

ON THE HYPERCENTER OF A NEAR-FIELD SPACE OVER A NEAR-FIELD

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ABSTRACT

The problems we are going to consider here arose in another context, that of near-field spaces with sub near-field spaces and where existing rings with polynomial identities similar to that of. However, author feel the subject matter has an independent interest, and Dr N V Nagendram develop the material here. In what follows N will always be an associative near-field space over a near-field, $Z(N)$ the center of N , and $J(N)$ the Jacobson radical sub near-field space of a near-field space over a near-field N . When there will be no danger of ambiguity, we shall write $Z(N)$ and $J(N)$ as Z and J , respectively throughout this paper. We now define a certain sub near-field space of a near-field R which is closely related to the center.

Keywords: sub near-field space, near-field space, hypercenter, division sub near-field space, semi simple near-field space.

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Definition: The hyper center $S(N)$ of N is $S(N) = \{a \in N / a x^n = x^n a, n = n(x, a) \geq 1, \text{ all } x \in N\}$. Here, too, we shall often write $S(N)$ as S whenever there is no confusion as to the near-field space over a near-field in question. The following (a), (b) and (c) three basic properties of S are trivial to verify. They will be used almost everywhere in the paper.

- (a). $S \supset Z$.
- (b). S is a sub near-field space over a near-field N .
- (c). If ϕ is an automorphism of N then $\phi(S) \subset S$.

Thus S need not be equal to Z is clear. If N is a non commutative nil near-field space over a near-field then $S = N$ but $Z \neq N$. Our claim is to see that how close S and Z are to each other. In view of the remark just made, the presence of nil sub near-field spaces under a near-field space over a near-field in N should interface with the equality of S and Z . In the absence of nil sub near-field spaces in N we will show that, indeed, $S = Z$.

Lemma 1: If D is division near-field space over a near-field $S(D) = Z(D)$.

Proof: When D is a division near-field space then it is trivial that only is S a sub near-field space of D , in fact, S is a sub division near-field space of D over a near-field. Since $\phi(S) \subset S$ for every automorphism of D , we have $S = D$ or $S \subset Z$. This second possibility, together with $Z \subset S \Rightarrow S = Z$ is the desired conclusion.

Suppose then that $S = D$, this means that, given $a, b \in D$ then $ab^n = b^n a$ for some $n = n(a, b) \geq 1$. By a theorem of Kaplansky as extended by Dr N V Nagendram we get D is commutative. In this case, of course, $D = Z$, hence certainly $S = Z$. This completes the proof of the lemma.

Having the result for division near-field spaces in hand, we follow the usual pattern of pushing the result through for semi-simple near-field spaces over a near-field N .

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Lemma 2: If N is a semi-simple near-field space over a near-field, then $S(N) = Z(N)$.

Proof: Given N is semi-simple near-field space over a near-field, it is a sub-direct product of primitive sub near-field spaces N_λ . Moreover, as is trivial, $S(N)$ maps into $S(N_\lambda)$ for each λ . Thus if we knew that $S(N_\lambda) = Z(N_\lambda)$ for each λ , we would get the required equality $S(N) = Z(N)$.

Hence, without loss of generality, we may assume that N is primitive sub near-field space over a near-field, therefore N is dense near-field space of linear transformations on a vector space V over a division near-field space D . If $\dim_D(V) = 1$ then $N = D$, and since N is a division near-field space over a near-field we would have $S(N) = Z(N)$ by lemma 1. Hence we may suppose $\dim_D(V) > 1$.

Let $s \neq 0$ be in S and suppose that for some $v \in V$, v and vs are linearly independent over D . By density of the action of N on V , there exists an $x \in N$ with $vx = 0$ and $vsx = vs$. Thus $vsx^m = vs$ for all $m \geq 1$. Since, $s \in S$, $sx^n = x^n s$ for some $n \geq 1$. Hence $vs = vx^n s = 0$, a contradiction.

Thus, given $v \in V$, $vs = \lambda(v)v$ where $\lambda(v) \in D$. If $v, w \in V$ are linearly independent over D then, we claim, $\lambda(v) = \lambda(w)$. For, $vs = \lambda(v)v$, $wt = \lambda(w)w$, and $(v + w)s = \lambda(v + w)(v + w)$.

These yield $[\lambda(v) - \lambda(v + w) + (\lambda(w) - \lambda(v + w))]w = 0$ by the independence of v and w over D we conclude that $\lambda(v) = \lambda(v + w) = \lambda(w)$. Since λ is constant on independent elements, and since $\dim_D(V) > 1$, we get that $\lambda(v) = \lambda$, λ independent of v , for all $v \in V$.

