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$(\alpha p)^*$ - CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce a new class of sets called $(\alpha p)^*$ -closed sets and a new class of generalized function called $(\alpha p)^*$ -continuous maps and $(\alpha p)^*$ -irresolute maps in topological spaces. Also we discuss some basic properties and applications of $(\alpha p)^*$ -closed sets, which define a new class of space $T_{(\alpha p)^*}$ spaces.

Key words: $(\alpha p)^*$ -closed sets, $(\alpha p)^*$ -continuous, $(\alpha p)^*$ - irresolute and $T_{(\alpha p)^*}$ spaces.

1. INTRODUCTION

In 1970 Levine [12] first introduced the concept of generalized closed (briefly g-closed) sets in topological spaces. S.P.Arya and T.Nour [2] defined gs-closed sets in 1990. Dontchev [9] introduced gsp-closed sets. Maki *et al.* [15] defined α g-closed sets in 1994. Levine [13], Mashhour *et al.* [14] introduced semi-open sets, pre-open sets respectively. Maki *et al.* [16] introduced g α -closed sets. The purpose of this paper is to introduce the concept of (αp)^{*} - closed sets, (αp)^{*} -continuous maps, (αp)^{*}-irresolute and $T_{(\alpha p)^*}$ spaces and investigate some of their properties.

2. PRELIMINARIES

Throughout this paper (X, τ) represents a non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space (X, τ) , cl(A) and int(A) denote the closure and interior of the subset A

Definition: 2.1 A subset A of a topological space (X,τ) is called,

- i. a pre- open set[14] if $A \subseteq int (cl(A))$ and pre- closed set if $cl (int(A)) \subseteq A$.
- ii. a semi-open set[13] if $A \subseteq cl$ (int(A)) and a semi -closed set if int (cl(A)) \subseteq A.
- iii. an α -open set[17] if A \subseteq int (cl (int(A))) and if an α -closed if cl (int (cl(A))) \subseteq A.
- iv. a semi-preopen set[1] if $A \subseteq cl$ (int (cl(A))) and a semi-preclosed set if int (cl (int(A))) \subseteq A.

Definition: 2.2 A subset A of a topological space (X, τ) is called,

- i. a generalized closed set [12] (briefly g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- ii. a generalized semi-preclosed set [9] (briefly gsp-closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- iii. a α -generalized closed set [15] (briefly αg -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- iv. a generalized α -closed set [16] (briefly α -closed) if α cl(A) \subseteq U whenever A \subseteq U and U is α -open in (X, τ).
- v. a semi-generalized closed set [4] (briefly sg-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ) .
- vi. a semi-pregeneralized closed set [21] (briefly spg-closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ) .
- vii. a generalized semiclosed set [2] (briefly gs-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- viii. a α -generalized semiclosed set [19] (briefly αgs -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in (X, τ) .

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- ix. a ψ -closed set [22] if scl(A) \subseteq U whenever A \subseteq U and U is sg-open in (X, τ).
- x. a $(sp)^*$ -closed set [10] cl(A) \subseteq U whenever A \subseteq U and U is semi-pre open in(X, τ).
- xi. a $(gsp)^*$ -closed set [18] cl(A) \subseteq U whenever A \subseteq U and U is gsp-open in (X, τ).

