**pg***- Connected space

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**ABSTRACT**

In this paper we introduce **pg***- connected space, **pg***-component, **pg***- connected modulo I space and establish results about the relation between them.

**Key words:** **pg***- connected space, **pg***-component, **pg***- connected modulo I space.

1. **INTRODUCTION**


2. **PRELIMINARIES**

**Definition 2.1:** A subset A of a topological space \((X, \tau)\) is called a pre-open set [4] if \(A \subseteq int(cl(A))\) and a pre-closed set if \(cl(int(A)) \subseteq A\).

**Definition 2.2:** A subset A of topological space \((X, \tau)\) is called

1. a generalized closed set (g-closed) [3] if \(cl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).
2. a g*-closed set [7] if \(cl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is g-open in \((X, \tau)\).
3. a g***-closed set [5] if \(cl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is g*-open in \((X, \tau)\).
4. a **pg***- closed set [6] if \(pcl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is g*-open in \((X, \tau)\).

**Definition 2.3:** A function \(f: (X, \tau) \rightarrow (Y, \sigma)\) is called

1. **pg***-irresolute[6] if \(f^{-1}(V)\) is a **pg***-closed set of \((X, \tau)\) for every **pg***-closed set \(V\) of \((Y, \sigma)\).
2. **pg***-continuous[6] if \(f^{-1}(V)\) is a **pg***-closed set of \((X, \tau)\) for every closed set \(V\) of \((Y, \sigma)\).
3. **pg***-resolute[6] if \(f(U)\) is **pg***-open in \(Y\) whenever \(U\) is **pg***-open in \(X\).

**Definition 2.4:** An ideal [2] \(I\) on a nonempty set \(X\) is a collection of subsets of \(X\) which satisfies the following properties.

1. \(A 
\in I, B 
\in I \Rightarrow A \cup B 
\in I\)
2. \(A 
\in I, B 
\subseteq A \Rightarrow B 
\in I\). A topological space \((X, \tau)\) with an ideal \(I\) on \(X\) is called an ideal topological space and is denoted by \((X, \tau, I)\).

3. **pg***- Connected space

**Definition 3.1:** Let \(X\) be a topological space. A **pg***-separation of \(X\) is a pair \(A\) and \(B\) of disjoint nonempty **pg***- open subsets of \(X\) whose union is \(X\). The space \(X\) is said to be **pg***- Connected if there does not exist a **pg***-separation of \(X\). If there exist a **pg***-separation then \(X\) is said to be **pg***-disconnected.

**Note:** If \(X = A \cup B\) is a **pg***-separation then \(A^c = B\) and \(B^c = A\) and hence \(A\) and \(B\) are **pg***- closed.

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Remark 3.2: A space $X$ is pg**-connected if and only if the only subsets of $X$ that are both pg**-open and pg**-closed in $X$ are the empty set and $X$ itself.

Proof is obvious.

Example 3.3: An infinite set with finite complement topology is pg**-connected since it is impossible to find two disjoint pg**-open sets.

Example 3.4: Any indiscrete topological space $(X, \tau)$ with more than one point is pg**-disconnected since every subset is pg**-open.

Theorem 3.5: Every pg**-connected space is connected but not conversely.

Proof: Obvious, since every open set is pg**-open.

Theorem 3.6: Every pg**-connected space is g**-connected but not conversely.

Proof: Obvious, since every g**-open set is pg**-open.

Example 3.7: The space in example (3.4) is connected but not pg**-connected.

Example 3.8: The space $X = \{a, b, c\}$ with topology $\tau = \{\varnothing, X, \{a, c\}\}$ is g**-connected but not pg**-connected.

Example 3.9: $\mathbb{R}$ with usual topology is connected and g**-connected but not pg**-connected.

Since $\mathbb{Q}$ and $\mathbb{Q}^c$ are pg**-open but not open and g**-open.

Theorem 3.10: Let $(X, \tau)$ be a topological space. The following conditions are equivalent:

(i) $X$ is pg**-connected.
(ii) If $A$ and $B$ are disjoint pg**-open subsets of $X$ with $X = A \cup B$, then either $A = \varnothing$ (hence $B = X$) or $B = \varnothing$ (hence $A = X$).
(iii) If $C$ and $D$ are disjoint pg**-closed subsets of $X$ with $X = C \cup D$, then either $C = \varnothing$ (hence $D = X$) or $D = \varnothing$ (hence $C = X$).