If $x \in N$ then, since $vx \in V$, $(vx)s = \lambda(vx)$, hence $v(xs) = \lambda(vx)$. On the other hand, $vs = \lambda v$ whence $(vs)x = (\lambda v)x = \lambda(vx)$ and so $v(sx) = \lambda(vx)$. The net result of this is that $v(xs) = v(sx)$, hence $v(xs - sx) = 0$ for all $v \in V$. Since N acts faithfully on V , we have $xs - sx = 0$ for all $x \in N$. this puts $s \in Z$. Thus we have that $S \subset Z$, and so, $S = Z$.

Lemma 3: Let N be any near-field space. If $a \in S(N)$ is nilpotent near-field space then aNa is a nil right sub near-field space of N . Hence a and aN lie in $J(N)$.

Proof: Let $a \neq 0$ in S be nilpotent sub near-field space. Then $a^n = 0$, $a^{n-1} \neq 0$ for some $n > 1$. Given $x \in N$, then since $a^{n-1} \in S$, $(ax)^m a^{n-1} = a^{m-1} (ax)^m = 0$, for a suitable $m \geq 1$. Pick j minimal such that $(ax)^u a^j = 0$ for some integer $u \geq 1$. If $j = 1$, this would yield $(ax)^{u+1} = 0$ and so ax is nilpotent, if $j > 1$, then since $x(ax)^u a^{j-1} = 0$, $(xa)^{u+1} a^{j-1} = 0$. Since $a^{j-1} \in S$, $a^{j-1} ((xa)^{u+1})^s = ((xa)^{u+1})^s a^{j-1} = 0$ for some $s \geq 1$. Thus $a^{j-1} (xa)^r = 0$, where $r = (u+1)s$. Hence $a^{j-1} (ax)^{r+1} = 0$. Since $a^{j-2} \in S$, $((ax)^{r+1})^v a^{j-2} = a^{j-2} ((ax)^{r+1})^v = 0$. But this contradicts the minimal nature of j , in short, $j = 1$ and ax is nilpotent for every x . therefore, aN is nil near-field space over a near-field. This completes the proof of the lemma.

Theorem 1: Let N be a prime near-field space with no non-zero nil sub near-field spaces. Then $S(N)$ has no nil potent sub near-field spaces.

Proof: Let M be the set of all nilpotent sub near-field spaces in S . Given $a, b \in S$ then $ab^n = b^n a$ for some $n = n(a, b) \geq 1$. Clearly N is a sub near-field space of S . In fact it is the maximal nil sub near-field space of S .

Suppose that $M \neq 0$. Since $\phi(S) \subset S$ for all automorphisms ϕ of N , we have that $\phi(M) \subset M$. By lemma 3, $M \subset (N)$.

If $x \in (N)$ then the mapping $\phi: N \rightarrow N$ defined by $\phi(y) = (1+x)y(1+x)^{-1}$ for $y \in N$ is an automorphism of N . Hence $(1+x)N(1+x)^{-1} \subset N$.

Suppose that $a \neq 0$ is in N where $a^2 = 0$. If $x \in N$ then ax is nilpotent sub near-field space by lemma 3, hence $(1+ax)^{-1} = 1 - ax + (ax)^2 - \dots$. Since $(1+ax)a(1+ax)^{-1} \in N$, we have that $(1+ax)a[1 - ax + (ax)^2 - \dots] \in N$. Because $a^2 = 0$, this last relation reduces to $(1+ax)a \in N$, and so $axa \in N \forall x \in N$. In short, $aNa \subset N$.

If $y \in J(N)$ and $b \in N$ then $(by - yb)(1+y)^{-1} = b - (1+y)b(1+y)^{-1}$ is in N . let $y = ax$ where $a^2 = 0$, $a \in N$. Hence $(axb - bax)(1 - ax)^{-1} \in N$. Multiply this from the left by a , using $a^2 = 0$ we arrive at $abax(1 - ax)^{-1} \in N \forall x \in N$. Given any $w \in J(N)$ true that any $w \in N$ then $w = x(1 - ax)^{-1}$ where $x = (wa + 1)^{-1}w$ is in $J(N)$. Thus we get $(aba)J(N) \subset N$.

If $y \in J(N)$ and $b \in N$ then $(by - yb)(1+y)^{-1} = b - (1+y)b(1+y)^{-1}$ is in N . Let $y = ax$ where $a^2 = 0$, $a \in N$.

Hence $(axb - bax)(1 - ax)^{-1} \in N$. on left multiplication by a using $a^2 = 0$ we arrive at $abax(1 - ax)^{-1} \in N \forall x \in N$. Given any $w \in J(N)$ in fact any $w \in N$ then $w = x(1 - ax)^{-1}$, where $x = (wa + 1)^{-1}w$ is in $J(N)$. Thus we get $(aba)J(N) \subset N$.