Definition: 2.3 A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is called

- i. an α -continuous [17] if $f^{-1}(V)$ is an α -closed set of (X, τ) for every closed set V of (Y, σ) .
- ii. a g-continuous [3] if $f^{-1}(V)$ is a g-closed set of (X, τ) for every closed set V of (Y, σ) .
- iii. a sg-continuous [4] if $f^{-1}(V)$ is a sg-closed set of (X, τ) for every closed set V of (Y, σ) .
- iv. a gs-continuous [6] if $f^{-1}(V)$ is a gs-closed set of (X, τ) for every closed set V of (Y, σ) .
- v. an αg -continuous [11] if $f^{-1}(V)$ is an αg -closed set of (X, τ) for every closed set V of (Y, σ) .
- vi. a g α -continuous [16] if $f^{-1}(V)$ is an $g\alpha$ -closed set of (X, τ) for every closed set V of (Y, σ) .
- vii. a gsp-continuous [9] if $f^{-1}(V)$ is a gsp-closed set of (X, τ) for every closed set V of (Y, σ) .
- viii. a spg-continuous [21] if $f^{-1}(V)$ is a spg-closed set of (X, τ) for every closed set V of (Y, σ) .
- ix. an *ags*-continuous [20] if $f^{-1}(V)$ is an *ags*-closed set of (X, τ) for every closed set V of (Y, σ) .
- x. a ψ -continuous [22] if $f^{-1}(V)$ is a ψ -closed set of (X, τ) for every closed set V of (Y, σ) .
- xi. $a(sp)^*$ -continuous [10] if $f^{-1}(V)$ is a $(sp)^*$ -closed set of (X, τ) for every closed set V of (Y, σ) .
- xii. $(gsp)^*$ -continuous [18] if $f^{-1}(V)$ is a $(gsp)^*$ -closed set of (X, τ) for every closed set V of (Y, σ) .

Definition: 2.4 A topological space (X, τ) is said to be,

- i. a $T_{1/2}$ space [12] if every g-closed set in it is closed.
- ii. a T_b space [7] if every gs-closed set in it is closed.
- iii. a T_d space [7] if every gs-closed set in it is g-closed.
- iv. $a_{\alpha}T_d$ space [8] if every αg -closed set in it is g-closed.
- v. a $_{\alpha}T_b$ space [8] if every αg -closed set in it is closed.
- vi. a $T_{g\alpha}$ space [5] if every $g\alpha$ -closed set in it is αg -closed.
- vii. T_{qsp} space [23] if every gsp-closed set in it is gs-closed.

3. BASIC PROPERTIES OF $(\alpha p)^*$ -CLOSED SETS

We introduce the following definition

Definition 3.1: A subset A of a topological space (X, τ) is called a $(\alpha p)^*$ -closed set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-preopen in X.

Theorem 3.2: Every closed set is $(\alpha p)^*$ - closed set.

Proof follows from the definition.

The following example supports that an $(\alpha p)^*$ -closed set need not be true in general.

Example 3.3: Let $X = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{a, b\}\}$. Let $A = \{b\}$ is $(\alpha p)^*$ -closed but not closed in (X, τ) .

Theorem 3.4: Every $(\alpha p)^*$ -closed set is gsp-closed set.

Proof: Let A be $(\alpha p)^*$ -closed. Let A \subseteq U and U be open. Then U is semi-preopen. Since A is $(\alpha p)^*$ - closed, then $spcl(A) \subseteq \alpha cl(A) \subseteq U$. Hence A is gsp-closed set.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.5: Let $X = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. Let $A = \{a\}$ is gsp-closed but not $(\alpha p)^*$ -closed in (X, τ) .

Theorem 3.6: Every $(\alpha p)^*$ -closed set is gs-closed set.

Proof follows from the definition.

The reverse implication does not hold as it can be seen from the following example.

Example 3.7: Let $X = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. Let $A = \{a\}$ is gs- closed but not $(\alpha p)^*$ -closed in (X, τ) .

Theorem 3.8 Every $(\alpha p)^*$ -closed set is a spg-closed set.

Proof: Let A be $(\alpha p)^*$ -closed set. Let A \subseteq U and U be open. Then U is semi-preopen. Since A is $(\alpha p)^*$ - closed, then $spcl(A) \subseteq \alpha cl(A) \subseteq U$. Hence A is spg-closed set.

The reverse implication does not hold as it can be seen from the following example.

Example 3.9: Let X = {*a*, *b*, *c*}, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. Let A= {*a*} is spg-closed but not $(\alpha p)^*$ -closed in (X, τ) .

Theorem 3.10: Every $(\alpha p)^*$ -closed set is sg-closed set.

Proof: Let A be $(\alpha p)^*$ -closed set. Let A \subseteq U and U be open. Then U is semi-preopen. Since A is $(\alpha p)^*$ - closed, then $scl(A) \subseteq \alpha cl(A) \subseteq U$. Hence A is sg-closed set.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.11: Let $X = \{a, b, c\}, \tau = \{\phi, X, \{b, c\}\}$. Let $A = \{a, b\}$ is a sg-closed but not $(\alpha p)^*$ -closed in (X, τ) .

