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# B - METRIC SPACE AND COMMON FIXED POINT THEOREMS FOR CONTRACTION MAPPINGS

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***ABSTRACT***

*In this paper, we have generalize and prove common fixed point results for mapping satisfying a general contractive condition in complete b- metric spaces. Our results are generalizations of results in [12].*

***Mathematics Subject Classification:*** *Primary 47H10; Secondary 54H25.*

***Key Words:*** *Fixed Point, Common fixed point theorem, Contractive condition, Complete b-Metric Space.*

## 1. INTRODUCTION

Fixed point is a beautiful mixture of Analysis, Topology and Geometry. For the last 50 years, fixed point theory has been revealed itself as a very powerful and important tool in the study of nonlinear phenomena. In particular, fixed point techniques have been applied in diverse fields such as in Biology, Chemistry, and Economics, Engineering, Game theory and Physics. Fixed point theory is rapidly moving into the mainstream of Mathematics mainly because of its applications in diverse fields which include numerical methods like Newton-Raphson method, establishing Picard's existence theorem, existence of solution of integral equations and a system of linear equations.

Fixed points are the points which remain invariant under a map/transformation. Fixed points tell us which parts of the space are pinned in plane, not moved, by the transformation. The fixed points of a transformation restrict the motion of the space under some restrictions. Pivotal results of functional Analysis.

In 1922, S. Banach The first important and significant result was proved a fixed point theorem for contraction mappings in complete metric space and also called it Banach fixed point theorem / Banach contraction principle which is considered as the mile stone in fixed point theory. This theorem states that, A mapping $T: X \to X$ where $(X, d)$ is a metric space, is said to be a contraction if there exists $k \in [0,1)$ such that

$$d(Tx, Ty) \leq kd(x, y) \text{ for all } x, y \in X. \tag{1.1}$$

If the metric space $(X, d)$ is complete the mapping satisfying (1.1) has a unique fixe point.

i.e every contraction map on a complete metric space has fixed point. Inequality (1.1) implies continuity of $T$. This theorem is very popular and effective tool in solving existence problems in many branches of mathematical analysis and engineering. There are a lot of generalizations of this principle has been obtained in several directions.

In 1989, Backhtin[1] introduced the concept of b- metric space. In 1993, Czerwik [2] extended the results of b-metric spaces that generalized the famous Banach contraction principle in metric space. Using this idea researcher presented generalization of the renowned Banach fixed point theorem in the b-metric space. Akkoochi, M. [4] Boriceanu[6], Mehmetkir *et al.* [7],Olatinwo, *et al.* [8], Pacurar [9] extended the fixed point theorem in b-metric space. A b- metric space was also called a metric type spaces in [10]. The fixed point theory in metric type spaces was investigated in [10] and [11].Recently, Pankaj *et al.* [14] gave some results related fixed point theorem in b-metric spaces. They have shown the extension theorem given by Reich [15], and Hardy and Rogers [16] to the b- metric spaces. In sequel, A.K. Dubey *et al.* [12] obtained unique fixed point results in b- metric spaces, which is generalized results of [7].

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In this paper, we extend common fixed point theorems for these mappings in b- metric spaces. Our results are generalization of results of [12].

## 2. PRELIMINARIES

In this section, at first, we recall some definitions and properties of their in b- metric spaces:

**Definition 2, 1([(1] & [5]): Let** $X$ be a non empty set and $s \geq 1$ be a given real number. A function $d: X \times X \to R_{+,}$ is called a b- metric provided that, for all $x, y, z \in X$,

1. $d(x, y) = 0$ iff $x = y$,
2. $d(x, y) = d(y, x)$ ,
3. $d(x, z) \leq s[d(x, y) + d(y, z)]$ .Then

A pair $(X, d)$ is called a b-metric space. It is clear that definition of b-metric space is a extension of usual metric space.

