CORDIAL AND PRODUCT CORDIAL LABELING FOR THE EXTENDED DUPLICATE GRAPH OF SPLITTING GRAPH OF PATH

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ABSTRACT

In this paper, we prove that the extended duplicate graph of splitting graph of path admits cordial, total cordial, product cordial and total product cordial labeling.

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Keywords: Graph labeling, Duplicate graph, Splitting graph, Cordial labeling, Product cordial labeling.

1. INTRODUCTION

The origin of graph labeling can be attributed to Rosa. E.Sampthkumar [1, 2] introduced the concept of splitting graph and duplicate graph. Gallian [3] provide the literature on survey of different types of graph labeling. The idea of cordial labeling was introduced by Cahit [4]. P.Lawrence Rozario Raj and S.Koilraj [5] have proved the cordial labeling for the splitting graph of some standard graphs. The concept of product cordial labeling and total product labeling was introduced by Sundaram and Somasundaram [6, 7]. K.Thirusangu, B.Selvam and P.P. Ulaganathan have proved that the extended duplicate graph of twig graphs is cordial and total cordial [8].

2. PRELIMINARIES

In this section, we give some basic definitions which are relevant to this paper. Let G(V, E) be a finite, simple and undirected graph with p vertices and q edges.

Definition 2.1 Splitting graph: For each vertex v of a graph G, take a new vertex v’. Join v’ to all the vertices of G adjacent to v. The graph Spl(G) thus obtained is called splitting graph of G.

Illustration 1: The Splitting graph of Path Spl (P₆)

Definition 2.2 Duplicate Graph: Let G (V, E) be a simple graph and the duplicate graph of G is DG = (V₁, E₁), where the vertex set V₁ = V ∪ V’ and V ∩ V’ = φ and f: V → V’ is bijective (for v ∈ V, we write f(v) = v’ for convenience) and the edge set E₁ of DG is defined as the edge ab is in E if and only if both ab’ and a’b are edges in E₁.
Definition 2.3 Extended duplicate graph of Splitting graph of Path: Let $DG = (V_1, E_1)$ be a duplicate graph of splitting graph of path $G(V,E)$. Extended duplicate graph of splitting graph of path is obtained by adding the edge $v_2v_2'$ to the duplicate graph. It is denoted by $EDG \text{Spl}(P_m)$. Clearly it has $4m$ vertices and $6m-5$ edges, $m \geq 2$.

Illustration 2: Extended duplicate graph of Splitting graph of Path.

$EDG(\text{Spl}(P_3))$

Definition 2.4 Cordial labeling: A function $f : V \rightarrow \{0, 1\}$ such that each edge $uv$ receives the label $|f(u) - f(v)|$ is said to be cordial labeling if the number of vertices labeled ‘0’ and the number of vertices labeled ‘1’ differ by at most one, and the number of edges labeled ‘0’ and the number of edges labeled ‘1’ differ by at most one.

Definition 2.5 Total cordial labeling: A function $f : V \rightarrow \{0,1\}$ such that each edge $uv$ receives the label $|f(u) - f(v)|$ is said to be total cordial labeling if the number of vertices and edges labeled ‘0’ and the number of vertices and edges labeled ‘1’ differ by at most one.

Definition 2.6 Product cordial labeling: A function $f : V \rightarrow \{0,1\}$ such that each edge $uv$ receives the label $f(u) \times f(v)$ is said to be product cordial labeling if the number of vertices labeled ‘0’ and the number of vertices labeled ‘1’ differ by at most one, and the number of edges labeled ‘0’ and the number of edges labeled ‘1’ differ by at most one.

Definition 2.7 Total product cordial labeling: A function $f : V \rightarrow \{0,1\}$ such that each edge $uv$ receives the label $f(u) \times f(v)$ is said to be total product cordial labeling if the number of vertices and edges labeled ‘0’ and the number of vertices and edges labeled ‘1’ differ by at most one.

3. MAIN RESULTS

3.1 Cordial labeling

In this section, we present an algorithm and prove the existence of cordial labeling for the extended duplicate graph of splitting graph of path $P_m$, $m \geq 2$.