If $y \in (N)$ then $(1 + y) (aba J(N)) (1 + y)^{-1} \subset (1 + y) N (1 + y)^{-1} \subset N$. Since $J(N) \cap (1 + y)^{-1} \subset J(N)$ and $aba J(N) \subset N \Rightarrow yaba (N) \subset N \forall y \in J(N)$. In other words, $J(N) aba J(N) \subset N$. But $J(N) aba J(N)$ is a sub near-field space of N , and since it lies in N , it must be nil sub near-field space of a near-field space over a near-field. We conclude that $J(N) aba J(N) = 0$. Since $aba \in (N)$ and $J(N)$ has no nilpotent sub near-field spaces, we get that $aba = 0$ for $b \in N$, i.e., $aNa = 0$. Summarizing, if $a^2 = 0$ where $a \in N$, then $aNa = 0$.

If $b \in N$ and $b^2 = 0$ then we have seen that $bNb \subset N$. So, if $a^2 = 0$ where $a \in N$, then $abNb \subset aNa = 0$. Since N is prime near-field space over a near-field, this yields that $ab = 0$ or $ba = 0$. In particular, if $x \in J(N)$ then $b = (1 + x) a (1 + x)^{-1}$ is in N and $b^2 = 0$. Hence $ab = 0$ or $ba = 0$. Now, $ab = 0$ yields $axa = 0$ and $ba = 0$ yields $a (1 + x)^{-1} a = 0$.

We claim that if $x \in J(N)$ is nilpotent near-field space over a near-field then $axa = 0$, we go by induction on the index of nil potency of x .

If $x^2 = 0$ then $(1 + x)^{-1} = 1 - x$, hence, by the above, either $axa = 0$ or $a (1 + x)^{-1} a = 0$; because $(1 + x)^{-1} = 1 - x$, this reads as $axa = 0$ or $a (1 - x)a = 0$. Since $a^2 = 0$, $a(1 - x)a = 0 \Rightarrow axa = 0$. Then either possibility leads to fact that $axa = 0$.

Suppose that $x \in J(N)$, $x^n = 0$. Since x^j for $j > 1$, has index nil potency less than n , by induction, $ax^j a = 0$ for $j > 1$. Now, we know that $axa = 0$ or $a (1 + x)^{-1} a = 0$. Since $(1 + x)^{-1} = 1 - x + x^2 - \dots \pm x^{n-1}$ and since $ax^j a = 0$ for $j > 1$, $a (1 + x)^{-1} a = 0$ yields $0 = a (1 - x + x^2 - \dots \pm x^{n-1})a = -axa$.

Thus, indeed $axa = 0$ follows, for all nilpotent $x \in J(N)$.

Let $a, b \in N$ with $a^2 = 0$ and $b^2 = 0$. If $r \in N$ then, by lemma 3, $br \in (N)$ and is nilpotent sub near-field space over a near-field. Thus, by the result derived above, $a(br)a = 0$ which is to say, $abNa = 0$. Since N is prime near-field space over a near-field, we get $ab = 0$. If $x \in J(N)$ let $b = a(1 + x) a(1 + x)^{-1} = axa (1 + x)^{-1}$ yields that $axa = 0$ and so $aJ(N)a = 0$. Because N is prime near-field space over a near-field and $J(N) \neq 0$ is a sub near-field space of N , we conclude that $a = 0$. Thus N is 0 and $S(N)$ has no nilpotent sub near-field spaces. This completes the proof of the theorem.

Lemma 4: Let N be a prime near-field space over a near-field with no nil sub near-field spaces. Then $S(N)$ is commutative and non zero sub near-field space of $S(N)$ is not a zero divisor sub near-field space in N .

Proof: Given $a, b \in S$ then $ab^n = b^n a$ for some $n \geq 1$. The Commutator sub near-field space of a near-field space over a near-field S of N is nil potent sub near-field space, according to theorem 1, S has no nilpotent sub near-field spaces. Thus the Commutator sub near-field space of S is 0, hence S must be commutative.

Suppose, that $a \neq 0$ is in S and $au = 0$ for some $u \in N$. If $x \in N$ then $y = uxa$ satisfies $y^2 = 0$ and $ay = 0$. But $(1 + y) a (1 + y)^{-1} = (1 + y) a (1 - y)$ is in S , i.e., $a + ya \in S$, whence $ya \in S$. However, $(ya)^2 = yaya = 0$. Since S has no nilpotent sub near-field spaces, we must have $ya = 0$. Recalling that $y = uxa$, we have $uxa^2 = 0 \forall x \in N$. Since $a^2 \neq 0$ and N is prime near-field space over a near-field, we get $u = 0$. Hence $au = 0 \Rightarrow u = 0$ and so a is not a zero divisor sub near-field space in N .

MAIN RESULT ON THE HYPERCENTER OF A NEAR-FIELD SPACE OVER A NEAR-FIELD

Theorem 2: Let N be a near-field space over a near-field with no nilpotent sub near-field spaces. Then $S(N) = Z(N)$.

