

CONTRA $sg\alpha$ - CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

A. DEVIKA*

Associate Professor P.S.G. College of Arts and Science, Coimbatore, India.

S. SATHYAPRIYA

M.Phil scholar, P.S.G. College of Arts and Science, Coimbatore, India.

(Received On: 23-02-17; Revised & Accepted On: 18-03-17)

ABSTRACT

In this paper, we introduce and investigate the notion of Contra $sg\alpha$ - Continuous Functions. We obtain fundamental properties and characterization of contra $sg\alpha$ -continuous functions and discuss the relation- ships between contra- $sg\alpha$ -continuity and other related functions.

Subject Classification: 54C05, 54C08, 54C10.

Keywords: $sg\alpha$ -closed sets, $sg\alpha$ -continuous functions, contra $sg\alpha$ -continuous functions.

1. INTRODUCTION

N. Levine [15] introduced generalized closed sets (briefly g-closed set) in 1970. N. Levine [14] introduced the concepts of semi-open sets in 1963. Bhattacharya and Lahiri [6] introduced and investigated semi-generalized closed (briefly sg- closed) sets in 1987. Arya and Nour [3] defined generalized semi-closed (briefly gs-closed) sets for obtaining some characterization of s-normal spaces in 1990. O.Njastad in 1965 defined α -open sets [22].

In 1996, Dontchev [10] introduced a new class of functions called contra- continuous functions. A new weaker form of this class of functions called contra semi-continuous function is introduced and investigated by Dontchev and Noiri [11].

In this paper, the notion of $sg\alpha$ -closed sets [8] in topological spaces is applied to introduce and study a new class of functions called contra $sg\alpha$ - continuous functions, as a new generalization of contra continuity, and to obtain some of their characterizations and properties. Also the relationships with some other functions are discussed.

2. PRELIMINARIES

Through this paper (X, τ) , (Y, σ) and (Z, η) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X . The closure of A and the interior of A are denoted by $cl(A)$ and $int(A)$ respectively. (X, τ) will be replaced by X if there is no chance of confusion. Let us recall the following definitions as pre requests.

A subset A of a topological space X is said to be open if $A \in \tau$. A subset.

A of a topological space X is said to be closed if the set $X-A$ is open.

The interior of a subset A of a topological space X is defined as the union of all open sets contained in A . It is denoted by $int(A)$. The closure of a subset A of a topological space X is defined as the intersection of all closed sets containing A . It is denoted by $cl(A)$.

Corresponding Author: A. Devika*

Associate Professor P.S.G. College of Arts and Science, Coimbatore, India.

Definitions 2.1: A subset A of a space (X, τ) is said to be

1. semi open [14] if $A \subseteq \text{cl}(\text{int}(A))$ and semi closed if $\text{int}(\text{cl}(A)) \subseteq A$.
2. α -open [22] if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ and α -closed if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.
3. β -open or semi pre-open [1] if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ and β -closed or semi pre-closed if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$. pre-open [20] if $A \subseteq \text{int}(\text{cl}(A))$ and pre-closed if $\text{cl}(\text{int}(A)) \subseteq A$.

The complement of a semi-open (resp. pre-open, α -open, β -open) set is called semi-closed (resp. pre-closed, α -closed, β -closed). The intersection of all semi-closed (resp. pre-closed, α -closed, β -closed) sets containing A is called the semi-closure (resp. pre-closure, α -closure, β -closure) of A and is denoted by $\text{scl}(A)$ (resp. $\text{pcl}(A)$, $\alpha\text{-cl}(A)$, $\beta\text{-cl}(A)$). The union of all semi-open (resp. pre-open, α -open, β -open) sets contained in A is called the semi-interior (resp. pre-interior, α -interior, β -interior) of A and is denoted by $\text{sint}(A)$ (resp. $\text{pint}(A)$, $\alpha\text{-int}(A)$, $\beta\text{-int}(A)$). The family of all semi-open (resp. pre-open, α -open, β -open) sets is denoted by $\text{SO}(X)$ (resp. $\text{PO}(X)$, $\alpha\text{-O}(X)$, $\beta\text{-O}(X)$). The family of all semi-closed (resp. pre-closed, α -closed, β -closed) sets is denoted by $\text{SCl}(X)$ (resp. $\text{PCl}(X)$, $\alpha\text{-Cl}(X)$, $\beta\text{-Cl}(X)$).