Proof:

(i) $\Rightarrow$ (ii): Let $X$ be pg**-connected and let $A$ and $B$ be pg**-open subsets of $X$ with $X = A \cup B$ and $A \cap B = \varnothing$. Since $X = A \cup B$, $A$ is also pg**-closed, so either $A = \varnothing$ or $A = X$, (ii) follows.

(ii) $\Rightarrow$ (i): Assume (ii) and let $G$ be a subset of $X$ which is both pg**-open and pg**-closed and hence $X \setminus G$ is also both pg**-open and pg**-closed. Since $X = G \cup X \setminus G$, (ii) gives that either $G = \varnothing$ or $G = X$.

(ii) $\iff$ (iii): This follows from the fact that if $A$ and $B$ are disjoint pg**-open sets with $X = A \cup B$, then $A$ and $B$ are also pg**-closed. Similarly if $A$ and $B$ are disjoint pg**-closed sets with $X = A \cup B$, then $A$ and $B$ are also pg**-open.

Definition 3.11: Let $Y$ be a subset of a topological space $X$. A pg**-separation of $Y$ is a pair of disjoint nonempty pg**-open subsets $A$ and $B$ of $X$ whose union is $Y$. The space $Y$ is said to be pg**-connected if there does not exist a pg**-separation of $Y$ is said to be pg**-disconnected if there exist a pg**-separation of $Y$.

Theorem 3.12: If the sets $A$ and $B$ form a pg**-separation of $X$, and if $Y$ is a pg**-open and pg**-connected subset of $X$, then $Y$ lies entirely within either $A$ or $B$.

Proof: $X = A \cup B$ is a pg**-separation of $X$. Suppose $Y$ intersects both $A$ and $B$ then $Y = (A \cap Y) \cup (B \cap Y)$ is a pg**-separation of $Y$ which is a contradiction.

Theorem 3.13: Let $C$ be a pg**-connected subset of a topological space $X$ and let $D$ be a subset such that $C \subset D \subset pg^{**}cl(C)$, then $D$ is pg**-connected.

Proof: Suppose $D$ is pg**-disconnected, then $D = A \cup B$ is a pg**-separation of $D$. Since $C$ is pg**-connected and $C \subset D = A \cup B$, then either $C \subset A$ or $C \subset B$. To be specific, that $C$ is disjoint from $B$. This implies $pg^{**}cl(C) \cap B = \varnothing$, and $D \subset pg^{**}cl(C)$. Therefore $D \cap B = \varnothing$, this is not true. Hence $D$ is pg**-connected.
Theorem 3.14: Let \( C \) be a pg*-connected subset of a topological space \( X \). Then pg ** cl(\( C \)) is also pg**-connected.

Proof follows from taking \( D = pg ** cl(C) \) in theorem (3.13).

Theorem 3.15: If \( C \) is a pg**-dense subset of a topological space \((X, \tau)\) and if \( C \) is also pg**-connected, then \( X \) is pg**-connected.

Proof: Follows from \( pg ** cl(C) = X \).

Theorem 3.16: Let \((X, \tau)\) and \((Y, \sigma)\) be two topological spaces and \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a function. Then,

1. \( f \) is onto, pg**- continuous and \( X \) is pg**- connected \( \Rightarrow \) \( Y \) is connected.
2. \( f \) is onto, continuous and \( X \) is pg**- connected \( \Rightarrow \) \( Y \) is connected.
3. \( f \) is strongly pg**- continuous and \( X \) is connected \( \Rightarrow \) \( Y \) is pg**- connected.
4. \( f \) is onto and pg**- irresolute then \( Y \) is pg**- connected \( \Rightarrow \) \( X \) is connected.
5. \( f \) is a bijection and pg**- connected \( \Rightarrow \) \( X \) is connected.
6. \( f \) is onto, pg**- irresolute and \( X \) is pg**- connected \( \Rightarrow \) \( Y \) is pg**- connected.
7. \( f \) is a bijection and pg**- irresolute then \( Y \) is pg**- connected \( \Rightarrow \) \( X \) is pg**- connected.

Proof: (1) Suppose \( Y = A \cup B \) is a separation of \( Y \) then \( X = f^{-1}(Y) = f^{-1}(A) \cup f^{-1}(B) \) is a pg**- separation of \( X \) which is a contradiction. Therefore \( Y \) is connected.

Proofs for (2) to (7) are similar to the above proof.