Theorem 3.12: Every $(\alpha p)^*$ -closed set is α g-closed set.

Proof: Let A be $(\alpha p)^*$ -closed set. Let A \subseteq U and U be open. Then U is semi-preopen. Since A is $(\alpha p)^*$ - closed, then $\alpha cl(A) \subseteq U$. Hence A is αg -closed set.

The reverse implication does not hold as it can be seen from the following example.

Example 3.13: Let X = {a, b, c}, $\tau = {\phi, X, {b, c}}$. Let A= {a, b} is α g-closed but not $(\alpha p)^*$ -closed in (X, τ).

Theorem 3.14: Every $(\alpha p)^*$ -closed set is $g\alpha$ -closed set.

Proof follows from the definition.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.15: Let $X = \{a, b, c\}, \tau = \{\phi, X, \{b, c\}\}$. Let $A = \{a, b\}$ is $g\alpha$ -closed but not $(\alpha p)^*$ -closed in (X, τ) .

Theorem 3.16: Every $(\alpha p)^*$ -closed set is a ψ -closed set.

Proof: Let A be $(\alpha p)^*$ -closed set. Let A \subseteq U and U be open. Then U is semi-preopen. Since A is $(\alpha p)^*$ - closed, then $scl(A) \subseteq \alpha cl(A) \subseteq U$. Hence A is ψ – closed set.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.17: Let $X = \{a, b, c\}, \tau = \{\phi, X, \{b, c\}\}$. Let $A = \{a, b\}$ is ψ -closed but not $(\alpha p)^*$ -closed in (X, τ) .

Theorem 3.18: Every $(sp)^*$ -closed set is a $(\alpha p)^*$ -closed set.

Proof: Let A be $(sp)^*$ -closed set. Let A \subseteq U and U be semi-preopen. Since A is $(sp)^*$ -closed, then $acl(A) \subseteq Cl(A) \subseteq U$. Hence A is $(\alpha p)^*$ - closed.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.19: Let X = {*a*, *b*, *c*}, $\tau = \{\phi, X, \{a\}, \{a, b\}\}$. Let A = {*b*} is $(\alpha p)^*$ -closed but not $(sp)^*$ -closed in (X, τ) .

Theorem 3.20: Every $(\alpha p)^*$ -closed set is αgs -closed set.

Proof follows from the definition.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.21: Let $X = \{a, b, c\}, \tau = \{\phi, X, \{b, c\}\}$. Let $A = \{a, b\}$ is αgs -closed but not $(\alpha p)^*$ -closed in (X, τ) .

Theorem 3.22: Every $(gsp)^*$ -closed set is a $(\alpha p)^*$ -closed set.

Proof follows from the definition.

Example 3.23: Let $X = \{a, b, c\}, \tau = \{\phi, X, \{a\}\}$. Let $A = \{b\}$ is $(\alpha p)^*$ -closed but not $(gsp)^*$ -closed in (X, τ) .

Theorem 3.24: A is $(\alpha p)^*$ -closed set of (X, τ) if and only if $\alpha cl(A) \setminus A$ does not contain any nonempty semi-pre closed set

Proof: Necessity: Let F be a semi-pre closed set of (X, τ) such that $F \subseteq \alpha cl(A) \setminus A$. Then $A \subseteq X \setminus F$. A is $(\alpha p)^*$ -closed and $X \setminus F$ is semi-preopen, $\alpha cl(A) \subseteq X \setminus F$. Since $F \subseteq X \setminus \alpha cl(A)$ So, $F \subseteq ((X/\alpha cl(A)) \cap \alpha cl(A)/A) = \phi$. Therefore $F = \phi$

Sufficiency: Let A be a subset of (X, τ) such that $\alpha cl(A) \setminus A$ does not contain any non-empty semi-pre closed set.Let U be a semi-pre open set of (X, τ) such that $A \subseteq U$. If $\alpha cl(A) \not\subset U$, then $\alpha cl(A) \cap U^c \neq \phi$. and $\alpha cl(A) \cap U^c$ is semi-pre closed. $\therefore \phi \neq \alpha cl(A) \cap U^c \subseteq \alpha cl(A) \setminus A$. $\therefore \alpha cl(A) \setminus A$ contains a non-empty semi-pre closed set, which is a contradiction. $\therefore \alpha cl(A) \subseteq U$ \therefore A is $(\alpha p)^*$ -closed set.