Definitions 2.2: A subset A of a space (X, τ) is called

1. g -closed [15] if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ) . The complement of a g -closed set is called g -open set.
2. gs -closed set [7] if $\text{scl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ) .
3. sg -closed set [6] if $\text{scl}(A) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in (X, τ) .
4. αg -closed [16] if $\alpha(\text{cl}(A)) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ) .
5. $g\alpha$ -closed [17] if $\alpha(\text{cl}(A)) \subseteq U$, whenever $A \subseteq U$ and U is α -open in (X, τ) .
6. gp -closed [18] if $\text{pcl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ) .

Definition 2.3: Let X and Y be topological spaces. A function $f: X \rightarrow Y$ is said to be

1. continuous [13] if for each open set V of Y the set $f^{-1}(V)$ is an open subset of X .
2. α -continuous [22] if $f^{-1}(V)$ is a α -closed set of (X, τ) for every closed set V of (Y, σ) .
3. β -continuous [1] if $f^{-1}(V)$ is a β -closed set of (X, τ) for every closed set V of (Y, σ) .
4. pre-continuous [20] if $f^{-1}(V)$ is a pre-closed set of (X, τ) for every closed set V of (Y, σ) .
5. semi-continuous [14] if $f^{-1}(V)$ is a semi-closed set of (X, τ) for every closed set V of (Y, σ) .

Definition 2.4: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be

1. g -continuous [15] if $f^{-1}(V)$ is a g -closed set of (X, τ) for every closed set V of (Y, σ) .
2. gs -continuous [7] if $f^{-1}(V)$ is a gs -closed set of (X, τ) for every closed set V of (Y, σ) .
3. sg -continuous [6] if $f^{-1}(V)$ is a sg -closed set of (X, τ) for every closed set V of (Y, σ) .
4. αg -continuous [16] if $f^{-1}(V)$ is a αg -closed set of (X, τ) for every closed set V of (Y, σ) .
5. $g\alpha$ -continuous [17] if $f^{-1}(V)$ is a $g\alpha$ -closed set of (X, τ) for every closed set V of (Y, σ) .
6. gp -continuous [18] if $f^{-1}(V)$ is a gp -closed set of (X, τ) for every closed set V of (Y, σ) .

Definitions 2.5 [21]: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be almost continuous if for every open set V of Y , $f^{-1}(V)$ is regular open in X .

Definitions 2.6 [8]: A subset A of space (X, τ) is called $sg\alpha$ -closed if $\text{scl}(A) \subseteq U$, whenever $A \subseteq U$ and U is α -open in X .

The family of all $sg\alpha$ -closed subsets of the space X is denoted by $\text{SG}\alpha\text{C}(X)$.

Definitions 2.7 [8]: The intersection of all $sg\alpha$ -closed sets containing a set A is called $sg\alpha$ -closure of A and is denoted by $\text{sg}\alpha\text{-cl}(A)$.

A set A is $sg\alpha$ -closed set if and only if $\text{sg}\alpha\text{Cl}(A) = A$.

Definitions 2.8 [8]: A subset A in X is called $sg\alpha$ -open in X if A^c is $sg\alpha$ -closed in X .

The family of a $sg\alpha$ -open sets is denoted by $SG\alpha O(X)$

Definitions 2.9 [8]: The union of all $sg\alpha$ -open sets containing a set A is called $sg\alpha$ -interior of A and is denoted by $sg\alpha\text{-Int}(A)$.

A set A is $sg\alpha$ -open set if and only if $sg\alpha\text{-Int}(A) = A$.

Lemma 2.10 [12]: The following properties hold for subsets A and B of a space X .

1. $x \in \ker(A)$ if and only if $A \cap F = \emptyset$ for any closed set F of X containing x .
2. $A \subset \ker(A)$ and $A = \ker(A)$ if A is open in X .
3. if $A \subset B$, then $\ker(A) \subset \ker(B)$

3. CONTRA – $sg\alpha$ -CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

In this section, the notion of a new class of functions called contra $sg\alpha$ -continuous functions is introduced and we obtain some of their characterizations and properties. Also, the relationships with some other related functions are discussed.

Definition 3.1: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called $sg\alpha$ -continuous if $f^{-1}(V)$ is $sg\alpha$ -closed in (X, τ) for every closed set V of (Y, σ) .

Definition 3.2: A function $f: X \rightarrow Y$ is said to be Contra $sg\alpha$ -Continuous if $f^{-1}(V)$ is $sg\alpha$ -closed in X for each open set V of Y .

