-2)/2] and $0 \leq$ j $\leq$1, the edges e$_{2+6i +j}$ receive label '0'**;**

for $0 \leq$ i $\leq$ [(m-3)/2] ,  the  edges e$_{4+6i}$ receive label  '0'  and  the edges e '$_{4+6i}$ receive label '1';

for $0 \leq$ i $\leq$ [(m-3)/2] and $0 \leq$ j $\leq$1, the edges e$_{5+6i +j}$ receive label '1'and  the edges e'$_{5+6i +j}$ receive label '0'**;**

for $0 \leq$ i $\leq$ [(m-4)/2] ,  the  edges e'$_{7+6i}$  receive label '0'**;**

for $0 \leq$ i $\leq$ [(m-4)/2] and $0 \leq$ j $\leq$1, the  edges e'$_{8+6i +j}$ receive label '1'**.**

Thus  all  the  6m-5 edges  are  labeled  such that  3m-3  edges  receive label '0' and  3m-2 edges  receive label '1' which  differ by  at most  one  and  satisfies the required condition.

Hence the extended duplicate graph of splitting graph of path Spl$(P_m)$,m $\geq$ 2  is cordial.

**Illustration 3:** *Cordial labeling for the extended duplicate graph of Splitting graph of Path $P_6$*



*EDG(Spl($P_6$))*
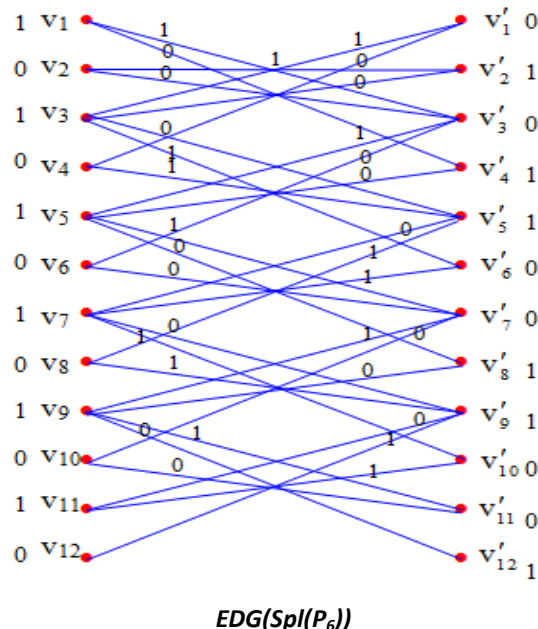
**Theorem 3.2:** The extended duplicate graph of splitting graph of path $Spl(P_m)$, $m \geq 2$ admits total cordial labeling.

**Proof:** In theorem 3.1, 2m vertices were assigned the label '0' and 2m vertices were assigned the label '1' and it has been proved that the number of edges labeled '0' is $(3m-3)$ and the number of edges labeled '1' is $(3m-2)$. From this, we conclude that the number of vertices and edges labeled '0' is $2m + (3m-3) = 5m-3$ and the number of vertices and edges labeled '1' is $2m+(3m-2) = 5m-2$, which differ by one. Hence the extended duplicate graph of splitting graph of path $Spl(P_m)$, $m \geq 2$ is total cordial.

## 3.2 Product cordial labeling

In this section, we present an algorithm and prove the existence of product cordial labeling for the extended duplicate graph of splitting graph of path $P_m$, $m \geq 2$.

**Algorithm: 3.2:** Procedure [Product cordial labeling for EDG $Spl(P_m)$, $m \geq 2$]
$V \leftarrow \{v_1, v_2, \ldots, v_{2m}, v'_1, v'_2, \ldots, v'_{2m}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{3m-3}, e_{3m-2}, e'_1, e'_2, \ldots, e'_{3m-3}\}$
$\quad\quad v_1 \leftarrow 0$, $v_2 \leftarrow 0$, $v'_1 \leftarrow 1$, $v'_2 \leftarrow 1$
for i = 0 to [(m-2)/2] do
$\quad\quad$ for j = 0 to 1 do
$\quad\quad\quad\quad v_{3+4i+j} \leftarrow 1$
$\quad\quad\quad\quad v'_{3+4i+j} \leftarrow 0$
$\quad\quad$ end for
end for
for i = 0 to [(m-3)/2] do
$\quad\quad$ for j = 0 to 1 do
$\quad\quad\quad\quad v_{5+4i+j} \leftarrow 0$
$\quad\quad\quad\quad v'_{5+4i+j} \leftarrow 1$
$\quad\quad$ end for
end for
end procedure

**Theorem 3.3:** The extended duplicate graph of splitting graph of path $Spl(P_m)$, $m \geq 2$ admits product cordial labeling.