**Example 2.2 (see [6]):** The space$l_p$ $(0 < P < 1)$,

$l_p$ =$\{(x_n) \subset R: \sum_{n=1}^{\infty} |x_n|^p < \infty\}$, together with the function$d: l_p \times l_p \to R$,

$d(x, y)$ = $(\sum_{n=1}^{\infty} |x_n - y_n|^p)^{\frac{1}{p}}$, where $x = x_n, y = y_n \in l_p$ is a b-metric space. By an elementary calculation We obtain that

$d(x, z) \leq 2^{\frac{1}{p}}[d(x, y) + d(y, z)]$ .

**Example 2.3 (see [5]):** Let $X = \{0,1,2\}$ and $d(2,0) = d(0,2) = m \geq 2$,

$d(0,1) = d(1,2) = d(1,0) = d(2,1) = 1$ $and$ $d(0,0) = d(1,1) = d(2,1) = 0$.Then

$d(x, y) \leq \frac{m}{2}[d(x, z) + d(z, y)]$ for all $x, y, z \in X$. If $m > 2$ then the triangle inequality does not hold.

**Example 2.4 (see [6]):** The $l_p$ [0, 1] where $(0 < p < 1)$ of all real functions$x(t), t \in [0,1]$ such that $\int_0^1 |x(t)|^p \, dt < \infty$, is a b- metric space if we take

$d(x, y)$ =$(\int_0^1 |x(t) - y(t)|^p \, dt)^{\frac{1}{p}}$, for each $x, y \in l_p[0,1]$.

**Definition 2.5[6]:**

(i) Let $(X, d)$be a b-metric space. Then a sequence $\{x_n\}$ in X is called a Cauchy sequence if and only if for all $\epsilon > 0$ there exists $n(\epsilon) \in N$ such that for each $n, m \geq n(\epsilon)$ we have $d(x_n, x_m) < \varepsilon$.

(ii) Let $(X, d)$be a b-metric space. Then a sequence $\{x_n\}$ in X is called a Convergent sequence if and only if there exists $x \in X$such that for all there exists $n(\epsilon) \in N$ such that for all $n, \geq n(\epsilon)$we have $d(x_n, x) < \varepsilon$.In this case $\lim_{n\to\infty} x_n = x$.

(iii) The b-metric space is complete if every Cauchy sequence convergent.

**Definition 2.5[17]:** Let $E$be a non empty set and $T: E \to E$ as self map. We say that $x \in E$ is a fixed point of T if $T(x) = x$ and denote by $FT$ or Fix ($T$) the set of all fixed points of $T$.

Let $E$be a non empty set and $T: E \to E$ as self map. For any given $x \in E$ we define $T^n(x)$ inductively by $T^0(x)$ = $x$ and $T^{n+1}(x) = T(T^n(x))$, we recall $T^n(x)$, the $n^{th}$iterative of $x$ under $T$. For any $x_0 \in X$, the sequence$\{x_n\}_{n\geq 0} \subset X$ given by

$x_n = Tx_{n-1} = T^n x_0$, n=1, 2……… is called the sequence of successive approximations with the initial value $x_0$. It is also known as the Picard iteration starting at $x_0$.

## 3. MAIN RESULTS

In this section, we shall prove common fixed point results for pair of mappings in b- metric spaces. The following theorems are extends and improve the theorem 1 & 2 from [12].

**Theorem 3.1:** Let $(X, d)$be a complete b-metric space with $s \geq 1$and $T_1, T_2: X \to X$be a self mappings satisfies the conditions

$$d(T_1x, T_2y) \leq a_1 d(x, y) + a_2 d(x, T_1x) + a_3 d(y, T_2y) + a_4[d(y, T_1x) + d(x, T_2y)] \quad (3.1)$$

Where$a_1 + 2a_2 s + a_3 + 2a_4 s \leq 1$, for all$x, y \in X$. then $T_1$ $and$ $T_2$ have a unique common fixed point.