Remark 3.3: From the following examples, it is clear that both contra $sg\alpha$ -continuous and $sg\alpha$ -continuous are independent notions of each other.

Example 3.4: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a, b\}\}$ be topologies on X and Y respectively. Define a function $f: X \rightarrow Y$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then f is $sg\alpha$ -continuous function but not contra $sg\alpha$ -continuous, because for the open set $\{a, b\}$ in Y , $f^{-1}(\{a, b\}) = \{a, b\}$ is not $sg\alpha$ -closed in X .

Example 3.5: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a, c\}\}$ be topologies on X and Y respectively. Define a function $f: X \rightarrow Y$ by $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is contra $sg\alpha$ -continuous function but not $sg\alpha$ -continuous, because for the open set $\{a\}$ in Y , $f^{-1}(\{a\}) = \{c\}$ is not $sg\alpha$ -open in X .

Theorem 3.6: If $f: X \rightarrow Y$ is contra continuous, then it is contra $sg\alpha$ -continuous.

Proof: Let V be an open set in Y . Since f is contra continuous, $f^{-1}(V)$ is closed in X . Since every closed set is $sg\alpha$ -closed, $f^{-1}(V)$ is $sg\alpha$ -closed in X . Therefore f is contra $sg\alpha$ -continuous.

Remark 3.7: Converse of the above theorem need be true in general as seen from the following examples.

Example 3.8: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a, c\}\}$ be topologies on X and Y respectively. Define a function $f: X \rightarrow Y$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then f is contra $sg\alpha$ -continuous function but not contra-continuous, because for the open set $\{a\}$ in Y , and $f^{-1}(\{a\}) = \{b\}$ is not closed in X .

Theorem 3.9: If $f: X \rightarrow Y$ is contra semi-continuous, then it is contra $sg\alpha$ -continuous.

Proof: Let V be an open set in Y . Since f is contra semi-continuous, $f^{-1}(V)$ is semi-closed in X . Since every semi-closed set is $sg\alpha$ -closed, $f^{-1}(V)$ is $sg\alpha$ -closed in X . Therefore f is contra $sg\alpha$ -continuous.

Remark 3.10: Converse of the above theorem need be true in general as seen from the following examples.

Example 3.11: Let τ and $\sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ be topologies on X and Y respectively. Define a function $f: X \rightarrow Y$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then f is contra $sg\alpha$ -continuous function but not contra semi-continuous, because for the open set $\{b\}$ in Y and $f^{-1}(\{b\}) = \{a\}$ is not semi-closed in X .

Theorem 3.12: The following are equivalent for a function $f: X \rightarrow Y$

1. f is contra $sg\alpha$ -continuous.
2. for every closed set F of Y , $f^{-1}(F)$ is $sg\alpha$ -open set of X .
3. for each $x \in X$ and each closed set F of Y containing $f(x)$, there exist $sg\alpha$ -open set U containing x such that $f(U) \subset F$.
4. for each $x \in X$ and each other open set F of Y containing $f(x)$, there exists $sg\alpha$ -closed set K not containing x such that $f^{-1}(V) \subset K$.
5. $f(sg\alpha - Cl(A)) \subset \ker(f(A))$ for every subset A of X .
6. $sg\alpha - Cl(f^{-1}(B)) \subset f^{-1}(\ker(B))$ for every subset B of Y .

Proof:

(1) \Rightarrow (2): Let F be a closed set in Y . Then $Y - F$ is an open set in Y . By (1), $f^{-1}(Y - F) = X - f^{-1}(F)$ is $sg\alpha$ -closed set in X . This implies $f^{-1}(F)$ is $sg\alpha$ -open set in X . Therefore (2) holds.

(2) \Rightarrow (1): Let G be an open set of Y . Then $Y - G$ is a closed set in Y . By (2), $f^{-1}(Y - G) = X - f^{-1}(G)$ is $sg\alpha$ -open set in X , which implies $f^{-1}(G)$ is $sg\alpha$ -closed set in X . Therefore (1) holds.

(2) \Rightarrow (3): Let F be a closed set in Y containing $f(x)$. Then $x \in f^{-1}(F)$. By (2), $f^{-1}(F)$ is $sg\alpha$ -open set in X containing x . Let $U = f^{-1}(F)$. Then $f(U) = f(f^{-1}(F)) \subset F$. Therefore (3) holds.