Theorem 3.25: If A is $(\alpha p)^*$ -closed set in X and $A \subseteq B \subseteq \alpha cl(A)$ then B is $also(\alpha p)^*$ -closed set in X.

Proof: It is given that A is $(\alpha p)^*$ -closed set in X.To prove B is also $(\alpha p)^*$ -closed set of X.Let U be semi-preopen set of X such that $B \subseteq U$. Since $A \subseteq B$, we have $A \subseteq U$. Since A is $(\alpha p)^*$ -closed and $\alpha cl(A) \subseteq U$. Now $\alpha cl(B) \subseteq \alpha cl(\alpha cl(A)) = \alpha cl(A) \subseteq U$.So that $\alpha cl(B) \subseteq U$. Hence B is $(\alpha p)^*$ -closed set in X.

Theorem 3.26: If A is both semi-preopen and $(\alpha p)^*$ -closed, then A is α -closed.

Proof: Let A be both semi-preopen and $(\alpha p)^*$ -closed. Let $A \subseteq A$, Where A is semi-preopen. Then $\alpha cl(A) \subseteq A$ as A is $(\alpha p)^*$ -closed in (X, τ) . But $A \subseteq \alpha cl(A)$ is always true. $\therefore A = \alpha cl(A)$ Hence A is α -closed set in (X, τ)

The above result can be represented as the following diagram



Where $A \rightarrow B$ represents A implies B and A $\rightarrow B$ represents A does not imply B

4. $(\alpha p)^*$ -CONTINUOUS AND $(\alpha p)^*$ -IRRESOLUTE MAPS

We introduce the following definition

Definition 4.1: A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is called $(\alpha p)^*$ -continuous if $f^{-1}(V)$ is a $(\alpha p)^*$ -closed set of (X, τ) for every closed set V of (Y, σ) .

Theorem 4.2: Every continuous map is $(\alpha p)^*$ - continuous.

The following example supports that the converse of the above theorem is not true.

Example 4.3: Let $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{a, b\}\}$ and $\sigma = \{\phi, Y, \{a, c\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by an identity mapping. $f^{-1}\{b\} = \{b\}$ is $(\alpha p)^*$ -closed but not closed in (X, τ) .

Theorem 4.4: Every $(\alpha p)^*$ -continuous map is gsp-continuous.

Proof: Let f: $(X, \tau) \to (Y, \sigma)$ be $(\alpha p)^*$ -continuous map. Let V be a closed set in (Y, σ) , then $f^{-1}(V)$ is $(\alpha p)^*$ -closed in (X, τ) . Since every $(\alpha p)^*$ -closed set is gsp-closed, $f^{-1}(V)$ is gsp-closed in (X, τ) . Therefore f is gsp-continuous in (X, τ) .

The following example support that the converse of the above theorem need not be true in general.

Example 4.5: Let $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{b, c\}\}$ and $\sigma = \{\phi, Y, \{b\}\}$ Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by an identity mapping. $f^{-1}\{a, c\} = \{a, c\}$ is gsp-closed but not $(\alpha p)^*$ -closed in (X, τ) .

Theorem 4.6: Every $(\alpha p)^*$ -continuous map is gs-continuous.

Proof: Let f: $(X, \tau) \to (Y, \sigma)$ be $(\alpha p)^*$ -continuous map. Let V be a closed set in (Y, σ) , then $f^{-1}(V)$ is $(\alpha p)^*$ -closed in (X,τ) . Since every $(\alpha p)^*$ -closed set is gs-closed, $f^{-1}(V)$ is gs-closed in (X,τ) . Therefore f is gs-continuous in (X,τ) .