Remark 3.17: The property of being “pg**- connected” is a pg**- topological property. This follows from (6) and (7) of theorem (3.16).

Theorem 3.18: A topological space \((X, \tau)\) is pg**- disconnected if and only if there exists a pg**- continuous map of \( X \) onto discrete two point space \( Y = \{0, 1\} \).

Proof: Let \( \cup A_{\alpha} = B \cup C \) be pg**- separation of \( \cup A_{\alpha} \). Then \( B \) and \( C \) are disjoint non empty pg**- open sets in \( X \). If \( p \in \cap A_{\alpha} \Rightarrow p \in B \) or \( p \in C \). Assume that \( p \in B \). Then by theorem (3.12), \( A_{\alpha} \) lies entirely within \( B \) for all \( \alpha \) (since \( p \in B \)). Therefore \( C \) is empty which is a contradiction.

Corollary 3.20: Let \( \{A_{\alpha}\} \) be a sequence of pg**- open pg**- connected subsets of \( X \) such that \( A_{\alpha} \cap A_{\alpha+1} \neq \varnothing \), for all \( n \). Then \( \cup A_{\alpha} \) is pg**- connected.

Proof: This can be proved by induction on \( n \). By theorem (3.14), the result is true for \( n = 2 \). Assume that the result to be true when \( n = k \). Now to prove the result when \( n = k + 1 \). By the hypothesis \( \cup_{i=1}^{k} A_{i} \) is pg**- connected. Now \( \cup_{i=1}^{k+1} A_{i+1} \neq \varnothing \). Therefore \( \cup_{i=1}^{k+1} A_{i} \) is pg**- connected. By induction hypothesis the result is true for all \( n \).

Corollary 3.21: Let \( \{A_{\alpha}\}_{\alpha \in \mathbb{A}} \) be an arbitrary collection of pg**-open pg**-connected subsets of \( X \). Let \( A \) be a pg**-open pg**- connected subset of \( X \). If \( A \cap A_{\alpha} \neq \varnothing \), for all \( \alpha \) then \( \cup \cup A_{\alpha} \) is pg**- connected.

Proof: Suppose that \( A \cup \cup A_{\alpha} = B \cup C \) be a pg**- separation of the subset \( A \cup \cup A_{\alpha} \). Since \( A \subseteq B \cup C \), by theorem (3.10) \( A \subseteq B \) or \( A \subseteq C \). Without loss of generality assume that \( A \subseteq B \). Let \( A_{\alpha} \subseteq B \cup C \Rightarrow A_{\alpha} \subseteq B \). But \( A \cap A_{\alpha} \neq \varnothing \Rightarrow A_{\alpha} \subseteq B \). Hence \( A \cup \cup A_{\alpha} \subseteq B \). Contradicting the fact that \( C \) is nonempty. Therefore \( A \cup \cup A_{\alpha} \) is pg**- connected.

Definition 3.22: A space \((X, \tau)\) is said to be totally pg**- disconnected if its only pg**- connected subsets are one point sets.
Example 3.23: Let \((X, \tau)\) be an indiscrete topological space with more than one point. Here all subsets are \(pg**\)-open. If \(A = \{x_1, x_2\}\) then \(A = \{x_1\} \cup \{x_2\}\) is a \(pg**\)-separation of \(A\). Therefore any subset with more than one point is \(pg**\)-disconnected. Hence \((X, \tau)\) is totally \(pg**\)-disconnected.

Example 3.24: An infinite subset with finite complement topology is not totally \(pg**\)-disconnected.

Remark 3.25: Totally \(pg**\)-disconnectedness implies \(pg**\)-disconnectedness.

Definition 3.26: A point \(x \in X\) is said to be in \(pg**\)-boundary of \(A\) (\(pg**Bd(A)\)) if every \(pg**\)-open set containing \(x\) intersects both \(A\) and \(X - A\).

Example 3.27: Any infinite subset \(A\) of \(\mathbb{R}\) whose complement is also infinite has every real number as its \(pg**\)-boundary point.

Theorem 3.28: Let \((X, \tau)\) be a topological space and let \(A\) be a subset of \(X\). If \(C\) is \(pg**\)-open \(pg**\)-connected subset of \(X\) that intersects both \(A\) and \(X - A\) then \(C\) intersects \(pg**Bd(A)\).