**Proof:** Let $x_0 \in X$ and define sequence $\{x_n\}$ in $X$ such that

$$x_{2n} = T_1 x_{2n-1} = {T_1}^{2n} x_0; n = 1,2, \dots \dots \dots \dots \quad (3.2)$$

Similarly, we can have

$$x_{2n+1} = T_2 x_{2n} = T_2^{\,2n+1} x_0;\ n = 1,2, \dots\dots\dots \tag{3.3}$$

Now, we show that $\{x_n\}$ is Cauchy sequence. Let $x = x_{2n-1}$ and $y = x_{2n}$ in (3.1), we have

$$\begin{aligned} d(x_{2n}, x_{2n+1}) &= d(T_1 x_{2n-1}, T_2 x_{2n}) \\ &\le a_1 d(x_{2n-1}, x_{2n}) + a_2 d(x_{2n-1}, T_1 x_{2n-1}) + a_3 d(x_{2n}, T_2 x_{2n}) \\ &\quad + a_4 [d(x_{2n}, T_1 x_{2n-1}) + d(x_{2n-1}, T_2 x_{2n})] \\ &\le a_1 d(x_{2n-1}, x_{2n}) + a_2 d(x_{2n-1}, x_{2n}) + a_3 d(x_{2n}, x_{2n+1}) \\ &\quad + a_4 [d(x_{2n}, x_{2n}) + d(x_{2n-1}, x_{2n+1})] \\ &\le a_1 d(x_{2n-1}, x_{2n}) + s a_2 [d(x_{2n-1}, x_{2n}) + d(x_{2n}, x_{2n+1})] + a_3 d(x_{2n}, x_{2n+1}) \\ &\quad + s a_4 [d(x_{2n-1}, x_{2n}) + d(x_{2n}, x_{2n+1})] \end{aligned}$$

$$d(x_{2n}, x_{2n+1}) \le (a_1 + s a_2 + s a_4)\, d(x_{2n-1}, x_{2n}) + (s a_2 + a_3 + s a_4) d(x_{2n}, x_{2n+1})$$

Implies that (1-$s a_2 - a_3 - s a_4) d(x_{2n}, x_{2n+1}) \le (a_1 + s a_2 + s a_4)\, d(x_{2n-1}, x_{2n})$

So $\quad d(x_{2n}, x_{2n+1}) \le \frac{(a_1 + s a_2 + s a_4)}{(1 - s a_2 - a_3 - s a_4)}\, d(x_{2n-1}, x_{2n})$

Where L = $\frac{(a_1 + s a_2 + s a_4)}{(1 - s a_2 - a_3 - s a_4)}$

So

$$d(x_{2n}, x_{2n+1}) \le L\, d(x_{2n-1}, x_{2n}) \tag{3.4}$$

Similarly, we obtain

$$d(x_{2n+1}, x_{2n+2}) \le \frac{(a_1 + s a_2 + s a_4)}{(1 - s a_2 - a_3 - s a_4)}\, d(x_{2n}, x_{2n+1})$$

$$d(x_{2n+1}, x_{2n+2}) \le L\, d(x_{2n}, x_{2n+1}) \tag{3.5}$$

Where L = $\frac{(a_1 + s a_2 + s a_4)}{(1 - s a_2 - a_3 - s a_4)} \le 1$, as $(a_1 + 2 s a_2 + 2 s a_4 \le 1$

$$\begin{aligned} &a_1 + s a_2 + s a_4 \le 1 - s a_2 - a_3 - s a_4 \\ &d(x_{2n}, x_{2n+1}) \le L\, d(x_{2n-1}, x_{2n}) \\ &\qquad \le L^2\, d(x_{2n-2}, x_{2n-1}) \\ &\qquad \le \ \dots\dots\dots\dots\dots\dots \end{aligned}$$