(3) \Rightarrow (2): Let F be a closed set in Y containing $f(x)$. Then $x \in f^{-1}(F)$. From (3), there exists $sg\alpha$ -open set U_x in X containing x such that $f(U_x) \subset F$. That is $U_x \subset f^{-1}(F)$. Thus $f^{-1}(F) = \bigcup \{U_x : x \in f^{-1}(F)\}$, which is union of $sg\alpha$ -open sets. Since union of $sg\alpha$ -open sets is a $sg\alpha$ -open sets, $f^{-1}(F)$ is $sg\alpha$ -open set of X .

(3) \Rightarrow (4): Let V be an open set in Y not containing $f(x)$. Then $Y - V$ is closed set in Y containing $f(x)$. From (3), there exists a $sg\alpha$ -open set U in X containing x such that $f(U) \subset Y - V$. This implies $U \subset f^{-1}(Y - V) = X - f^{-1}(V)$. Hence, $f^{-1}(V) \subset X - U$. Set $K = X - U$, then K is $sg\alpha$ -closed set not containing x in X such that $f^{-1}(V) \subset K$.

(4) \Rightarrow (3): Let F be a closed set in Y containing $f(x)$. Then $Y - F$ is an open set in Y not containing $f(x)$. From (4), there exists $sg\alpha$ -closed set K in X not containing x such that $f^{-1}(Y - F) \subset K$. This implies $X - f^{-1}(F) \subset K$. Hence, $X - K \subset f^{-1}(F)$, that is $f(X - K) \subset F$. Set $U = X - K$, then U is $sg\alpha$ -open set containing x in X such that $f(U) \subset F$.

(2) \Rightarrow (5): Let A be any subset of X . Suppose $y \notin \ker(f(A))$. Then by lemma 4.1, there exists a closed set F in Y containing y such that $f(A) \cap F = \emptyset$. Thus, $A \subset f^{-1}(F) = \emptyset$. Therefore $A \subset X - f^{-1}(F)$. By (2), $f^{-1}(F)$ is $sg\alpha$ -open set in X and hence $X - f^{-1}(F)$ is $sg\alpha$ -closed set in X . Therefore, $sg\alpha - Cl(X - f^{-1}(F)) = X - f^{-1}(F)$. Now $A \subset X - f^{-1}(F)$, which implies $sg\alpha - Cl(A) \subset sg\alpha - Cl(X - f^{-1}(F)) = X - f^{-1}(F)$. Therefore $sg\alpha - Cl(A) \cap f^{-1}(F) = \emptyset$, which implies $f(sg\alpha - Cl(A)) \cap F = \emptyset$ and hence $y \notin sg\alpha - Cl(A)$. Therefore $f(sg\alpha - Cl(A)) \subset \ker(f(A))$ for every subset A of X .

(5) \Rightarrow (6): Let $B \subset Y$. Then $f^{-1}(B) \subset X$. By (4) and lemma 2.10, $f(sg\alpha - Cl(f^{-1}(B))) \subset \ker(f(f^{-1}(B))) \subset \ker(B)$. Thus $sg\alpha - cl(f^{-1}(B)) \subset f^{-1}(\ker(B))$ for every subset B of Y .

(6) \Rightarrow (1): Let V be any open subset of Y . Then by (6) and lemma 2.10, $sg\alpha-C1(f^{-1}(V)) \subset f^{-1}(\ker(V)) = f^{-1}(V)$ and $sg\alpha-C1(f^{-1}(V)) = f^{-1}(V)$. Therefore $f^{-1}(V)$ is $sg\alpha$ -closed set in X . This shows that f is contra $sg\alpha$ -continuous.

Theorem 3.13: If a function $f: X \rightarrow Y$ is contra $sg\alpha$ -continuous and Y is regular, then f is $sg\alpha$ -continuous.

Proof: Let $x \in X$ and V be an open set in Y containing $f(x)$. Since Y is regular, there exists an open set W in Y containing $f(x)$ such that $C1(W) \subset V$. Since f is contra $sg\alpha$ -continuous, by theorem 3.12 (3), there exists $sg\alpha$ -open set U in X containing x such that $f(U) \subset C1(W)$. Then $f(U) \subset C1(W) \subset V$. Therefore f is $sg\alpha$ -continuous.

Theorem 3.14: If a function $f: X \rightarrow Y$ is contra $sg\alpha$ -continuous and X is $Tsg\alpha$ -space, then f is contra continuous.

Proof: Let U be an open set in Y . Since f is contra $sg\alpha$ -continuous, $f^{-1}(U)$ is $sg\alpha$ -closed in X . Since X is $Tsg\alpha$ -space $f^{-1}(U)$ is a closed set in X . Therefore f is contra continuous.