**Proof:** Given that \(C \cap A \neq \emptyset\) and \(C \cap A^c \neq \emptyset\). Now \(C = (C \cap A) \cup (C \cap A^c)\) is a nonempty disjoint union. Suppose both are \(pg**\)-open then it is a contradiction to the fact that \(C\) is \(pg**\)-connected. Hence either \(C \cap A\) or \(C \cap A^c\) is not \(pg**\)-open. Suppose that \(C \cap A\) is not \(pg**\)-open. Then there exist \(x \in C \cap A\) which is not \(pg**\)-interior point of \(C \cap A\). Let \(U\) be a \(pg**\)-open set containing \(x\). Then \(U \cap A\) is a \(pg**\)-open set containing \(x\) and hence \((U \cap C ) \cap (C \cap A^c) \neq \emptyset\). This implies \(U\) intersects both \(A\) and \(A^c\) and therefore \(x \in pg**Bd(A)\). Hence \(C \cap pg**Bd(A) \neq \emptyset\).

Next we extend the intermediate value theorem for \(pg**\)-connected space.

Theorem 3.29: (Generalisation of Intermediate value theorem) Let \(f : X \rightarrow \mathbb{R}\) be a \(pg**\)-continuous map, where \(X\) is \(pg**\)-connected space and \(\mathbb{R}\) with usual topology. If \(x, y\) are two points of \(X\) and \(a = f(x)\) and \(b = f(y)\) then for every real number \(r\) between \(a\) and \(b\), there exists a point \(c\) of \(X\) such that \(f(c) = r\).

**Proof:** Assume the hypothesis of the theorem. Suppose there is no point \(c\) of \(X\) such that \(f(c) = r\), then \(A = (-\infty, r)\) and \(B = (r, \infty)\) are disjoint open sets in \(\mathbb{R}\) and \(X = f^{-1}(A) \cup f^{-1}(B)\) which is a \(pg**\)-separation of \(X\), contradicting the fact that \(X\) is \(pg**\)-connected. Therefore there exists \(c \in X\) such that \(f(c) = r\).

Remark 3.30: The above theorem holds even if,
- \(f\) is continuous and \(X\) is \(pg**\)-connected.
- \(f\) is \(pg**\)-irresolute and \(X\) is \(pg**\)-connected.
- \(f\) is \(pg**\)-continuous and \(X\) is \(pg**\)-connected.
- \(f\) is strongly \(pg**\)-continuous and \(X\) is \(pg**\)-connected.

4. \(pg**\)-components

Definition 4.1: Let \((X, \tau)\) be a topological space. Define an equivalence relation on \(X\) by setting \(x \sim y\) if and only if there exists a \(pg**\)-connected subset of \(X\) containing both \(x\) and \(y\). The equivalence classes are called \(pg**\)-components of \(X\). A \(pg**\)-component containing \(x\) is denoted by \(C_x = \{y \in X \mid x \sim y\}\).

(i) \(x \sim x\), since \((x)\) is \(pg**\)-connected. Hence \(\sim\) is reflexive.

(ii) If \(x \sim y\), then there exists a \(pg**\)-connected subset of \(X\) containing both \(x\) and \(y\) and hence \(y \sim x\). Therefore \(\sim\) is symmetric.

(iii) Let \(x \sim y\) and \(y \sim z\). Then there exists a \(pg**\)-connected subset \(A\) of \(X\) containing both \(x\) and \(y\) and a \(pg**\)-connected subset \(B\) of \(X\) containing both \(y\) and \(z\). Since \(A\) and \(B\) are \(pg**\)-connected have a point \(y\) in common \(A \cup B\) is a \(pg**\)-connected subset of \(X\) containing \(x, y\) and \(z\). Therefore \(\sim\) is transitive.

Example 4.2: Let \((X, \tau)\) be an indiscrete topological space with more than one point. Then each \(pg**\)-component of \(X\) consists of a single point.

Theorem 4.3: Any two \(pg**\)-components are either identical or disjoint.

**Proof:** Follows from the definition of \(pg**\)-component.

Theorem 4.4: The \(pg**\)-components of \(X\) are \(pg**\)-connected subsets of \(X\) whose union is \(X\), such that each nonempty \(pg**\)-connected subset of \(X\) intersects only one of the \(pg**\)-components.
Proof: Each pg*-connected subset $A$ of $X$ intersects only one of the pg*-components. For, if $A$ intersects the pg*-components $C_1$ and $C_2$ of $X$, say in points $x_1$ and $x_2$ then $x_1 \sim x_2$, this implies $C_1 = C_2$. To prove the pg*-component $C$ is pg*-connected, choose a point $x_0 \in C$.