Algorithm: 3.1: Procedure [Cordial labeling for EDG $\text{Spl}(P_m)$, $m \geq 2$]

$V \leftarrow \{v_1, v_2, ..., v_{2m}, v_1', v_2', ..., v_{2m}'\}$
$E \leftarrow \{e_1, e_2, ..., e_{3m-3}, e_{3m-2}, e_1', e_2', ..., e_{3m-3}'\}$
$v_1 \leftarrow 1, v_2 \leftarrow 0, v_1' \leftarrow 0, v_2' \leftarrow 1$

for $i = 0$ to $(m-2)$ do
for $i = 0$ to $[(m-2)/2]$ do

for $i = 0$ to $[(m-3)/2]$ do

end for

end for

end for

for $i = 0$ to $[(m-3)/2]$ do

$e_{3+4i} \leftarrow 0$
$e_{4+4i} \leftarrow 1$

end for

end for

end procedure
Theorem 3.1: The extended duplicate graph of splitting graph of path $\text{Spl}(P_m)$, $m \geq 2$ admits cordial labeling.

**Proof:** Let $\text{Spl}(P_m)$, $m \geq 2$ be a splitting graph of path. Let $\text{EDG Spl}(P_m)$, $m \geq 2$ be the extended duplicate graph of splitting graph of path.

To label the vertices, define a function $f: V \rightarrow \{0, 1\}$ as given in algorithm 3.1.

The vertices $v_1$, $v_2$, $v'_1$ and $v'_2$ receive label ‘1’, ‘0’, ‘0’ and ‘1’ respectively;

for $0 \leq i \leq (m – 2)$, the vertices $v_{3+2i}$ receive label ‘1’ and the vertices $v_{4+2i}$ receive label ‘0’;

for $0 \leq i \leq [(m-2)/2]$, the vertices $v'_{3+4i}$ receive label ‘0’ and the vertices $v'_{4+4i}$ receive label ‘1’;

for $0 \leq i \leq [(m-3)/2]$, the vertices $v'_{5+4i}$ receive label ‘1’ and the vertices $v'_{6+4i}$ receive label ‘0’.

Hence the entire $4m$ vertices are labeled such that the number of vertices labeled ‘0’ is $2m$ which is same as the number of vertices labeled ‘1’ and satisfies the required condition.

To obtain the label for edges, we define the induced function $f^*: E \rightarrow \{0, 1\}$ such that

$$f^*(v_i, v_j) = |f(v_i) - f(v_j)|; \quad v_i, v_j \in V$$

The induced function yields the label ‘0’ for the edges $e'_{2}$, $e'_{3}$ and the label ‘1’ for the edges $e'_{1}$ and $e_{3m-2}$;

for $0 \leq i \leq [(m-2)/2]$ , the edges $e_{1+6i}$ receive label ‘1’;

for $0 \leq i \leq [(m-2)/2]$ and $0 \leq j \leq 1$, the edges $e_{2+6i+j}$ receive label ‘0’;

for $0 \leq i \leq [(m-3)/2]$ , the edges $e_{4+6i}$ receive label ‘0’ and the edges $e'_{4+6i}$ receive label ‘1’;

for $0 \leq i \leq [(m-3)/2]$ and $0 \leq j \leq 1$, the edges $e_{5+6i+j}$ receive label ‘1’ and the edges $e'_{5+6i+j}$ receive label ‘0’;

for $0 \leq i \leq [(m-4)/2]$ , the edges $e'_{7+6i}$ receive label ‘0’;

for $0 \leq i \leq [(m-4)/2]$ and $0 \leq j \leq 1$, the edges $e'_{8+6i+j}$ receive label ‘1’.

Thus all the $6m-5$ edges are labeled such that $3m-3$ edges receive label ‘0’ and $3m-2$ edges receive label ‘1’ which differ by at most one and satisfies the required condition.

Hence the extended duplicate graph of splitting graph of path $\text{Spl}(P_m)$, $m \geq 2$ is cordial.

Illustration 3: Cordial labeling for the extended duplicate graph of Splitting graph of Path $P_6$

![Diagram of EDG(Spl(P_6))](image_url)
Theorem 3.2: The extended duplicate graph of splitting graph of path $\text{Spl}(P_m)$, $m \geq 2$ admits total cordial labeling.

Proof: In theorem 3.1, $2m$ vertices were assigned the label ‘0’ and $2m$ vertices were assigned the label ‘1’ and it has been proved that the number of edges labeled ‘0’ is $(3m-3)$ and the number of edges labeled ‘1’ is $(3m-2)$. From this, we conclude that the number of vertices and edges labeled ‘0’ is $2m + (3m-3) = 5m-3$ and the number of vertices and edges labeled ‘1’ is $2m+(3m-2) = 5m-2$, which differ by one. Hence the extended duplicate graph of splitting graph of path $\text{Spl}(P_m)$, $m \geq 2$ is total cordial.

