BIPOLAR VALUED MULTI FUZZY SUBSEMIGROUPS OF A SEMIGROUP

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ABSTRACT
In this paper, we study some of the properties of bipolar valued multi fuzzy subsemigroup and prove some results on these.

Key Words: Bipolar valued fuzzy subset, bipolar valued multi fuzzy subset, bipolar valued multi fuzzy subsemigroup, product, pseudo bipolar valued multi fuzzy coset.

INTRODUCTION
In 1965, Zadeh [12] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets etc [5]. Lee [7] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy subset, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0 ) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [7, 8]. We introduce the concept of bipolar valued multi fuzzy subsemigroup and established some results.

1. PRELIMINARIES

1.1 Definition: A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form A = \{< x, A^+(x), A^-(x) >/ x ∈ X\}, where A^+: X→ [0, 1] and A^-: X→ [−1, 0]. The positive membership degree A^+(x) denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree A^-(x) denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

1.2 Example: A = {< a, 0.4, −0.7 >, < b, 0.8, −0.5 >, < c, 0.7, −0.4 >} is a bipolar valued fuzzy subset of X = {a, b, c}.

1.3 Definition: A bipolar valued multi fuzzy set (BVMFS) A in X is defined as an object of the form A = {< x, A^+_i(x), A^-_i(x) >/ x ∈ X}, where A^+_i: X→ [0, 1] and A^-_i: X→ [−1, 0]. The positive membership degrees A^+_i(x) denote the satisfaction degree of an element x to the property corresponding to a bipolar valued multi fuzzy set A and the negative membership degrees A^-_i(x) denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued multi fuzzy set A.

1.4 Example: A = {< a, 0.3, 0, 2, 0.3, −0.3, −0.7, −0.4 >, < b, 0.2, 0.5, 0.6, −0.7, −0.2, −0.6 >, < c, 0.5, 0.4, 0.7, −0.4, −0.2, −0.1 >} is a bipolar valued multi fuzzy subset with order three of X = {a, b, c}.

1.5 Definition: Let S be a semigroup. A bipolar valued multi fuzzy subset A of S is said to be a bipolar valued multi fuzzy subsemigroup of S if the following conditions are satisfied
(i) A^+_i(xy) ≥ min \{A^+_i(x), A^+_i(y)\}
(ii) A^-_i(xy) ≤ max \{A^-_i(x), A^-_i(y)\} for all x and y in S.

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1.6 Example: Let \( S = \{1, -1, i, -i\} \) be a semigroup with respect to the ordinary multiplication. Then \( A = \{< 1, 0.6, 0.6, 0.5, -0.6, -0.7, -0.3>, < -1, 0.5, 0.5, 0.4, -0.5, -0.6, -0.2>, < i, 0.3, 0.2, 0.3, -0.4, -0.5, -0.1>, < -i, 0.3, 0.2, 0.3, -0.4, -0.5, -0.1> \} \) is a bipolar valued multi fuzzy subsemigroup of \( S \).

1.7 Definition: Let \( A = \langle A_i^+, A_i^- \rangle \) and \( B = \langle B_i^+, B_i^- \rangle \) be any two bipolar valued multi fuzzy subsets of sets \( G \) and \( H \), respectively. The product of \( A \) and \( B \), denoted by \( AB \), is defined as \( AB = \{ (x, y), (A_i^+, B_i^+) (x, y), (A_i^-, B_i^-) (x, y) \} / \) for all \( x \) in \( G \) and \( y \) in \( H \) where \( (A_i^+, B_i^+) (x, y) = min \{ A_i^+(x), B_i^+(y) \} \) and \( (A_i^-, B_i^-) (x, y) = max \{ A_i^-(x), B_i^-(y) \} \) for all \( x \) in \( G \) and \( y \) in \( H \).

1.8 Definition: Let \( A = \langle A_i^+, A_i^- \rangle \) be a bipolar valued multi fuzzy subset in a set \( S \), the strongest bipolar valued multi fuzzy relation on \( S \), that is a bipolar valued multi fuzzy relation on \( A \) is \( V = \{(x, y), V_i^+(x, y), V_i^-(x, y)\} / x \) and \( y \) in \( S \) given by \( V_i^+(x, y) = min \{ A_i^+(x), A_i^+(y) \} \) and \( V_i^-(x, y) = max \{ A_i^-(x), A_i^-(y) \} \) for all \( x \) and \( y \) in \( S \).