Now for every $x \in C$, $x_0 \sim x$. Therefore there exists a pg*-connected subset $A_x$ containing $x$ and $x_0$, implies $A_x \subset C$. Therefore $\bigcup_{x \in A_x} A_x = C$. Since $A_x$ are pg*-connected subsets having the point $x_0$ in common, $C$ is pg*-connected.

Corollary 4.5: $C_x$ is the union of all pg*-connected sets containing $x$.

Theorem: $C_x$ is the largest pg*-connected set containing $x$. If there is another pg*-connected subset $A$ of $X$ such that $x \in A$, then $A \subset C_x$.

Proof: Let $t \in A \Rightarrow x, t \in A$, where $A$ is pg*-connected, this implies $t \sim x$. Therefore $t \in C_x$ and hence $x \in C_x$. Hence $C_x$ is the largest pg*-connected set containing $x$.

Theorem 4.6: Let $(X, \tau)$ be a topological space, then the following are true.

(i) Each point in $X$ is contained in exactly one pg*-component of $X$.
(ii) Each pg*-connected subset of $X$ is contained in a pg*-component of $X$.
(iii) A pg*-connected subset of $X$ which is pg*-open is a pg*-component of $X$.
(iv) If $C$ is pg*-component of $X$ then $C = pg^* \text{cl}(C)$. If $(X, \tau)$ is a pg*-multiplicative space then every pg*-component is pg*-closed.

Proof:

(i) Let $x \in X$ and consider the collection $\{C_i\}$ of all pg*-connected subsets of $X$ containing $x$, this collection is non-empty since $\{x\}$ itself is pg*-connected. $C = \bigcup C_i$ is a maximal pg*-connected subset of $X$ which contains $x$ and therefore a pg*-component of $X$. Suppose $C^*$ is another pg*-component of $X$ containing $x$, it clearly among the $C_i$’s and is therefore contained in $C$, since $C^*$ is also pg*-component we must have $C = C^*$.
(ii) A pg*-connected subset of $X$ is contained in the pg*-component which contains any one of its points.
(iii) Let $A$ be a pg*-connected subset of $X$ which is pg*-open, then (by (ii)) $A$ is contained in some pg*-component $C$. If $A$ is a proper subset of $C$, then $C \cap A$ and $C \cap A^\circ$ forms a pg*-separation of $C$ which is a contradiction to the fact that $C$, being a pg*-component, is pg*-connected. Therefore $A = C$.
(iv) If the pg*-component $C \neq pg^* \text{cl}(C)$ then its pg*-closure ($pg^* \text{cl}(C)$) is a pg*-connected subset (3.14) subset of $X$ which properly contains $C$, this is the contradiction to the maximality of $C$ as pg*-connected subset of $X$. Hence $C = pg^* \text{cl}(C)$. If $X$ is a pg*-multiplicative space, then $C$ is pg*-closed.

Theorem 4.7: Let $X$ be a totally pg*-disconnected space. Then $C_x = \{x\}$, where $C_x$ is a pg*-component of $x$.

Proof: Let $X$ be a totally pg*-disconnected space, and then its only pg*-connected subsets are one point sets. Suppose $y \in C_x$ such that $x \neq y$ then $C_y$ is not pg*-connected which is contradiction to the fact that the pg*-components of $X$ are pg*-connected subsets of $X$ (4.4). Therefore in a totally pg*-disconnected space the pg*-component of $x$ is $\{x\}$.

5. pg*-connected modulo $I$

Definition 5.1: Let $(X, \tau, I)$ be an ideal topological space then $X = A \cup B$ is said to be pg*-separation modulo $I$ if $A$ and $B$ are non empty pg*-open sub sets of $X$ such that $A \cap B \in I$. $(X, \tau, I)$ is said to be pg*-connected modulo $I$ if there is no pg*-separation modulo $I$ for $X$.

Definition 5.2: Let $Y$ be a subset of $X$. $Y = A \cup B$ is said to be pg*-separation modulo $I$ of $Y$ if $A$ and $B$ are non empty pg*-open sub sets of $X$ and $A \cap B \in I$.

If there is no pg*-separation modulo $I$ for $Y$ then we say $Y$ is pg*-connected modulo $I$ subset.

Theorem 5.3: $X = A \cup B$ is pg*-separation of $X$ implies $X = A \cup B$ is a pg*-separation modulo $I$ of $X$ for any ideal $I$.