Theorem 3.15: If a function $f: X \rightarrow Y$ is contra $sg\alpha$ -continuous and X is $sg\alpha$ $T1/2$ -space, then f is contra semi continuous.

Proof: Let U be an open set in Y . Since f is contra $sg\alpha$ -continuous, $f^{-1}(U)$ is $sg\alpha$ -closed in X . Since X is $sg\alpha$ $T1/2$ -space, $f^{-1}(U)$ is a semi closed set in X . Therefore f is contra semi continuous.

Definition 3.16: A space X is called locally $sg\alpha$ -indiscrete if every $sg\alpha$ -open set is closed in X .

Theorem 3.17: If a function $f: X \rightarrow Y$ is contra $sg\alpha$ -continuous and X is locally $sg\alpha$ -indiscrete space, then f is continuous.

Proof: Let U be an open set in Y . Since f is contra $sg\alpha$ -continuous and X is locally $sg\alpha$ -indiscrete space, $f^{-1}(U)$ is an open set in X . Therefore f is continuous.

Definition 3.18: If a function $f: X \rightarrow Y$ is called almost $sg\alpha$ -continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in SG\alpha O(X, x)$ such that $f(U) \subset \text{Int}(cl(V))$.

Definition 3.19: If a function $f: X \rightarrow Y$ is called quasi $sg\alpha$ -open if image of every $sg\alpha$ -open set of X is open set in Y .

Theorem 3.20: If a function $f: X \rightarrow Y$ is contra $sg\alpha$ -continuous, quasi $sg\alpha$ -open, then f is almost $sg\alpha$ -continuous function.

Proof: Let x be any arbitrary point of X and V be an open set in Y containing $f(x)$. Then $C1(V)$ is a closed set in Y containing $f(x)$. Since f is contra $sg\alpha$ -continuous, then by theorem 3.12 (3), there exists $U \in SG\alpha O(X, x)$ such that $f(U) \subset cl(V)$. Since f is quasi $sg\alpha$ -open, $f(U)$ is open in Y . Therefore $f(U) = \text{Int}(C1(U))$. Thus, $f(U) \subset \text{Int}(f(V))$. This shows that f is almost $sg\alpha$ -continuous function.

Definition 3.21: If a function $f: X \rightarrow Y$ is called weakly $sg\alpha$ -continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in SG\alpha O(X, x)$ such that $f(U) \subset scl(V)$.

Theorem 3.22: If a function $f: X \rightarrow Y$ is contra $sg\alpha$ -continuous, then f is weakly $sg\alpha$ -continuous function.

Proof: Let V be an open set in Y . Since $C1(V)$ is closed in Y , by theorem 3.12 (2), $f^{-1}(C1(V))$ is $sg\alpha$ -open set in X . Set $U = f^{-1}(C1(V))$ then $f(U) \subset f(f^{-1}(C1(V))) \subset C1(V)$. This shows that f is almost weakly $sg\alpha$ -continuous function.

Definition 3.23: Let A be a subset of X . Then $sg\alpha-C1(A)$ - $sg\alpha$ - $\text{Int}(A)$ is called $sg\alpha$ -frontier of A and is denoted by $sg\alpha-Fr(A)$

Theorem 3.24: The set of all points of x of X at which $f: X \rightarrow Y$ is not contra $sg\alpha$ -continuous is identical with the union of $sg\alpha$ -frontier of the inverse images of closed sets of Y containing $f(x)$.

Proof: Assume that f is not contra $sg\alpha$ -continuous at $x \in X$. Then by theorem 3.12(3), there exists $F \in C(Y, f(x))$ such that $f(U) \cap (Y - F) = \emptyset$. For every $U \in SG\alpha O(X, x)$. This implies $U \cap f^{-1}(Y - F) = \emptyset$, for every $U \in SG\alpha O(X, x)$. Therefore, $x \in sg\alpha - Cl(f^{-1}(Y - F)) = sg\alpha - Cl(X - f^{-1}(F))$. Also $x \in f^{-1}(F) \subset sg\alpha - Cl(f^{-1}(F))$. Thus, $x \in sg\alpha - Cl(f^{-1}(F)) \cap sg\alpha - Cl(X - f^{-1}(F))$. This implies $x \in sg\alpha - Cl(f^{-1}(F)) = sg\alpha - Int(f^{-1}(F))$. Therefore, $x \in sg\alpha - Fr(f^{-1}(F))$.

