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ON SUPRA T-CLOSED SETS

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ABSTRACT

In this paper we introduce a new class of set namely T^{μ}-closed set in supra topological space. We further discuss the concept of T^{μ}-continuity and obtained their applications.

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1. INTRODUCTION

In 1970, Levine [6] introduced the concept of generalized closed sets in topological space and a class of topological spaces called $T_{1/2}$ spaces. Extensive research on generalizing closedness was done in recent years by many Mathematicians [3, 4, 6, 7, 8]. Andrijevic [1] introduced a new class of generalized open sets in a topological space, the so-called b-open sets. This type of sets was discussed by Ekici and Caldas [5] under the name of γ - open sets.

In 1983, A. S. Mashhour et al [8] introduced the notion of supra topological spaces and studied S-S continuous functions and S^* - continuous functions. In 2010, O. R. Sayed and Takashi Noiri [9] introduced supra b - open sets and supra b - continuity on topological spaces. In this paper we introduce the concept of T^{μ}-closed set and also studied some of their basic properties. Further the notion of T^{μ} -continuity is also studied. We also note that the class of T^{μ}-closed sets is properly placed between supra closed sets and g^{μ} b – closed sets.

2. PRELIMINARIES

Definition: 2.1 [8] A subclass $\tau^* \subset P(X)$ is called a supra topology on X if $X \in \tau^*$ and τ^* is closed under arbitrary union.(X, τ^*) is called a supra topological space (or supra space).The members of τ^* are called supra open sets.

Definition: 2.2 [8] The supra closure of a set A is defined as $Cl^{\mu}(A) = \cap \{B : B \text{ is supra closed and } A \subseteq B\}$

The supra interior of a set A is defined as $Int^{\mu}(A) = \cup \{B : B \text{ is supra open and } A \supseteq B\}$

Definition 2.3 [9] Let (X,μ) be a supra topological space. A set A is called a supra b - open set if
$A \subseteq Cl^{\mu}(Int^{\mu}(A)) \cup Int^{\mu}(Cl^{\mu}(A))$ .The complement of a supra b - open set is called a supra b - closed set.

Definition: 2.4 [2] Let (X,μ) be a supra topological space. A set A of X is called supra generalized b - closed set (simply g^{μ} b - closed) if $bcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open. The complement of supra generalized b - closed set is supra generalized b - open set.

Definition: 2.5 A Subset A of (X,μ) is said to be supra regular open if $A = Int^{\mu}(Cl^{\mu}(A))$ and supra regular closed if $A = cl^{\mu}(Int^{\mu}(A))$.

3. BASIC PROPERTIES OF T^{μ}-CLOSED SETS

Definition: 3.1 A subset A of (X,τ) is called T^{μ}-closed set if $bcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is $g^{\mu}b$-open in (X,τ).

Theorem: 3.2
(a)Every supra-closed set is T^{μ}-closed.

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(b)Every $regular^{\mu}$-closed set is T^{μ}-closed.
(c)Every T^{μ}-closed set is $g^{\mu}b$-closed.
(d) Every b^{μ} -closed set is T^{μ}-closed.

Proof: It is obvious.

Remark: 3.3 The converse of the above theorem is not true and it is shown by the following example.

Example: 3.4 Let X= {a, b, c}; τ= {ϕ, X, {a}}
(a){c} is T^{μ}-closed but it is not supra closed.
(b){a, b} is $g^{\mu}b$-closed but it is not T^{μ}-closed.

Example: 3.5 Let X={a, b}; τ={ϕ,X,{a}}(c) {a} is T^{μ}-closed but it is not $regular^{\mu}$-closed.

Example: 3.6 Let X={a, b, c}; τ={ϕ,X,{a},{b},{a,b}}(d)){c} is T^{μ}-closed but it is not b^{μ}- closed.

Remark: 3.7
1. The sets T^{μ}-closed and g^{μ}-closed are independent of each other.
2. The sets T^{μ}-closed and sg^{μ}-closed are independent of each other.
3. The sets T^{μ}-closed and gs^{μ}-closed are independent of each other.
4. The sets T^{μ}-closed and αg^{μ}-closed are independent of each other.
5. The sets T^{μ}-closed and $g\alpha^{\mu}$-closed are independent of each other.

The above remark is shown by the following examples.

Example: 3.8 Let X= {a, b, c}; τ= {ϕ, X,{a}}
1. {a, b} is g^{μ}-closed and sg^{μ}-closed but not T^{μ}-closed.
2. {a, c} is αg^{μ}-closed but not T^{μ}-closed.