**Proof:** Let $Spl(P_m)$, $m \geq 2$ be a splitting graph of path. Let EDG $Spl(P_m)$, $m \geq 2$ be the extended duplicate graph of splitting graph of path.

To label the vertices, define a function $f : V \rightarrow \{0, 1\}$ as given in algorithm 3.2.

The vertices $v_1$, $v_2$, $v'_1$ and $v'_2$ receive label '0', '0', '1' and '1' respectively;

for $0 \leq i \leq [(m-2)/2]$ and $0 \leq j \leq 1$, the vertices $v_{3+4i+j}$ receive label '1' and the vertices $v'_{3+4i+j}$ receive label '0';

for $0 \leq i \leq [(m-3)/2]$ and $0 \leq j \leq 1$, the vertices $v_{5+4i+j}$ receive label '0' and the vertices $v'_{5+4i+j}$ receive label '1'.

Hence the entire 4m vertices are labeled such that the number of vertices labeled '0' is 2m which is same as the number of vertices labeled '1' and satisfies the required condition.

To obtain the label for edges, we define the induced function $f * : E \rightarrow \{0,1\}$ such that
$\quad\quad f * (v_i v_j) = f(v_i) \times f(v_j)$; $v_i, v_j \in V$

The induced function yields the label '0' for the edge $e_{3m-2}$;

for $0 \leq i \leq [(m-2)/2]$ and $0 \leq j \leq 2$, the edges $e_{1+6i+j}$ receive label '0' and the edges $e'_{1+6i+j}$ receive label '1';

for $0 \leq i \leq [(m-3)/2]$ and $0 \leq j \leq 2$, the edges $e_{4+6i+j}$ receive label '1' and the edges $e'_{4+6i+j}$ receive label '0'.

Thus all the 6m-5 edges are labeled such that 3m-2 edges receive label '0' and 3m-3 edges receive label '1' which differ by at most one and satisfies the required condition.

Hence the extended duplicate graph of splitting graph of path $Spl(P_m)$, $m \geq 2$ is product cordial.

**Illustration 4:** Product cordial labeling for the extended duplicate graph of Splitting graph of Path $P_6$



*EDG(Spl($P_6$))*
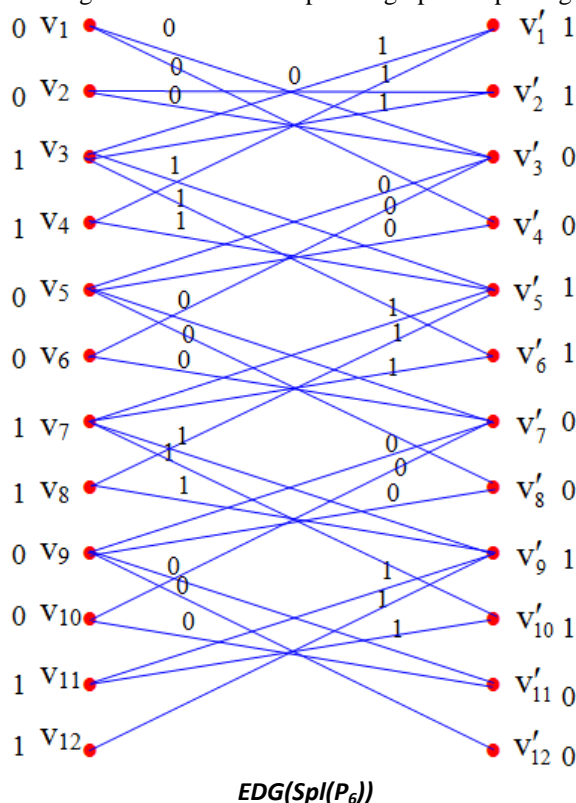
**Theorem 3.4:** The extended duplicate graph of splitting graph of path Spl($P_m$), m ≥ 2 admits total product cordial labeling.

**Proof:** In theorem 3.3, 2m vertices were assigned the label '0' and 2m vertices were assigned the label '1' and it has been proved that the number of edges labeled '0' is (3m-2) and the number of edges labeled '1' is (3m-3) . From this, we conclude that the number of vertices and edges labeled '0' is 2m + (3m-2) = 5m-2 and the number of vertices and edges labeled '1' is 2m+(3m-3) = 5m-3, which differ by one. Hence the extended duplicate graph of splitting graph of path Spl($P_m$), m ≥ 2 is total product cordial.

### 4. CONCLUSION

In this paper, we presented algorithms and prove that the extended duplicate graph of splitting graph of path $P_m$, m ≥ 2 admits cordial , total cordial, product cordial and total product cordial labeling .

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