Continue this process, we get

$$d(x_{2n}, x_{2n+1}) \le L^{2n} d(x_0, x_1) \tag{3.6}$$

Let $m, n > 0$ with $m > n$,

$$\begin{aligned} d(x_{2n}, x_{2m}) &\le s d(x_{2n}, x_{2n+1}) + s^2 d(x_{2n+1}, x_{2n+2}) + s^3 d(x_{2n+2}, x_{2n+3}) + \dots\dots\dots \\ &. \le s L^{2n} d(x_1, x_0) + \le s^2 L^{2n+1} d(x_1, x_0) + \dots\dots\dots\dots s^{2m} L^{2n+2m-1} d(x_1, x_0) \\ &\le s L^{2n} d(x_1, x_0)[1 + (sk) + (sk)^2 + \cdots \dots\dots\dots\dots\dots\dots . + (sk)^{2m-1} \\ &\le s L^{2n} d(x_1, x_0)[\ \frac{1 - (sk)^{2n-2m+1}}{1 - sk}\ ]. \end{aligned}$$

When we take $m, n \to \infty$. Then $\lim_{n\to\infty} d(x_{2n}, x_{2m}) = 0$.Thus$\{x_n\}$ is a Cauchy sequence in X.

Since X is complete, there exist some $u \in X$ such that $x_n \to u$ as $n \to \infty$.Assum not, then there exist $z \in X$ such that

$$d(u, T_1 u) = z > 0 \tag{3.7}$$

So by using triangular inequality and (3.1), we get

$$\begin{aligned} z &= d(u, T_1 u) \\ &\le s[d(u, x_{2n+1}) + d(x_{2n+1}, T_1 u)] \\ &\le sd(d(u, x_{2n+1}) + sd(T_2 x_{2n}, T_1 u) \\ &\le sd(u, x_{2n+1}) + s a_1 d(x_{2n}, u) + s a_2\, d(x_{2n}, T_2 x_{2n}) + s a_3 d(u, T_1 u) \\ &\qquad + s a_4 [d(u, T_2 x_{2n}) + d(x_{2n,} T_1 u)] \\ &\le sd(u, x_{2n+1}) + s a_1 d(x_{2n}, u) + s a_2\, d(x_{2n}, x_{2n+1}) + s a_3 d(u, T_1 u) \\ &\qquad + s a_4 d(u, T_2 x_{2n}) + s a_4\, d(x_{2n,} T_1 u) \\ &\le sd(u, x_{2n+1}) + s a_1 d(x_{2n}, u) + s^2 a_2\, d(x_{2n}, u) + s^2 a_2 d(u, x_{2n+1}) + s a_3 d(u, T_1 u) \\ &\qquad + s a_4 d(u, x_{2n+1}) + s^2 a_4\, [d(x_{2n,} u) + d(u, T_1 u)] \end{aligned}$$

=>$(1 - s a_3 - s^2 a_4) d(u, T_1 u) \le (s + s^2 a_2 + s a_4) d(x_{2n+1}, u) + (s a_1 + s^2 a_2 + s^2 a_4) d(x_{2n,} u)$

 

$$=> \ d(u,T_1u) \le \frac{(s+s^2a_2+sa_4)}{(1-sa_3-s^2a_4)}\, d(x_{2n+1},u) + \frac{(sa_1+s^2a_2+s^2a_4)}{(1-sa_3-s^2a_4)}\, d(x_{2n,}u) \tag{3.8}$$

Taking the limit of (3.8) as $n \to \infty$, we get that $z = \ d(u,T_1u) \le 0$, a contradiction with (3.7).

So, $z = 0$. Hence $u = \ T_1u$. Similarly, it can easily be proved that $u = \ T_2u$.Therefore, $u$ is common fixed point of $T_1$ and $T_2$.

Now, to prove the uniqueness of common fixed point.

Let $u^*$ be another common fixed point of $T_1$ and $T_{2.}$ $i.e.\ u^* = T_1u^* = T_2u^*$. Then

$$\begin{aligned} d(u,u^*) &= d(T_1u,T_2u^*) \\ &\le a_1d(u,u^*) + a_2d(u,T_1u) + a_3d(u^*,T_2u^*) + a_4[d(u^*,T_1u) + d(u,T_2u^*)] \\ &\le a_1d(u,u^*) + a_2d(u,u) + a_3d(u^*,u^*) + a_4[d(u^*,u) + d(u,u^*)] \\ &\le a_1d(u,u^*) + a_4[d(u^*,u) + d(u,u^*)] \\ &\le (a_1 + 2a_4)d(u,u^*), \text{ which is a contradiction, so that } u = u^*. \end{aligned}$$

This completes the proof.