3.2 Product cordial labeling

In this section, we present an algorithm and prove the existence of product cordial labeling for the extended duplicate graph of splitting graph of path $P_m$, $m \geq 2$.

Algorithm 3.2: Procedure [Product cordial labeling for $\text{EDG \ Spl}(P_m)$, $m \geq 2$]

1. $V \leftarrow \{v_1, v_2, \ldots, v_{2m}, v'_1, v'_2, \ldots, v'_{2m}\}$
2. $E \leftarrow \{e_1, e_2, \ldots, e_{3m-3}, e'_{3m-3}\}$
3. $v_1 \leftarrow 0, v_2 \leftarrow 0, v'_1 \leftarrow 1, v'_2 \leftarrow 1$
4. for $i = 0$ to $[(m-2)/2]$ do
   for $j = 0$ to 1 do
      $v_{3+4i+j} \leftarrow 1$
      $v'_{3+4i+j} \leftarrow 0$
   end for
end for
5. for $i = 0$ to $[(m-3)/2]$ do
   for $j = 0$ to 1 do
      $v_{5+4i+j} \leftarrow 0$
      $v'_{5+4i+j} \leftarrow 1$
   end for
end for

Theorem 3.3: The extended duplicate graph of splitting graph of path $\text{Spl}(P_m)$, $m \geq 2$ admits product cordial labeling.

Proof: Let $\text{Spl}(P_m)$, $m \geq 2$ be a splitting graph of path. Let $\text{EDG \ Spl}(P_m)$, $m \geq 2$ be the extended duplicate graph of splitting graph of path.

To label the vertices, define a function $f: V \rightarrow \{0, 1\}$ as given in algorithm 3.2.

The vertices $v_1, v_2, v'_1$ and $v'_2$ receive label ‘0’, ‘0’, ‘1’ and ‘1’ respectively;

for $0 \leq i \leq [(m-2)/2]$ and $0 \leq j \leq 1$, the vertices $v_{3+4i+j}$ receive label ‘1’ and the vertices $v'_{3+4i+j}$ receive label ‘0’;

for $0 \leq i \leq [(m-3)/2]$ and $0 \leq j \leq 1$, the vertices $v_{5+4i+j}$ receive label ‘0’ and the vertices $v'_{5+4i+j}$ receive label ‘1’.

Hence the entire $4m$ vertices are labeled such that the number of vertices labeled ‘0’ is $2m$ which is same as the number of vertices labeled ‘1’ and satisfies the required condition.

To obtain the label for edges, we define the induced function $f^*: E \rightarrow \{0, 1\}$ such that $f^*(v_i, v_j) = f(v_i) \times f(v_j)$; $v_i, v_j \in V$

The induced function yields the label ‘0’ for the edge $e_{3m-2}$;

for $0 \leq i \leq [(m-2)/2]$ and $0 \leq j \leq 2$, the edges $e_{1+6i+j}$ receive label ‘0’ and the edges $e'_{1+6i+j}$ receive label ‘1’;

for $0 \leq i \leq [(m-3)/2]$ and $0 \leq j \leq 2$, the edges $e_{4+6i+j}$ receive label ‘1’ and the edges $e'_{4+6i+j}$ receive label ‘0’.

Thus all the $6m-5$ edges are labeled such that $3m-2$ edges receive label ‘0’ and $3m-3$ edges receive label ‘1’ which differ by at most one and satisfies the required condition.

Hence the extended duplicate graph of splitting graph of path $\text{Spl}(P_m)$, $m \geq 2$ is product cordial.
Theorem 3.4: The extended duplicate graph of splitting graph of path \( \text{Spl}(P_m) \), \( m \geq 2 \) admits total product cordial labeling.

Proof: In theorem 3.3, 2m vertices were assigned the label '0' and 2m vertices were assigned the label '1' and it has been proved that the number of edges labeled '0' is \( (3m-2) \) and the number of edges labeled '1' is \( (3m-3) \). From this, we conclude that the number of vertices and edges labeled '0' is \( 2m + (3m-2) = 5m-2 \) and the number of vertices and edges labeled '1' is \( 2m + (3m-3) = 5m-3 \), which differ by one. Hence the extended duplicate graph of splitting graph of path \( \text{Spl}(P_m) \), \( m \geq 2 \) is total product cordial.

4. CONCLUSION

In this paper, we presented algorithms and prove that the extended duplicate graph of splitting graph of path \( P_m \), \( m \geq 2 \) admits cordial, total cordial, product cordial and total product cordial labeling.

5. REFERENCES


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