The following example support that the converse of the above theorem need not be true in general.

Example 4.7: Let $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{b, c\}\}$ and $\sigma = \{\phi, Y, \{b\}\}$. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be defined by an identity mapping $f^{-1}\{a, c\} = \{a, c\}$ is gs-closed but not $(\alpha p)^*$ - closed in (X, τ) .

Theorem 4.8: Every $(\alpha p)^*$ -continuous map is spg-continuous.

Proof: Let f: $(X, \tau) \to (Y, \sigma)$ be $(\alpha p)^*$ -continuous map. Let V be a closed set in (Y, σ) , then $f^{-1}(V)$ is $(\alpha p)^*$ -closed in (X, τ) . Since every $(\alpha p)^*$ -closed set is spg-closed, $f^{-1}(V)$ is spg-closed in (X, τ) . Therefore f is spg-continuous in (X, τ) .

The following example support that the converse of the above theorem need not be true in general.

Example 4.9: Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b, c\}\}$ and $\sigma = \{\phi, Y, \{b\}\}$. Let $f(X, \tau) \rightarrow (Y, \sigma)$ be defined by an identity mapping $f^{-1}\{a, c\} = \{a, c\}$ is spg-closed but not $(\alpha p)^*$ -closed in (X, τ) .

Theorem 4.10: Every $(\alpha p)^*$ -continuous map is sg-continuous.

Proof: Let f: $(X, \tau) \to (Y, \sigma)$ be $(\alpha p)^*$ -continuous map. Let V be a closed set in (Y, σ) , then $f^{-1}(V)$ is $(\alpha p)^*$ -closed in (X, τ) . Since every $(\alpha p)^*$ -closed set is sg-closed, $f^{-1}(V)$ is sg-closed in (X, τ) . Therefore f is sg-continuous in (X, τ) .

The following example support that the converse of the above theorem need not be true in general.

Example 4.11: Let $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{b, c\}\}$ and $\sigma = \{\phi, Y, \{b\}\}$. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be defined by an identity mapping. $f^{-1}\{a, c\} = \{a, c\}$ is sg-closed but not $(\alpha p)^*$ -closed in (X, τ) .

Theorem 4.12: Every $(\alpha p)^*$ -continuous map is αg -continuous.

Proof: Let f: $(X, \tau) \to (Y, \sigma)$ be $(\alpha p)^*$ -continuous map. Let V be a closed set in (Y, σ) , then $f^{-1}(V)$ is $(\alpha p)^*$ -closed in (X, τ) . Since every $(\alpha p)^*$ -closed set is αg -closed, $f^{-1}(V)$ is αg -closed in (X, τ) . Therefore f is αg -continuous in (X, τ) .

The following example support that the converse of the above theorem need not be true in general.

Example 4.13: Let $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{b, c\}\}$ and $\sigma = \{\phi, Y, \{b\}\}$. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be defined by an identity mapping $f^{-1}\{a, c\} = \{a, c\}$ is αg – closed set but not a $(\alpha p)^*$ -closed in (X, τ) .

Theorem 4.14: Every $(\alpha p)^*$ -continuous map is $g\alpha$ -continuous.

Proof: Let f: $(X, \tau) \to (Y, \sigma)$ be $(\alpha p)^*$ -continuous map. Let V be a closed set in (Y, σ) , then $f^{-1}(V)$ is $(\alpha p)^*$ -closed in (X, τ) . Since every $(\alpha p)^*$ -closed set is $g\alpha$ -closed, $f^{-1}(V)$ is $g\alpha$ -closed in (X, τ) . Therefore f is $g\alpha$ -continuous in (X, τ) .

The following example support that the converse of the above theorem need not be true in general.

Example 4.15: Let $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{b, c\}\}$ and $\sigma = \{\phi, Y, \{b\}\}$. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be defined by an identity mapping. $f^{-1}\{a, c\} = \{a, c\}$ is $g\alpha$ – closed set but not $(\alpha p)^*$ - closed in (X, τ) .

Theorem 4.16: Every $(\alpha p)^*$ -continuous map is ψ -continuous.