Example: 3.9 Let X = {a, b, c, d}; τ ={ϕ, X, {a},{b},{a,b}}
1. {a} is T^{μ}-closed but not g^{μ}-closed and αg^{μ}-closed.
2. {a, c, d} is T^{μ}-closed but not sg^{μ}-closed.

Theorem: 3.10 The union of two T^{μ}-closed sets is T^{μ}-closed.

Proof: Let A and B be two T^{μ}-closed sets. Let $A \cup B \subseteq G$, where G is $g^{\mu}b$-open. Since A and B are T^{μ}-closed sets, $bcl^{\mu}(A) \cup bcl^{\mu}(B) \subseteq G$. Thus $bcl^{\mu}(A \cup B) \subseteq G$. Hence $A \cup B$ is T^{μ}-closed set.

Theorem: 3.11 Let A be T^{μ}-closed set of (X,τ), then $bcl^{\mu}(A) - A$ does not contain any non-empty $g^{\mu}b$-closed set.

Proof: Let A be T^{μ}-closed set. Suppose $F \neq \phi$ is a $g^{\mu}b$-closed set of $bcl^{\mu}(A) - A$, then $F \subseteq bcl^{\mu}(A) - A$. This implies that $F \subseteq bcl^{\mu}(A)$ and $F \subseteq A^{C}$. This implies $A \subseteq F^{C}$. Since A is T^{μ}-closed, $bcl^{\mu}(A) \subseteq F^{C}$. Then $F \subseteq [bcl^{\mu}(A)]^{C}$. Therefore $F \subseteq bcl^{\mu}(A) \cap [bcl^{\mu}(A)]^{C} = \phi$.

Theorem: 3.12 If A is T^{μ}-closed set in a supra topological space (X,τ) and $A \subset B \subset bcl^{\mu}(A)$ then B is also T^{μ}-closed set.

Proof: Let U be $g^{\mu}b$-open in (X,τ) such that$B \subseteq U$. Since $A \subseteq B$ implies $A \subseteq U$ and since A is T^{μ}-closed set in (X,τ), $bcl^{\mu}(A) \subseteq U$. Since $B \subset bcl^{\mu}(A)$ then$bcl^{\mu}(B) \subseteq U$. Therefore B is also T^{μ}-closed in (X,τ).

Theorem: 3.13 Let A be T^{μ}-closed set then A is b^{μ}-closed iff $bcl^{\mu}(A) - A$ is $g^{\mu}b$-closed.

Proof: Let A be T^{μ}-closed set. If A is b^{μ}-closed we have $bcl^{\mu}(A) - A = \phi$ which is $g^{\mu}b$-closed. Conversely,
Let $bcl^{\mu}(A) - A$ is $g^{\mu}b$-closed. Then by theorem 3.11, $bcl^{\mu}(A) - A$ does not contain any non-empty $g^{\mu}b$-closed set then $bcl^{\mu}(A) - A = \phi$. This implies that A is b^{μ}-closed.

Theorem: 3.14 A subset $A \subseteq X$ is T^{μ}-open iff $F \subseteq bInt^{\mu}(A)$ whenever F is $g^{\mu}b$-closed set and $F \subseteq A$.

Proof: Let A be T^{μ}-open set and suppose $F \subseteq A$ where F is $g^{\mu}b$-closed set. Then X-A is T^{μ}-closed set contained in the $g^{\mu}b$-open set X−F. Hence $bcl^{\mu}(X - A) \subseteq X - F$.

Thus $F \subseteq bInt^{\mu}(A)$.Conversely, if F is $g^{\mu}b$-closed set with $F \subseteq bInt^{\mu}(A)$ and $F \subseteq A$, then
$X - bInt^{\mu}(A) \subseteq X - F$. This implies that $bcl^{\mu}(X - A) \subseteq X - F$. Hence X−A is T^{μ}-closed. Therefore A is T^{μ}-open.

Theorem: 3.15 If B is $g^{\mu}b$-open and T^{μ}-closed set in X,then B is b^{μ}-closed.

Proof: Since B is $g^{\mu}b$-open and T^{μ}-closed then $bcl^{\mu}(B) \subseteq B$, but $B \subseteq bcl^{\mu}(B)$.

Thus,$B = bcl^{\mu}(B)$. Therefore B is b^{μ}-closed.

Corollary: 3.16 If B is supra open and T^{μ}-closed set in X, then B is b^{μ}-closed.

Theorem: 3.17 Let A be supra open and T^{μ}-closed set. Then $A \cap F$ is g^{μ} b - closed whenever $F \in bc1^{\mu}(X)$.