**Theorem 3.2:** Let $(X,d)$be a complete b-metric space with $s \ge 1$and $T_1,T_2\colon X \to X$be a self mappings satisfies the conditions

$$d(T_1x,T_2y) \le a_1d(x,y) + a_2[d(x,T_1x) + d(y,T_2y)] \tag{3.9}$$

Where$a_1,a_2 \in [0,\frac{1}{3})$ for all$x,y \in X$. then $T_1 \ \ and \ \ T_2$ have a unique common fixed point.

**Proof:** Let $x_0 \in X$ and define sequence $\{x_n\}$ in $X$ such that

$$x_{2n} = \ T_1x_{2n-1} = \ {T_1}^{2n}x_0;\ n = 1,2, \dots\dots\dots \tag{3.10}$$

Similarly, we can have

$$x_{2n+1} = \ T_2x_{2n} = \ {T_2}^{2n+1}x_0;\ n = 1,2, \dots\dots\dots \tag{3.11}$$

Now, we show that $\{x_n\}$ is Cauchy sequence. Let $x = x_{2n-1}$ and $y = \ x_{2n}$ in (3.1), we have

$$\begin{aligned} d(x_{2n},x_{2n+1}) &= \ d(T_1x_{2n-1},T_2x_{2n}) \\ &\le a_1d(x_{2n-1},x_{2n}) + a_2[d(x_{2n-1},T_1x_{2n-1}) + d(x_{2n},T_2x_{2n})\ ] \\ &\le a_1d(x_{2n-1},x_{2n}) + a_2[d(x_{2n-1},x_{2n}) + d(x_{2n},x_{2n+1})\ ] \\ &\le (a_1 + \ a_2)\ d(x_{2n-1},x_{2n}) + a_2\ d(x_{2n},x_{2n+1}) \end{aligned}$$

$$=> \ (1-a_2)\ d(x_{2n},x_{2n+1}) \le (a_1 + \ a_2)\ d(x_{2n-1},x_{2n})$$

So $\quad d(x_{2n},x_{2n+1}) \ \le \frac{(a_1+a_2)}{(1-a_2)}\ d(x_{2n-1},x_{2n})$

Where L = $\frac{(a_1+a_2)}{(1-a_2)}$

$$\text{So} \quad d(x_{2n},x_{2n+1}) \ \le L\ d(x_{2n-1},x_{2n}) \tag{3.12}$$

Similarly, we obtain

$$d(x_{2n+1},x_{2n+2}) \ \le \frac{(a_1 + \ a_2)}{(1-a_2)}\ \ d(x_{2n},x_{2n+1})$$

$$d(x_{2n+1},x_{2n+2}) \ \le L\ d(x_{2n},x_{2n+1}) \tag{3.13}$$

Where L = $\frac{(a_1+a_2)}{(1-a_2)} \ \le 1$, as $(a_1 + \ 2a_2 \le 1$

$$a_1 + a_2 \le 1 - a_2,$$

$$\frac{(a_1 + \ a_2)}{(1-a_2)} \le 1$$

$$\begin{aligned} d(x_{2n},x_{2n+1}) &\le L\ d(x_{2n-1},x_{2n}) \\ &\le L^2\ d(x_{2n-2},x_{2n-1}) \\ &\le \quad \dots\dots\dots\dots\dots\dots\dots \end{aligned}$$

Continue this process, we get

$$d(x_{2n},x_{2n+1}) \le L^{2n}d(x_0,x_1) \tag{3.14}$$

 