Proof: It follows since $\tau \in I$.

Theorem 5.4: $(X, \tau, I)$ is pg*-connected modulo $I$ for some ideal $I$ implies $(X, \tau)$ is pg*-connected. Equivalently If $(X, \tau)$ is pg*-disconnected then $(X, \tau, I)$ is pg*-disconnected modulo $I$ for some ideal $I$.

Proof follows from theorem (5.3).
Remark 5.5: The converse is false as seen in the following example.

Example 5.6: Let \((X, \tau)\) be infinite cofinite topological space and \(I = p(X)\). Then \(X\) is \(pg^*-\) connected. On the other hand \(X - \{x\}, X - \{y\}\) are \(pg^*-\)open and non empty, and \((X - \{x\}) \cup (X - \{y\})\) is a \(pg^*-\)separation modulo \(I\) of \(X\). Therefore \((X, \tau, I)\) is not \(pg^*-\) connected modulo \(I\).

Theorem 5.7: Let \((X, \tau, I)\) be an ideal topological space, \(X = A \cup B\) is a \(pg^*-\)separation modulo \(I\) of \(X\) and \(Y\) is \(pg^*-\)open \(pg^*-\) connected subset of \(X\) modulo \(I\) then \(Y\) lies entirely within either \(A\) or \(B\).

Proof: \(X = A \cup B\) is a \(pg^*-\)separation of \(X\) modulo \(I\). Therefore \(A\) and \(B\) are nonempty \(pg^*-\) open sets and \(A \cap B \in I\).

Now \(Y = (Y \cap A) \cup (Y \cap B), (Y \cap A)\) and \((Y \cap B)\) are \(pg^*-\)open sets and \((Y \cap A) \cap (Y \cap B) = Y \cap (A \cap B) \in I\). If \((Y \cap A)\) and \((Y \cap B)\) are both non empty then \(Y = (Y \cap A) \cup (Y \cap B)\) is a \(pg^*-\)separation of \(Y\) modulo \(I\) which is a contradiction. Therefore \((Y \cap A) = \emptyset\) or \((Y \cap B) = \emptyset\) and hence \(Y\) lies entirely within either \(A\) or \(B\).

Theorem 5.8: Let \((X, \tau, I)\) and \((Y, \sigma, J)\) be two ideal topological spaces and \(f: (X, \tau, I) \rightarrow (Y, \sigma, J)\) be a bijection where \(J = f(I)\), then

1. \(f\) is \(pg^*-\) continuous and \(X\) is \(pg^*-\) connected modulo \(I\) \(\Rightarrow\) \(Y\) is connected modulo \(J\).
2. \(f\) is continuous and \(X\) is \(pg^*-\) separated modulo \(I\) \(\Rightarrow\) \(Y\) is connected modulo \(J\).
3. \(f\) is strongly \(pg^*-\) continuous and \(X\) is connected \(\Rightarrow\) \(Y\) is \(pg^*-\) connected modulo \(J\).
4. \(f\) is \(pg^*-\) irresolute then \(Y\) is \(pg^*-\) connected modulo \(J\) \(\Rightarrow\) \(X\) is connected modulo \(f\).
5. \(f\) is a bijection and open then \(Y\) is \(pg^*-\) connected modulo \(J\) \(\Rightarrow\) \(X\) is connected modulo \(I\).
6. \(f\) is \(pg^*-\) irresolute and \(X\) is \(pg^*-\) connected modulo \(I\) \(\Rightarrow\) \(Y\) is \(pg^*-\) connected modulo \(J\).
7. \(f\) is \(pg^*-\) irresolute then \(Y\) is \(pg^*-\) connected modulo \(J\) \(\Rightarrow\) \(X\) is \(pg^*-\) connected modulo \(I\).

Proof: (1) Assume that \(Y\) is not connected modulo \(J\). Let \(Y = A \cup B\) be a \(pg^*-\) separation modulo \(J\). Therefore \(A\) and \(B\) are nonempty \(pg^*-\) open subsets of \(Y\) such that \(A \cap B \notin J\). Then \(X = f^{-1}(A) \cup f^{-1}(B)\) is a \(pg^*-\) separation modulo \(I\) since \(f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) \subseteq I\) which is a contradiction. Therefore \(Y\) is connected modulo \(J\).

Proofs for (2) to (7) are similar to the above proof.

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