Proof: Let A be supra open and T^{μ}-closed set then $bc1^{\mu}(A) \subseteq A$ and also $A \subseteq bc1^{\mu}(A)$. Therefore $bc1^{\mu}(A) = A$. Hence A is supra b - closed. Since F is supra b - closed.

Therefore $A \cap F$ is supra b - closed in X. Therefore $A \cap F$ is g^{μ} b - closed in X.

4. T^{μ}-Continuous Functions

Definition: 4.1 A function $f: (X, \tau) \to (Y, \sigma)$ is said to be T^{μ}–continuous if $f^{-1}(V)$ is T^{μ} - closed in (X, τ) for every supra closed set V of (Y, σ).

Definition: 4.2 A function $f: (X, \tau) \to (Y, \sigma)$ is said to be T^{μ} –irresolute if $f^{-1}(V)$ is T^{μ} - closed in (X, τ) for every T^{μ} – closed V of (Y, σ).

Definition: 4.3 A function $f: (X, \tau) \to (Y, \sigma)$ is said to be regular$^{\mu}$ –continuous if $f^{-1}(V)$ is regular$^{\mu}$ - closed in (X, τ) for every supra closed V of (Y, σ).

Definition: 4.4 A function $f: (X, \tau) \to (Y, \sigma)$ is said to be b^{μ} –continuous if $f^{-1}(V)$ is b^{μ} - closed in (X, τ) for every supra closed V of (Y, σ).

Theorem: 4.5
(a) Every supra continuous function is T^{μ}– continuous.
(b) Every T^{μ} –irresolte function is T^{μ}– continuous.
(c) Every regular$^{\mu}$ –continuous function is T^{μ}– continuous.
(d) Every b^{μ} –continuous function is T^{μ}– continuous.

Proof: It is obvious.

Remark: 4.6 The converse of the above theorem is not true and shown by the following examples.

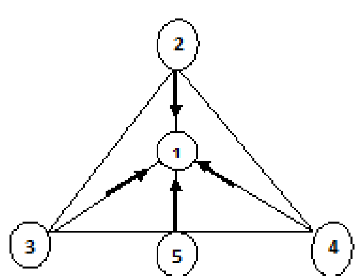
Example: 4.7 Let X = {a, b, c, d}; τ ={ϕ, X, {a},{b},{a,b}}. Let $f: (X, \tau) \to (X, \tau)$ be the function defined by

$$f(a) = b, f(b) = f(c) = d \text{ and } f(d) = a.$$

(a) $f^{-1}\{c, d\} = \{b, c\}$ Which is T^{μ}– continuous but not supra continuous.
(b) $f^{-1}\{b, c\} = \{a, b\}$ Which is T^{μ}– continuous but not T^{μ}– irresolute.

Example: 4.8 Let X = {a, b, c}; $\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$. Let $f: (X, \tau) \to (X, \tau)$ be an identity function.
(a) $f^{-1}\{b\} = \{b\}$ Which is T^{μ}– continuous but not regular$^{\mu}$ –continuous.

From the above theorem and examples we have the following diagram



Here the numbers 1- 5 represent the following:
1. T^{μ}– continuous 2.Supra continuous 3. regular$^{\mu}$ –continuous 4. T^{μ}– irresolute 5. b^{μ}– continuous

Theorem: 4.9 Let $f:(X,\tau)\to(Y,\sigma)\ and\ g:(Y,\sigma)\to(Z,\gamma)$ be any two functions then
(i) g ∘ f is T^{μ}– continuous if g is supra continuous and f is T^{μ} – continuous.
(ii) g ∘ f is T^{μ}– irresolute if g is T^{μ}– irresolute and f is T^{μ}– irresolute .
(iii) g ∘ f is T^{μ}– continuous if g is T^{μ} – continuous and f is T^{μ}– irresolute .

Proof: It is obvious.

Remark: 4.10 The composition of two T^{μ} – continuous functions need not be T^{μ} – continuous and it is shown by the following example.

Example: 4.11 Let X = {a, b, c}; $\tau=\{\phi,X,\{a\},\{b\},\{a,b\}\}$and $\sigma=\{\phi,X,\{a\},\{c\},\{a,c\}\}$.Define $f:(X,\tau)\to(X,\tau)\ by\ f(a)=b,f(b)=c,f(c)=d\ and\ f(d)=a.$

Define $g:(X,\tau)\to(X,\sigma)by\quad g(a)=b,\ g(b)=c,g(c)=d\ and\ g(d)=a.$.Then f and g are T^{μ} – continuous. Since {b, c, d} is Supra closed in (X,σ), $(g\circ f)^{-1}\{b,c,d\}=\{a,b,d\}$ which is not T^{μ}–closed in (X,τ).Therefore g ∘ f is not T^{μ} – continuous.