Let $m, n > 0$ with $m > n$,

$$d(x_{2n}, x_{2m}) \leq s[d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2m})]$$
$$\leq sd(x_{2n}, x_{2n+1}) + s^2 d(x_{2n+1}, x_{2n+2}) + s^3 d(x_{2n+2}, x_{2n+3})$$
$$+ \ldots\ldots\ldots\ldots + s^{2m} d(x_{2n+2m-1}, x_{2m})$$
$$.\leq sL^{2n} d(x_1, x_0) + \leq s^2 L^{2n+1} d(x_1, x_0) + \ldots\ldots\ldots\ldots\ldots s^{2m} L^{2n+2m-1} d(x_1, x_0)$$
$$\leq sL^{2n} d(x_1, x_0)[1 + (sk) + (sk)^2 + \cdots \ldots\ldots\ldots\ldots\ldots + (sk)^{2m-1}]$$

$$\leq sL^{2n} d(x_1, x_0)\left[\frac{1-(sk)^{2n-2m+1}}{1-sk}\right].$$

When we take $m, n \to \infty$. Then $\lim_{n\to\infty} d(x_{2n}, x_{2m}) = 0$.

Thus$\{x_n\}$ is a Cauchy sequence in X.

Since X is complete, there exist some $u \in X$ such that $x_n \to u$ as $n \to \infty$.Assum not, then there exist $z \in X$ such that

$$d(u, T_1 u) = z > 0 \tag{3.15}$$

So by using triangular inequality and (3.9), we get

$$z = d(u, T_1 u)$$
$$\leq s[d(u, x_{2n}) + d(x_{2n}, T_1 u)]$$
$$\leq s[d(u, x_{2n}) + d(T_1 x_{2n-1}, T_1 u]$$
$$\leq sd(u, x_{2n}) + sd(T_1 x_{2n-1}, T_1 u)$$
$$\leq sd(u, x_{2n}) + sa_1 d(x_{2n-1}, u) + sa_2[d(x_{2n-1,} T_1 x_{2n-1}) + d(u, T_1 u)]$$
$$\leq sd(u, x_{2n}) + sa_1 d(x_{2n-1}, u) + sa_2[d(x_{2n-1,} T_1 x_{2n-1})] + sa_2[d(u, T_1 u)]$$
$$\leq sd(x_{2n}, u) + sa_1 d(x_{2n-1}, u) + s^2 a_2[d(x_{2n-1,} u) + d(u, x_{2n})] + sa_2[d(u, T_1 u)]$$
$$\leq sd(x_{2n}, u) + sa_1 d(x_{2n-1}, u) + s^2 a_2 d(x_{2n-1,} u) + s^2 d(u, x_{2n}) + sa_2[d(u, T_1 u)]$$

$$\Rightarrow (1 - sa_2) d(u, T_1 u) \leq sd(x_{2n}, u) + sa_1 d(x_{2n-1}, u) + s^2 a_2 d(x_{2n-1,} u) + s^2 a_2 d(x_{2n}, u) \tag{3.16}$$

Taking the limit of (3.16) as $n \to \infty$, we get that $z = d(u, T_1 u) \leq 0$, a contradiction with (3.15).

So, $z = 0$. Hence $u = T_1 u$. Similarly, it can easily be proved that $u = T_2 u$. Therefore, $u$ is common fixed point of $T_1$ and $T_2$.

Now, to prove the uniqueness of common fixed point.

Let $u^*$ be another common fixed point of $T_1$ and $T_{2.}$ $i.e.\ u^* = T_1 u^* = T_2 u^*$. Then

$$d(u, u^*) = d(T_1 u, T_2 u^*)$$
$$\leq a_1 d(u, u^*) + a_2[d(u, T_1 u) + d(u^*, T_2 u^*)]$$
$$\leq a_1 d(u, u^*)$$

$$(1 - a_1) d(u, u^*) \leq 0$$

Which is a contradiction, so that $u = u^*$.

This completes the proof.

## CONCLUSION

From our result, we improve and extend well – known results in the literature which is very useful for further research work.

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