Proof: Let f: $(X, \tau) \to (Y, \sigma)$ be $(\alpha p)^*$ -continuous map. Let V be a closed set in (Y, σ) , then $f^{-1}(V)$ is $(\alpha p)^*$ -closed in (X, τ) . Since every $(\alpha p)^*$ -closed set is ψ -closed, $f^{-1}(V)$ is ψ -closed in (X, τ) . Therefore f is ψ -continuous in (X, τ) .

The following example support that the converse of the above theorem need not be true in general.

Example 4.17: Let $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{b, c\}\}, \sigma = \{\phi, Y, \{c\}\}$. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be defined by an identity mapping $f^{-1}\{a, b\} = \{a, b\}$ is ψ – closed set but not $(\alpha p)^*$ -closed in (X, τ) .

Theorem 4.18: Every $(\alpha p)^*$ -continuous map is αgs -continuous.

Proof: Let f: $(X, \tau) \to (Y, \sigma)$ be $(\alpha p)^*$ -continuous map. Let V be a closed set in (Y, σ) , then $f^{-1}(V)$ is $(\alpha p)^*$ -closed in (X, τ) . Since every $(\alpha p)^*$ -closed set is αgs -closed, $f^{-1}(V)$ is αgs -closed in (X, τ) . Therefore f is αgs -continuous in (X, τ) .

The following example support that the converse of the above theorem need not be true in general

Example 4.19: Let $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{b, c\}\}$ and $\sigma = \{\phi, Y, \{b\}\}$. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be defined by an identity $f^{-1}\{a, c\} = \{a, c\}$ is $\alpha gs - closed$ set but not $(\alpha p)^*$ - closed in (X, τ) .

Theorem 4.20: Every $(sp)^*$ -continuous map is $(\alpha p)^*$ -continuous.

Proof: Let f: $(X, \tau) \to (Y, \sigma)$ be $(sp)^*$ -continuous map. Let V be a closed set in (Y, σ) , then $f^{-1}(V)$ is $(sp)^*$ -closed in (X, τ) . Since every $(sp)^*$ -closed set is $(\alpha p)^*$ -closed, $f^{-1}(V)$ is $(\alpha p)^*$ -closed in (X, τ) . Therefore f is $(\alpha p)^*$ -continuous in (X, τ) .

Example 4.21: Let $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{a, b\}\}$ and $\sigma = \{\phi, Y, \{a, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by an identity mapping $f^{-1}\{b\} = \{b\}$ is $(\alpha p)^*$ - closed set but not $(sp)^*$ - closed in (X, τ) .

Theorem 4.22: Every $(gsp)^*$ -continuous is $(\alpha p)^*$ -continuous.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be $(gsp)^*$ -continuous map. Let V be a closed set in (Y, σ) , then $f^{-1}(V)$ is $(gsp)^*$ -closed in (X, τ) . Since every $(gsp)^*$ -closed set is (αp) -closed, $f^{-1}(V)$ is (αp) -closed in (X, τ) . Therefore f is (αp) -continuous in (X, τ) .

Example 4.23: Let $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a\}\}$ and $\sigma = \{\phi, Y, \{a, c\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by an identity mapping $f^{-1}\{b\} = \{b\}$ is $(\alpha p)^*$ – closed set but not $(gsp)^*$ -closed in (X, τ) .

Definition 4.24: A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is called $(\alpha p)^*$ -irresolute if $f^{-1}(V)$ is a $(\alpha p)^*$ -closed set of (X, τ) for every $(\alpha p)^*$ -closed set V of (Y, σ)

Theorem 4.25: Every $(\alpha p)^*$ -irresolute is $(\alpha p)^*$ -continuous

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an $(\alpha p)^*$ - irresolute. Let V be a closed set in (Y, σ) Every closed set is $(\alpha p)^*$ -closed. Therefore V is $(\alpha p)^*$ -closed. Then $f^{-1}(V)$ is $(\alpha p)^*$ -closed since f is $(\alpha p)^*$ -irresolute and hence f is $(\alpha p)^*$ - continuous. The converse of the above theorem is not true in general as it can be seen from the following example