Proof: Since N is sub direct product of prime near-field spaces N_β with no nil sub near-field spaces, Since $S(N)$ maps into $S(N_\beta)$, if we could prove that $S(N_\beta) = Z(N_\beta)$ for each β , we would get that $S(N) = Z(N)$. With loss of generality (w. g. l.), N is a prime near-field space with no nil sub near-field spaces. Also, if $J(N) = 0$ then by lemma 2, $S(N) = Z(N)$, therefore we may assume that $J(N) \neq 0$.

Since N prime near-field space and $J \neq 0$, the centralizer near-field space of J in N must be precisely $Z(N)$. In particular, $Z(J) \subset Z(N)$. Thus to prove that $S = Z$, it is enough to show that S centralizes J .

Suppose then that $a \in S$ and $x \in J$ and $ax - xa \neq 0$.

Now $0 \neq (ax - xa) (1 + x)^{-1} = a - (1 + x) a (1 + x)^{-1} \in S$.

Clearly, since $x \in J$, $(ax - xa) (1 + x)^{-1} \in Z$. Therefore, $0 \neq (ax - xa) (1 + x)^{-1}$ is in $S \cap J$, and so $S \cap J \neq 0$. If we could show that $S \cap J \subset Z$. Then we would have $(ax - xa) (1 + x)^{-1} \in Z$. Also if $b \in S$, then $b(ax - xa) (1 + x)^{-1} \in S \cap J \subset Z$ and since both $0 \neq (ax - xa) (1 + x)^{-1} \in Z$ and $b(ax - xa) (1 + x)^{-1} \in Z$ and since sub near-field spaces of Z are not zero

divisor sub near-field spaces of a near-field space N , these relations would imply that $b \in Z$ and we would get the desired conclusion $S(N) = Z(N)$.

It is obvious that if N is a prime near-field space with no nilpotent sub near-field spaces of near-field space over a near-field and $J(N) = N$ then $S(N) = Z(N)$.

So we suppose that N is prime near-field space, no nil potent sub near-field spaces and $J(N) = N$.

Suppose that $0 \neq a \in S$, $x \in N$ and $x \in N$ is such that $xa = ax$ and $zx = xz$. Then since $(1+x)a(1+x)^{-1}$ and $(1+zx)a(1+zx)^{-1}$ are in N , we have (i) $(1+x)a = a_1(1+x)$ (ii) $(1+zx)a = a_2(1+zx)$ where $a_1, a_2 \in S$. Multiply (i) by z and subtract (ii); since z commutes with x and with a we get, (iii) $za - a = xa_1 - a_2 + (a_1 + a_2)zx$.

Now $xa - a$ commutes with a , since z does. Also since $a_1, a_2 \in S$ and S is commutative, $za_1 - a_2$ commutes with a . Thus from (iii) we deduce that $(a_1 - a_2)zx$ commutes with a . This gives $(a_1 - a_2)z(xa - ax) = 0$.

If $a_1 - a_2 \neq 0$, since $a_1 - a_2 \in S$ it is not a zero divisor sub near-field space in N . So we conclude that $z(xa - ax) = 0$. On the other hand, if $a_1 = a_2$, remaining to (iii) we see that $za - a = za_1 - a_1$ and so $(1-z)(a - a_1) = 0$. Since $z \in N = J$, $(1-z)(a - a_1) = 0$ forces $a = a_1$. But then (i) tells us that $xa = ax$ in which case certainly $z(xa - ax) = 0$. Hence if $z \in N$ commutes with both $a \in S$ and $x \in N$ then $z(xa - ax) = 0$.

Now if $a \neq 0$ is in S , and $x \in N$ then $ax^n = x^n a$ for some $n \geq 1$. If $z = x^n$ then $za = az$ and $zx = xz$ whence $z(xa - ax) = 0$ i.e., $x^n(xa - ax) = 0$. But then $x^n(ax - xa)(1+x)^{-1} = 0$. Since $(ax - xa)(1+x)^{-1} \in S$, if it is not 0, it is not zero divisor sub near-field space. So if $xa - ax \neq 0$ then $x^n = 0$. Therefore a commutes with all nilpotent sub near-field spaces in N over a near-field.

Consequently, if $ay - ya \neq 0$, $ay - ya$ is not a zero divisor near-field space, so certainly cannot be nilpotent sub near-field space. Thus a must commute with $ay - ya$. If $ay - ya = 0$ then a certainly commutes with $ay - ya$. Therefore a commutes with all $ax - xa$, $x \in N$. if $\text{char } N \neq 2$ it is well known result that this forces a to be in Z . So if $\text{char } N \neq 2$, then $S \subset Z$, whence $S(N) = Z(N)$. on the other hand, if $\text{char } N = 2$, since $a(ax + xa) = (ax + xa)a$ for all $x \in N$, we get $a^2 \in Z$. Thus $z = a^2$ commutes with a and any x . In consequence, $0 = z(ax + xa) = a^2(ax + xa)$. But a is not a zero divisor near-field space of N , we consequently get that $ax = xa$ and so $a \in Z$. Thus here too we end up with $S \subset Z$ and so $S(N) = Z(N)$. we have now succeeded in proving that $S(N) = Z(N)$ thereby establishing the theorem. This completes the proof of the theorem.