5. APPLICATIONS

Definition: 5.1 A space (X,τ) is called T^{μ}_{gb}-space if every $g^{\mu}b$-closed set is T^{μ}-closed.

Theorem: 5.2 Let (X,τ) be a supra topological space then
(i) $T^{\mu}O(\tau)\subset G^{\mu}bO(\tau)$
(ii) A space (X,τ) is T^{μ}_{gb}-space iff $T^{\mu}O(\tau)=G^{\mu}bO(\tau)$.

Proof:
(i) Let A be T^{μ}-open. Then X− A is T^{μ}-closed and so $g^{\mu}b$-closed.This implies that A is $g^{\mu}b$-open. Hence $T^{\mu}O(\tau)\subset G^{\mu}bO(\tau)$.

(ii) Let (X,τ) be T^{μ}_{gb}-space. Let $A\in G^{\mu}bO(\tau)$ then X− A is $g^{\mu}b$-closed.By hypothesis, X− A is T^{μ}-closed and thus $A\in T^{\mu}O(\tau)$.Hence $T^{\mu}O(\tau)=G^{\mu}bO(\tau)$. Conversely, Let $T^{\mu}O(\tau)=G^{\mu}bO(\tau)$.Let A be $g^{\mu}b$-closed then X− A is $g^{\mu}b$-open. Hence X− A is T^{μ}-open. Thus X is T^{μ}-closed .This implies that (X,τ) is T^{μ}_{gb}-space.

Theorem: 5.3 If (X,τ) is T^{μ}_{gb}-space then for each $x\in X$, { x } is either $g^{\mu}b$-closed set or T^{μ}-open.

Proof: Suppose (X,τ) is T^{μ}_{gb}-space. Let $x\in X$ and assume that { x } is not T^{μ}-open then X−{ x } is not T^{μ}-closed set. Then X−{ x } is $g^{\mu}b$-closed. Since (X,τ) is called T^{μ}_{gb}-space then X−{ x } is T^{μ}-closed or equivalently { x } is T^{μ}-open.

Definition: 5.4 A space (X,τ) is called ${T^{\mu}}_{C}$-space if every T^{μ}-closed set is supra closed.

Theorem: 5.5 Let (X,τ) be a supra topological space then
(i) $O^{\mu}(\tau)\subset T^{\mu}O(\tau)$
(ii) A space (X,τ) is T^{μ}_{C}-space iff $O^{\mu}(\tau)=\ T^{\mu}O(\tau)$.

Proof: It is obvious.

Theorem: 5.6 If (X,τ) is ${T^{\mu}}_{C}$-space then for each $x\in X$, { x } is either T^{μ}-closed or supra open.

Proof: It is obvious.

Definition: 5.7 A space (X,τ) is called T^{μ}_{R}-space if every T^{μ}-closed set is $regular^{\mu}$-closed.

Theorem: 5.8 Let (X,τ) be a supra topological space then
(i) $R^{\mu}O(\tau)\subset T^{\mu}O(\tau)$
(ii) A space (X,τ) is T^{μ}_{R}-space iff $R^{\mu}O(\tau)=\ T^{\mu}O(\tau)$.

Proof: It is obvious.

Theorem: 5.9 If (X,τ) is T^{μ}_{R}-space then for each $x\in X$, { x } is either T^{μ}-closed or $regular^{\mu}$-open.

Proof: It is obvious.

Definition: 5.10 A space (X,τ) is called T_B^{μ}-space if every T^{μ}-closed set is b^{μ}-closed.

Theorem: 5.11 Let (X,τ) be a supra topological space then
(i) $B^{\mu}O(\tau) \subset T^{\mu}O(\tau)$
(ii) A space (X,τ) is T_B^{μ}-space iff $B^{\mu}O(\tau) = T^{\mu}O(\tau)$.

Proof: It is obvious.

Theorem: 5.12 If (X,τ) is T_B^{μ}-space then for each $x \in X$, $\{ x \}$ is either T^{μ}-closed or b^{μ}-open.

Proof: It is obvious.

Theorem: 5.13
(a) Every T_R^{μ}-space is T_C^{μ}-space.
(b) Every T_C^{μ}-space is T_B^{μ}-space.
(c) Every T_R^{μ}-space is T_B^{μ}-space.

Proof: It is obvious.

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