1.9 Definition: Let \( A = \langle A_i^+, A_i^- \rangle \) be a bipolar valued multi fuzzy subsemigroup of a semigroup \( S \) and \( a \) in \( S \). Then the pseudo bipolar valued multi fuzzy coset \((aA)^p = \langle (aA_i^+, aA_i^-) \rangle \) is defined by \( (aA_i^+, aA_i^-) (x) = p(a) A_i^+(x) \) and \( (aA_i^-, aA_i^-) (x) = p(a)A_i^-(x) \), for every \( x \) in \( S \) and for some \( p \) in \( P \).

2. PROPERTIES

2.1 Theorem: Let \( A = \langle A_i^+, A_i^- \rangle \) be a bipolar valued multi fuzzy subsemigroup of a semigroup \( S \).

(i) If \( A_i^+(xy) = 0 \), then either \( A_i^+(x) = 0 \) or \( A_i^+(y) = 0 \) for \( x \) and \( y \) in \( S \).

(ii) If \( A_i^-(xy) = 0 \), then either \( A_i^-(x) = 0 \) or \( A_i^-(y) = 0 \) for \( x \) and \( y \) in \( S \).

Proof: Let \( x \) and \( y \) in \( S \).

(i) By the definition \( A_i^+(xy) \geq \min \{ A_i^+(x), A_i^+(y) \} \) which implies that \( 0 \geq \min \{ A_i^+(x), A_i^+(y) \} \). Therefore either \( A_i^+(x) = 0 \) or \( A_i^+(y) = 0 \).

(ii) By the definition \( A_i^-(xy) \leq \max \{ A_i^-(x), A_i^-(y) \} \) which implies that \( 0 \leq \max \{ A_i^-(x), A_i^-(y) \} \). Therefore either \( A_i^-(x) = 0 \) or \( A_i^-(y) = 0 \).

2.2 Theorem: If \( A = \langle A_i^+, A_i^- \rangle \) is a bipolar valued multi fuzzy subsemigroup of a semigroup \( S \), then \( H = \{ x \in S | A_i^+(x) = 1, A_i^-(x) = -1 \} \) is either empty or a subsemigroup of \( S \).

Proof: If no element satisfies this condition, then \( H \) is empty. If \( x \) and \( y \) in \( H \), then \( A_i^+(xy) \geq \min \{ A_i^+(x), A_i^+(y) \} = \min \{ 1, 1 \} = 1 \). Therefore \( A_i^+(x) = 1 \). And \( A_i^-(xy) \leq \max \{ A_i^-(x), A_i^-(y) \} = \max \{ -1, -1 \} = -1 \). Therefore \( A_i^-(x) = -1 \). That is \( xy \in H \). Hence \( H \) is a subsemigroup of \( S \). Hence \( H \) is either empty or a subsemigroup of \( S \).

2.3 Theorem: If \( A = \langle A_i^+, A_i^- \rangle \) is a bipolar valued multi fuzzy subsemigroup of \( S \), then \( H = \{ x \in S | A_i^+(x) = H(A_i^+) \) and \( A_i^-(x) = H(A_i^-) \} \) is a subsemigroup of \( S \).

Proof: Here \( H = \{ x \in S | A_i^+(x) = H(A_i^+) \) and \( A_i^-(x) = H(A_i^-) \} \). Let \( x, y \) in \( H \). Then \( A_i^+(xy) \geq \min \{ A_i^+(x), A_i^+(y) \} \geq \min \{ H(A_i^+), H(A_i^+) \} = H(A_i^+) \). Hence \( A_i^+(xy) = H(A_i^+) \). Also \( A_i^-(xy) \leq \max \{ A_i^-(x), A_i^-(y) \} = \max \{ H(A_i^-), H(A_i^-) \} = H(A_i^-) \). Hence \( A_i^-(xy) = H(A_i^-) \). Therefore \( xy \in H \). Hence \( H \) is a subsemigroup of \( S \).

2.4 Theorem: If \( A = \langle A_i^+, A_i^- \rangle \) and \( B = \langle B_i^+, B_i^- \rangle \) are two bipolar valued multi fuzzy subsemigroups of a semigroup \( S \), then their intersection \( A \cap B \) is a bipolar valued multi fuzzy subsemigroup of \( S \).