MAIN RESULT ON PROPERTY OF $S(N)$ IN GENERAL CASE OF THE HYPERCENTER OF A NEAR-FIELD SPACE OVER A NEAR-FIELD.

Theorem 3: Let N be a near-field space and $S(N)$ its hypercenter. If $a \in S(N)$ and $x \in N$ then $ax - xa$ generates a nil potent sub near-field space of N . In particular, $ax - xa$ is nilpotent sub near-field space over a near-field for every $x \in N$.

Proof: Unfortunately the property that $ax - xa$ generate a nil potent sub near-field space for all $x \in N$ is not sufficient to force a to lie in $S(N)$. If $N = \left\{ \begin{bmatrix} \alpha & \beta \\ 0 & \gamma \end{bmatrix} / \alpha, \beta, \gamma \text{ integers} \right\}$ then $a = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ has the property that $ax - xa$ generate a nil sub near-field space of N over a near-field. Yet a fails to commute with any power of $x = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ hence $a \notin S(N)$. This indicates that the theorem is about as much as we can say in the general case.

Let B be a commutative sub near-field space such that given $x \in N$, then $x^{n(x)} \in B$. Then the nilpotent sub near-field spaces of N form a sub near-field space N and N/N is commutative. We had done this for the case $B = Z$. This result by author of Dr N V Nagendram is a very special case of this research article of advance in near-field spaces over a near-field under algebra of mathematics. To prove this result it is trivial to reduce to the case when N has no nil potent sub near-field spaces. Since $B \subset S$, and since $S = Z$, when N has no nilpotent sub near-field spaces, we have $B \subset Z$. Thus given $x \in N$, $x^{n(x)} \in B \subset Z$. N must be commutative. This completes the proof of the theorem.