Example 4.26: Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b, c\}\}$ and $\sigma = \{\phi, Y, \{c\}\}$ Define f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = c, f(b) = a and f(c) = b. $f^{-1}\{a, b\} = \{b, c\}$ is $(\alpha p)^*$ closed set in (X, τ) . Therefore f is $(\alpha p)^*$ -continuous. $\{b, c\}$ is $(\alpha p)^*$ closed set in (Y, σ) . $f^{-1}\{a\} = \{b\}$, $f^{-1}\{b\} = \{c\}$ is not $(\alpha p)^*$ closed set in (X, τ) . Therefore f is not $(\alpha p)^*$ -irresolute.

Theorem 4.27: Let f: $(X, \tau) \to (Y, \sigma)$ and g: $(Y, \sigma) \to (Z, \eta)$ be any two functions then, (i).g \circ f: $(X, \tau) \to (Z, \eta)$ is $(\alpha p)^*$ -continuous if f is $(\alpha p)^*$ -irresolute and g is $(\alpha p)^*$ -continuous. (ii).g \circ f: $(X, \tau) \to (Z, \eta)$ is $(\alpha p)^*$ - irresolute if f and g are $(\alpha p)^*$ - irresolute. (iii).g \circ f: $(X, \tau) \to (Z, \eta)$ is $(\alpha p)^*$ -continuous if f is $(\alpha p)^*$ -continuous and g is continuous.

Proof: (i).Let F be closed set in (Z, η) .Then $g^{-1}(F)$ is $(\alpha p)^*$ -closed in (Y, σ) implies $f^1(g^{-1}(F))$ is $(\alpha p)^*$ -closed in (X, τ) . Therefore $(f \circ g)^{-1}(F)$ is $(\alpha p)^*$ -closed in (X, τ) . Hence $(f \circ g)$ is $(\alpha p)^*$ -continuous. (ii).Let F be $(\alpha p)^*$ -closed in (Z, η) . Then $g^{-1}(F)$ is $(\alpha p)^*$ -closed in (Y, σ) .Therefore $f^1(g^{-1}(F))$ is $(\alpha p)^*$ -closed in (X, τ) . Therefore $(f \circ g)^{-1}(F)$ is $(\alpha p)^*$ -closed in (X, τ) . Therefore $(f \circ g)^{-1}(F)$ is $(\alpha p)^*$ -closed in (X, τ) . Therefore $(f \circ g)^{-1}(F)$ is $(\alpha p)^*$ -closed in (X, τ) . Therefore $f^1(g^{-1}(F))$ is $(\alpha p)^*$ -closed in (X, τ) . Therefore $(f \circ g)^{-1}(F)$ is $(\alpha p)^*$ -closed in (X, τ) . Therefore $(f \circ g)^*$ -closed in (X, τ) . Therefore $(f \circ g)^{-1}(F)$ is $(\alpha p)^*$ -closed in (X, τ) . Hence $(f \circ g)$ is $(\alpha p)^*$ -closed in (X, τ) . Therefore $(f \circ g)^{-1}(F)$ is $(\alpha p)^*$ -closed in (X, τ) . Hence $(f \circ g)^*$ -closed in (X, τ) . Therefore $(f \circ g)^{-1}(F)$ is $(\alpha p)^*$ -closed in (X, τ) . Hence $(f \circ g)^*$ -closed in (X, τ) . Therefore $(f \circ g)^{-1}(F)$ is $(\alpha p)^*$ -closed in (X, τ) . Hence $(f \circ g)$ is $(\alpha p)^*$ -closed in (X, τ) . Hence $(f \circ g)$ is $(\alpha p)^*$ -closed in (X, τ) . Hence $(f \circ g)^*$ is $(\alpha p)^*$ -closed in (X, τ) . Hence $(f \circ g)$ is $(\alpha p)^*$ -closed in (X, τ) .

Thus we have the following diagram.



Where $A \rightarrow B$ represents A implies B and $A \rightarrow B$ represents A does not imply B

5. APPLICATIONS OF $(\alpha p)^*$ -CLOSED SETS

As applications of $(\alpha p)^*$ - closed set, a new space $T_{(\alpha p)^*}$ is introduced.