Conversely, Suppose $x \in sg\alpha - Fr(f^{-1}(F))$ for some $F \in C(Y, f(x))$ and f is contra $sg\alpha$ -continuous at $x \in X$, then there exists $U \in SG\alpha O(X, x)$ such that $f(U) \subset F$. Therefore, $x \in U \subset f^{-1}(F)$ and hence $x \in sg\alpha - Int(f^{-1}(F)) \subset X - sg\alpha - Fr(f^{-1}(F))$. This contradicts the fact that $x \in sg\alpha - Fr(f^{-1}(F))$. Therefore f is not contra $sg\alpha$ -continuous.

REFERENCES

1. ME.Abd. Monsef, S.N.El. Deeb, R.A. Mahmoud, β -open sets and β -continuous mappings, Bull. Fac. Assint. Univ 12(1983), 77-90.
2. D. Andrijevic, Semi-pre-open sets, Mat.Vesnik, 38(1986), 24-32.
3. S.P.Arya and T.M. Tour, Characterization of s -normal spaces, Indian J.Pure-Appl.Math, 21(1990),717-719.
4. K. Balachandran, P. Sundaram and H. Maki, On generalized continuous maps in topological sets, Mem.Fac.Sci.Kochi.Univ.ser.A.Math,12(1991), 5-13.
5. S.S.Benchalli, Umadevi.I.Nelli, Gurupad P. Siddapur, Contra $g\delta s$ continuous functions in topological spaces, International Journal of Applied Mathematics vol (25) no.4, (2012), 452-471.
6. P.Bhattachayya and B.K.Lahari, Semi-generalized closed sets in topology, Indian.J.Math.,29(3) (1987), 375-382.
7. P. Devi and K. Balachandran and H. Maki, Semi-generalized homeomorphism and generalized semi-homeomorphism in topological spaces, Indian. J. pure.Appl.Math.26(3)(1995), 271-284.
8. A. Devika and S. Sathyapriya, $sg\alpha$ -closed sets in Topological Spaces, Communicated.
9. J. Dontchev, On generalizing semi-pre open sets, Mem.Fac.Sci.Kochi.Univ.ser.A.Math,16(1995), 35-48.
10. J.Dontchev, Contra continuous functions and strongly S -closed mappings, Int.J.Math.Sci.19 (1996), 303-310.
11. J.Dontchev, T.Noiri, Contra semi-continuous functions, Mathematica pannonica,10 no(2)(1999), 159-168.
12. S.Jafari, T.Noiri, On contra pre-continuous functions, Bull of. Malaysian Mathematical sci.soc, (25) (2002), 115-128.
13. R.James Munkres, Topology, Pearson Education Inc., Prentice Hall (2013).
14. N.Levine, Semi-open sets and Semi-continuity in topological spaces, American Mathematical Monthly70 (1963), 36-41.
15. N. Levine, Generalized closed sets in topological spaces, Rend.Circ.Mat.Palermo, vol 19(2), (1970), 89-96.
16. H.Maki, R. Devian and K. Balachandran, Generalized α -closed sets in topology, Bull.Fukuoka Univ, Ed. partIII., 42(1993), 13-21.
17. H.Maki, R.Devi and K.Balachandran, Associated topologies of generalized α -closed sets and α -generalized closed sets, Mem.Sci.Kochi Univ.sev.A.Math.,15(1994), 51-63.
18. H.Maki, J.Umehara and T.Noiri, Every Topological space is pre- $T_{1/2}$, Mem. Fac. Sci. Kochi. Univ. ser. A. Math, 17(1996), 33-42.
19. A.S. Mashlour, I.A.Hasanein and S.N.El.Deeb, α -continuous and α -open mappings, Acth. Math, Hung, 41, No-s 3-4, (1983), 213-218.
20. A.S.Mashlour, M.E.Abd.El-Monsef, S.N.El.Deeb, On pre-continuous and weak pre-continuous mappings, Proc. Math and phys.soc. Egypt, 53, (1982), 47-53.
21. T.Noiri, On almost continuous functions, TInd.J.of pure and Appl.Maths., (20)(1989),571-576.
22. O.Njastad, On Some classes of nearly open sets, Pacific.J.Math., 15(1965), 961-970.
23. M.H. Stone, Application of the theory of Boolean rings to general topology, Trans.Amer.Math.Soc., 41 (1937), 375-381.

Source of support: Nil, Conflict of interest: None Declared.

[Copy right © 2017. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]