Proof: Let \( A = \langle < x, A_i^+(x), A_i^-(x) > / x \in S \rangle \) and \( B = \langle < x, B_i^+(x), B_i^-(x) > / x \in S \rangle \). Let \( C = A \cap B \) and \( C = \langle < x, C_i^+(x), C_i^-(x) > / x \in S \rangle \). Now \( C_i^+(x) = \min \{ A_i^+(x), B_i^+(x) \} \geq \min \{ A_i^+(x), A_i^-(y) \} \), \( \min \{ B_i^+(x), B_i^-(y) \} \) \geq \( \min \{ A_i^+(x), B_i^+(x) \} \), \( \min \{ A_i^-(y), B_i^-(y) \} \) \geq \( \min \{ C_i^+(x), C_i^-(y) \} \). Therefore \( C_i^+(x) \geq \min \{ C_i^+(x), C_i^+(y) \} \). Also \( C_i^-(x) \leq \max \{ A_i^-(x), B_i^-(x) \} \leq \max \{ A_i^-(x), A_i^-(y) \} \), \( \max \{ B_i^-(x), B_i^-(y) \} \). Therefore \( C_i^-(x) \leq \max \{ C_i^-(x), C_i^-(y) \} \). Hence \( A \cap B \) is a bipolar valued multi fuzzy subsemigroup of \( S \).

2.5 Theorem: The intersection of a family of bipolar valued multi fuzzy subsemigroups of a semigroup \( S \) is a bipolar valued multi fuzzy subsemigroup of \( S \).

Proof: The Theorem is true by Theorem 2.4.

2.6 Theorem: If \( A = \langle A_i^+, A_i^- \rangle \) and \( B = \langle B_i^+, B_i^- \rangle \) are any two bipolar valued multi fuzzy subsemigroups of the semigroups \( S_1 \) and \( S_2 \) respectively, then \( A \times B = \langle (A_i^+ \times B_i^+), (A_i^- \times B_i^-) \rangle \) is a bipolar valued multi fuzzy subsemigroup of \( S_1 \times S_2 \).
Proof: Let A and B be two bipolar-valued multi fuzzy subsemigroups of the semigroups $S_1$ and $S_2$ respectively. Let $x_1$ and $x_2$ in $S_1$, $y_1$ and $y_2$ in $S_2$. Then $(x_1, y_1)$ and $(x_2, y_2)$ are in $S_1 \times S_2$. Now, $(A \times B)^+(x_1, y_1, x_2, y_2) = (A^+ \times B^+)(x_1, y_1)+ (A^- \times B^-)(x_2, y_2)$.

$2.7 \text{ Theorem:}$ Let $A = \langle A^+, A^- \rangle$ be a bipolar valued multi fuzzy subset of a semigroup $(S, \cdot, 1)$ and $V = \langle V^+, V^- \rangle$ be the strongest bipolar valued multi fuzzy relation of $S$. If $A$ is a bipolar valued multi fuzzy subsemigroup of $S$, then $V$ is a bipolar valued multi fuzzy subsemigroup of $S \times S$.

Proof: Suppose that $A$ is a bipolar valued multi fuzzy subsemigroup of $S$. Then for any $(x_1, x_2)$ and $(y_1, y_2)$ are in $S \times S$. We have $V^+_i((x_1, x_2)(y_1, y_2)) = \max \{V^+_i(x_1, y_1), A^+_i \times B^+_i(x_2, y_2)\} \geq \max \{A^+_i(x_1, y_1), A^+_i(x_2, y_2)\} \geq \max \{A^+_i(x_1, y_1), A^+_i(x_2, y_2)\}$. Also we have $V^-_i((x_1, x_2)(y_1, y_2)) = \max \{V^-_i(x_1, y_1), A^-_i \times B^-_i(x_2, y_2)\} = \max \{A^-_i(x_1, y_1), A^-_i(x_2, y_2)\}$. Therefore $(A^+_i \times B^+_i)(x_1, y_1) \times (A^-_i \times B^-_i)(x_2, y_2)$.

$2.8 \text{ Theorem:}$ Let $A = \langle A^+, A^- \rangle$ be a bipolar valued multi fuzzy subsemigroup of a semigroup $S$. Then the pseudo bipolar valued multi fuzzy coset $(aA)^p = \langle (a A^-)^p, (a A^+)^p \rangle$ is a bipolar valued multi fuzzy subsemigroup of the semigroup $S$, for every $a$ in $S$ and $p$ in $P$.

Proof: Let $A$ be a bipolar valued multi fuzzy subsemigroup of the semigroup $S$. For every $x$ and $y$ in $S$, we have $(aA)^p(x, y) = p(a)A^p(x, y)$ and $(aA)^p(x, y) = p(a)A^-p(x, y)$ for $x$ and $y$ in $S$. And $(aA)^p(x, y) = p(a)A^p(x, y) \leq p(a) \max \{A^-(x, y)\}$. Therefore $(aA)^p(x, y) \leq \max \{aA^-(x, y), (aA^+)^p(x, y)\}$ for $x$ and $y$ in $S$. Hence $(aA)^p$ is a bipolar valued multi fuzzy subsemigroup of the semigroup $S$.

REFERENCES


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