Definition 5.1: A space (X,τ) is called a $T_{(\alpha p)^*}$ space if every $(\alpha p)^*$ - closed set is closed.

Theorem 5.2: Every T_b space is a $T_{(\alpha p)^*}$ space.

Proof follows from the definition of T_b space and $T_{(\alpha p)^*}$ space.

The reverse implication does not hold as it can be seen from the following example.

Example 5.3: Let $X = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, then (X, τ) is a $T_{(\alpha p)^*}$ space. A={a} is gs-closed set but not a closed set. Therefore (X, τ) is a T_b space.

Theorem 5.4: Every $_{\alpha}T_b$ space is a $T_{(\alpha p)^*}$ space.

Proof follows from the definition of $_{\alpha}T_b$ space and $T_{(\alpha p)^*}$ space.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 5.5: Let $X = \{a, b, c\}, \tau = \{\phi, X, \{b, c\}\}$. $(\alpha p)^*$ -closed sets are $\phi, X, \{a\}$ and αg -closed sets are $\phi, X, \{a\}, \{a, b\}, \{a, c\}$. Since every $(\alpha p)^*$ -closed set is αg -closed, the space (X, τ) is a $T_{(\alpha p)^*}$ space. A = $\{a, b\}$ is αg -closed but not closed. Therefore the space is not a $_{\alpha}T_b$ space.

Theorem 5.6: Every $T_{(\alpha p)^*}$ space is a $T_{(gsp)^*}$ space.

Proof follows from the definition of $T_{(\alpha p)^*}$ space and $T_{(gsp)^*}$ space.

Example 5.7: Let $X = \{a, b, c\}, \tau = \{\phi, X, \{a\}\}. (gsp)^*$ -closed sets are ϕ , $X, \{b, c\}$ and $(\alpha p)^*$ -closed sets are ϕ , $X, \{b\}, \{c\}, \{b, c\}$. Since every $(gsp)^*$ -closed set is $(\alpha p)^*$ -closed, the space (X, τ) is a $T_{(gsp)^*}$ space. A={b} is $(\alpha p)^*$ -closed but not closed. Therefore the space is not a $T_{(\alpha p)^*}$ space.

Theorem 5.8: A space (X, τ) which is both $_{\alpha}T_b$ and $T_{g\alpha}$ is a $T_{(\alpha p)^*}$ space.

Theorem 5.9: A space (X, τ) which is both T_b and T_{gsp} is a $T_{(\alpha p)^*}$ space.

Theorem 5.10: A space (X, τ) which is both $T_{1/2}$ and $_{\alpha}T_d$ is a $T_{(\alpha p)^*}$ space.

Theorem 5.11: A space (X, τ) which is both $T_{1/2}$ and T_d is a $T_{(\alpha p)^*}$ space.

Theorem 5.12: Let (X, τ) be a $T_{(\alpha p)^*}$ space and $f: (X, \tau) \to (Y, \sigma)$ be $an(\alpha p)^*$ - irresolute.then f is continuous.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an $(\alpha p)^*$ - irresolute. Let V be a closed set of (Y, σ) . Every closed set is $(\alpha p)^*$ -closed set. Then $f^{-1}(V)$ is an $(\alpha p)^*$ -closed since f is $(\alpha p)^*$ -irresolute. Every $(\alpha p)^*$ -closed set is closed in X. Since (X, τ) is a $T_{(\alpha p)^*}$ space. Therefore $f^{-1}(V)$ is closed and hence f is continuous.

Theorem 5.13: Let (X, τ) be a $T_{(\alpha p)^*}$ space and f: $(X, \tau) \rightarrow (Y, \sigma)$ be continuous. Then f is $(\alpha p)^*$ - irresolute.

Proof: Let A be an $(\alpha p)^*$ - closed set in Y. Then A is closed, Since (Y, σ) is a $T_{(\alpha p)^*}$ space. Then $f^{-1}(A)$ is closed, since f is continuous. Every closed set is $(\alpha p)^*$ - closed set. Therefore f is an $(\alpha p)^*$ - irresolute.

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