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REFERENCES

1. A I Lihtman , Rings that are radical over a commutative subring, math, Sbormik (N.S.) 83,(1970) 513-523.
2. I N Herstein, two remarks on the commutativity of rings, contd. Math 7, 1955, 411-412.
3. I N Herstein Topiccs in ring theory Chicago lectures in Mathematics, University of Chicago press 11 1969.
4. I kaplansky A theorem on division rings contd., J Math, N.S. 83, 1970, 513 – 523.
5. N V Nagendram,T V Pradeep Kumar and Y V Reddy On “Semi Noetherian Regular Matrix δ -Near Rings and their extensions”, International Journal of Advances in Algebra (IJAA), Jordan, ISSN 0973 - 6964, Vol.4, No.1, (2011), pp.51-55.
6. N V Nagendram,T V Pradeep Kumar and Y V Reddy “A Note on Bounded Matrices over a Noetherian Regular Delta Near Rings”, (BMNR-delta-NR) published in International Journal of Contemporary Mathematics, Vol.2, No.1, June 2011, Copyright@MindReader Publications, ISSNNo:0973-6298, pp.13-19.
7. N V Nagendram,T V Pradeep Kumar and Y V Reddy “A Note on Boolean Regular Near-Rings and Boolean Regular δ -Near Rings”, (BR-delta-NR) published in International Journal of Contemporary Mathematics,IJCM Int. J. of Contemporary Mathematics ,Vol. 2, No. 1, June 2011,Copyright @ Mind Reader Publications, ISSN No: 0973-6298, pp. 29 - 34.
8. N V Nagendram,T V Pradeep Kumar and Y V Reddy “on p-Regular δ -Near-Rings and their extensions”, (PR-delta-NR) accepted and to be published in int. J. Contemporary Mathematics (IJCM),0973-6298,vol.1, no.2, pp.81-85,June 2011.
9. N V Nagendram, T V Pradeep Kumar and Y V Reddy “On Strongly Semi –Prime over Noetherian Regular δ -Near Rings and their extensions”,(SSPNR-delta-NR) published in International Journal of Contemporary Mathematics, Vol.2, No.1, June 2011, pp.83-90.
10. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Structure Theory and Planar of Noetherian Regular δ -Near-Rings (STPLNR-delta-NR)”,International Journal of Contemporary Mathematics, IJCM ,published by IJSMA, pp.79-83, Dec, 2011.
11. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Matrix’s Maps over Planar of Noetherian Regular δ -Near-Rings (MMPLNR-delta-NR)”,International Journal of Contemporary Mathematics, IJCM, published by IJSMA, pp.203-211, Dec, 2011.
12. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On IFP Ideals on Noetherian Regular- δ -Near Rings(IFPINR-delta-NR)”, Int. J. of Contemporary Mathematics, Copyright @ Mind Reader Publications, ISSN No: 0973-6298, Vol. 2, No. 1, pp.53-58, June 2011.
13. N V Nagendram, B Ramesh paper "A Note on Asymptotic value of the Maximal size of a Graph with rainbow connection number $2*(AVM-SGR-CN2*)$ " published in an International Journal of Advances in Algebra(IJAA) Jordan @ Research India Publications, Rohini , New Delhi , ISSN 0973-6964 Volume 5, Number 2 (2012), pp. 103-112.
14. N V Nagendram research paper on "Near Left Almost Near-Fields (N-LA-NF)" communicated to for 2nd intenational conference by International Journal of Mathematical Sciences and Applications, IJMSA@ mindreader publications, New Delhi on 23-04-2012 also for publication.
15. N V Nagendram, T Radha Rani, Dr T V Pradeep Kumar and Dr Y V Reddy “A Generalized Near Fields and (m, n) Bi-Ideals over Noetherian regular Delta-near rings (GNF-(m, n) BI-NR-delta-NR)", published in an International Journal of Theoretical Mathematics and Applications (TMA),Greece,Athens,dated 08-04-2012.
16. N V Nagendram, Smt.T.Radha Rani, Dr T V Pradeep Kumar and Dr Y V Reddy "Applications of Linear Programming on optimization of cool freezers (ALP-on-OCF)" Published in International Journal of Pure and Applied Mathematics, IJPAM-2012-17-670 ISSN-1314-0744 Vol-75 No-3(2011).
17. N V Nagendram "A Note on Algebra to spatial objects and Data Models(ASO-DM)" Published in international Journal American Journal of Mathematics and Mathematical Sciences, AJMMS, USA, Copyright @ Mind Reader Publications, Rohini , New Delhi, ISSN. 2250-3102, Vol.1, No.2 (Dec. 2012), pp. 233 – 247.
18. N V Nagendram, Ch Padma, Dr T V Pradeep Kumar and Dr Y V Reddy "A Note on Pi-Regularity and Pi-S-Unitality over Noetherian Regular Delta Near Rings(PI-R-PI-S-U-NR-Delta-NR)" Published in International Electronic Journal of Pure and Applied Mathematics, IeJPAM-2012-17-669 ISSN-1314-0744 Vol-75 No-4(2011).
19. N V Nagendram, Ch Padma, Dr T V Pradeep Kumar and Dr Y V Reddy "Ideal Comparability over Noetherian Regular Delta Near Rings(IC-NR-Delta-NR)" Published in International Journal of Advances in Algebra (IJAA, Jordan),ISSN 0973-6964 Vol:5, NO: 1(2012), pp.43-53@ Research India publications, Rohini, New Delhi.
20. N. V. Nagendram, S. Venu Madava Sarma and T. V. Pradeep Kumar, “A Note On Sufficient Condition Of Hamiltonian Path To Complete Graphs (SC-HPCG)”, IJMA-2(11), 2011, pp.1-6.
21. N V Nagendram,Dr T V Pradeep Kumar and Dr Y V Reddy “On Noetherian Regular Delta Near Rings and their Extensions(NR-delta-NR)”, IJCMS,Bulgaria,IJCMS-5-8-2011, Vol.6,2011, No.6,255-262.
22. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Semi Noetherian Regular Matrix Delta Near Rings and their Extensions(SNRM-delta-NR)”, Jordan@ResearchIndiaPublications, Advancesin Algebra ISSN 0973-6964 Volume 4, Number 1 (2011), pp.51-55© Research India Publications, pp.51-55.

23. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On Boolean Noetherian Regular Delta Near Ring(BNR-delta-NR)",International Journal of Contemporary Mathematics, IJCM Int. J. of Contemporary Mathematics ,Vol. 2, No. 1-2,Jan-Dec 2011 , Mind Reader Publications,ISSN No: 0973-6298, pp. 23-27.
24. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On Bounded Matrix over a Noetherian Regular Delta Near Rings (BMNR-delta-NR)",Int. J. of Contemporary Mathematics,Vol. 2, No. 1-2, Jan-Dec 2011, Copyright @ Mind Reader Publications, ISSN No: 0973-6298,pp.11-16.
25. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On Strongly Semi Prime over Noetherian Regular Delta Near Rings and their Extensions(SSPNR-delta-NR)", Int. J. of Contemporary Mathematics,Vol. 2, No. 1, Jan-Dec 2011 ,Copyright @ Mind Reader Publications ,ISSN No: 0973-6298,pp.69-74.
26. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On IFP Ideals on Noetherian Regular Delta Near Rings(IFPINR-delta-NR)", Int. J. of Contemporary Mathematics,Vol. 2, No. 1-2, Jan-Dec 2011 ,Copyright @ Mind Reader Publications, ISSN No: 0973-6298,pp.43-46.
27. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On Structure Thoery and Planar of Noetherian Regular delta-Near-Rings (STPLNR-delta-NR)",International Journal of Contemporary Mathematics, IJCM ,accepted for Ist international conference conducted by IJSMA, New Delhi December 18,2011, pp:79-83, Copyright @ Mind Reader Publications and to be published in the month of Jan 2011.
28. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On Matrix's Maps over Planar of Noetherian Regular delta-Near-Rings (MMPLNR-delta-NR)",International Journal of Contemporary Mathematics, IJCM, accepted for Ist international conference conducted by IJSMA, New Delhi December 18,2011, pp:203-211, Copyright @ Mind Reader Publications and to be published in the month of Jan 2011.
29. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "Some Fundamental Results on P- Regular delta-Near-Rings and their extensions (PNR-delta-NR)", International Journal of Contemporary Mathematics , IJCM, Jan-December'2011,Copyright@MindReader Publications,ISSN:0973-6298, vol.2, No.1-2, PP.81-85.
30. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "A Generalized ideal based-zero divisor graphs of Noetherian regular Delta-near rings (GIBDNR- d-NR)", International Journal of Theoretical Mathematics and Applications (TMA)accepted and published by TMA, Greece, Athens,ISSN:1792- 9687 (print),vol.1, no.1, 2011, 59-71, 1792-9709 (online),International Scientific Press, 2011.
31. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "Inversive Localization of Noetherian regular Delta-near rings (ILNR- Delta-NR)", International Journal of Pure And Applied Mathematics published by IJPAM-2012-17-668, ISSN.1314-0744 vol-75 No-3, SOFIA, Bulgaria.
32. N V Nagendram, N Chandra Sekhara Rao2 "Optical Near field Mapping of Plasmonic Nano Prisms over Noetherian Regular Delta Near Fields (ONFMPN-NR-Delta-NR)" accepted for 2nd international Conference by International Journal of Mathematical Sciences and Applications, IJMSA @ mind reader publications, New Delhi going to conduct on 15 – 16 th December 2012 also for publication.
33. N V Nagendram, K V S K Murthy (Yoga), "A Note on Present Trends on Yoga Apart From Medicine Usage and Its Applications (PTYAFMUIA)" Published by the International Association of Journal of Yoga Therapy, IAYT 18 th August, 2012.
34. N V Nagendram, B Ramesh, Ch Padma, T Radha Rani and S V M Sarma research article "A Note on Finite Pseudo Artinian Regular Delta Near Fields (FP AR-Delta-NF)" communicated to International Journal of Advances in Algebra, IJAA, Jordan on 22 nd August 2012.
35. N V Nagendram "Amenability for dual concrete complete near-field spaces over a regular delta near-rings (ADC-NFS-R- δ -NR)" accepted for 3rd international Conference by International Journal of Mathematical Sciences and Applications, IJMSA @ mind reader publications, New Delhi going to conduct on 15 – 16 th December 2014 also for publication.
36. N V Nagendram "Characterization of near-field spaces over Baer-ideals" accepted for 4th international Conference by International Journal Conference of Mathematical Sciences and Applications, IJCMSA @ mind reader publications, New Delhi going to conduct on 19–20 th December 2015 at Asian Institute of Technology AIT, Klaung Lange 12120, Bangkok, Thailand.
37. N V Nagendram, S V M Sarma Dr T V Pradeep Kumar "A note on sufficient condition of Hamiltonian path to Complete Graphs" published in International Journal of Mathematical archive IJMA, ISSN 2229-5046, Vol.2, No..2, Pg. 2113 – 2118, 2011.
38. N V Nagendram, S V M Sarma, Dr T V Pradeep Kumar "A note on Relations between Barnette and Sparse Graphs" published in an International Journal of Mathematical Archive (IJMA), An International Peer Review Journal for Mathematical, Science & Computing Professionals, 2(12), 2011, pg no.2538-2542, ISSN 2229 – 5046.
39. N V Nagendram "On Semi Modules over Artinian Regular Delta Near Rings (S Modules-AR-Delta-NR)" Accepted and published in an International Journal of Mathematical Archive (IJMA)", An International Peer Review Journal for Mathematical, Science & Computing Professionals ISSN 2229-5046, IJMA-3-474, 2012.
40. N V Nagendram "A note on Generating Near-field efficiently Theorem from Algebraic K - Theory" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.3, No.10, Pg. 1–8, 2012.
41. N V Nagendram and B Ramesh on "Polynomials over Euclidean Domain in Noetherian Regular Delta Near Ring Some Problems related to Near Fields of Mappings (PED-NR-Delta-NR & SPR-NF)" Accepted and published in an International Journal of Mathematical Archive (IJMA), An International Peer Review Journal for Mathematical, Science & Computing Professionals ISSN 2229-5046, vol.3,no.8, pp no. 2998-3002, 2012.

42. N V Nagendram "Semi Simple near-fields Generating efficiently Theorem from Algebraic K-Theory" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.3, No.12, Pg. 1 – 7, 2012.
43. N V Nagendram "-----" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.3, No.10, Pg. 3612 – 3619, 2012.
44. N V Nagendram, E Sudeeshna Susila, "Applications of linear infinite dimensional system in a Hilbert space and its characterizations in engg. Maths (AL FD S HS & EM)", IJMA, ISSN.2229-5046, Vol.4, No.7, Pg. 1– 11 (19 – 29), 2013.
45. N V Nagendram, Dr T V Pradeep Kumar, "Compactness in fuzzy near-field spaces (CN-F-NS)", IJMA, ISSN. 2229 – 5046, Vol.4, No.10, Pg. 1 – 12, 2013.
46. N V Nagendram, Dr T V Pradeep Kumar and Dr Y Venkateswara Reddy, " Fuzzy Bi- Γ ideals in Γ semi near – field spaces (F Bi-Gamma I G)" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.4, No.11, Pg. 1 – 11, 2013.
47. N V Nagendram," EIFP Near-fields extension of near-rings and regular delta near-rings (EIFP-NF-E-NR)" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229 - 5046, Vol.4, No.8, Pg. 1 –11, 2013.
48. N V Nagendram, E Sudeeshna Susila, "Generalization of $(\in, \in Vqk)$ fuzzy sub near-fields and ideals of near-fields (GF-NF-IO-NF)", IJMA, ISSN.2229-5046, Vol.4, No.7, Pg. 1 – 11, 2013.
49. N V Nagendram,Dr T V Pradeep Kumar," A note on Levitzki radical of near-fields(LR-NF)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.4, No.4, Pg.288 – 295, 2013.
50. N V Nagendram, "Amalgamated duplications of some special near-fields(AD-SP-N-F)",Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.4, No.2, Pg.1 – 7, 2013.
51. N V Nagendram," Infinite sub near-fields of infinite near-fields and near-left almost near-fields(IS-NF-INF-NL-A-NF)",Published by International Journal of Mathematical Archive, IJMA,ISSN. 2229-5046, Vol.4, No.2, Pg. 90 – 99, 2013.
52. N V Nagendram "-----" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.4, No.7, Pg. 1 – 11, 2013
53. N V Nagendram, E Sudeeshna Susila, Dr T V Pradeep Kumar "Some problems and applications of ordinary differential equations to Hilbert Spaces in Engg mathematics (SP-O-DE-HS-EM)", IJMA, ISSN.2229-5046, Vol.4, No.4,Pg. 118 – 125, 2013.
54. N V Nagendram,Dr T V Pradeep Kumar and D Venkateswarlu, " Completeness of near-field spaces over near-fields (VNFS-O-NF)" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.5, No.2, Pg. 65 – 74, 2014
55. Dr N V Nagendram "A note on Divided near-field spaces and ϕ -pseudo – valuation near-field spaces over regular δ -near-rings (DNF- ϕ -PVNFS-O- δ -NR)" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.6, No.4, Pg. 31 – 38, 2015.
56. Dr. N V Nagendram "A Note on B_1 -Near-fields over R-delta-NR(B_1 -NFS-R- δ -NR)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.6, No.8, Pg. 144 – 151, 2015.
57. Dr. N V Nagendram " A Note on TL-ideal of Near-fields over R-delta-NR(TL-I-NFS-R- δ -NR)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.6, No.8, Pg. 51 – 65, 2015.
58. Dr. N V Nagendram "A Note on structure of periodic Near-fields and near-field spaces (ANS-P-NF-NFS)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.7, No.4, Pg. 1 – 7, 2016.
59. Dr. N V Nagendram "Certain Near-field spaces are Near-fields(C-NFS-NF)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.7, No.4, Pg. 1 – 7, 2016.
60. Dr. N V Nagendram "Sum of Annihilators Near-field spaces over Near-rings is Annihilator Near-field space (SA-NFS-O-A-NFS)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.7, No.1, Pg. 125 – 136, 2016.
61. Dr. N V Nagendram "A note on commutativity of periodic near-field spaces", Published by IJMA, ISSN. 2229 - 5046, Vol.7, No. 6, Pg. 27 – 33, 2016.
62. Dr N V Nagendram "Densely Co-Hopfian sub near-field spaces over a near-field, IMA, ISSN No.2229-5046, 2016, Vol.7, No.10, Pg 1-12.
63. Dr N V Nagendram, "Closed (or open) sub near-field spaces of commutative near-field space over a near-field", 2016, Vol.7, No, 9, ISSN NO.2229 – 5046, Pg No.57 – 72.

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