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PRIME LABELING OF DESARGUES GRAPHS

S. CHANDRAKALA*

Assistant professor of Mathematics,
T. D. M. N. S College, T. Kallikulam, Tamilnadu, India.

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ABSTRACT

A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers 1,2,…|V| such that for edge xy the labels assigned to x and y are relatively prime. A graph which admits prime labeling is called a prime graph. In this paper I investigate prime labeling for some special graphs namely Desargues graph. I also discuss prime labeling in the context of graph operations namely duplication and switching in Desargues graph.

Keywords: *Prime labeling, Desargues graph, Duplication, switching.*

I. INTRODUCTION

In this paper, I consider only finite simple undirected graph. The graph G has vertex set V =V (G) and edge set E = E (G). The set of vertices adjacent to a vertex u of G is denoted by N(u). For notations and terminology we refer to Bondy and Murthy [1].

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout [6].Two integers a and b are said to be relatively prime if their greatest common divisor is 1.Many researchers have studied prime graph. Fu.H [3] has proved that the path P_n on n vertices is a prime graph. Deretsky *et al.* [2] have proved that the cycle C_n on n vertices is a prime graph. Around 1980 Roger Entringer conjectured that all trees have prime labeling which is not settled today.

In [7] S.K. Vaidhya and K.K. Kanmani have proved that the graphs obtained by identifying any two vertices duplicating arbitrary vertex and switching of any vertex in cycle C_n admit prime labeling. In [5] Meena and Vaithilingam have proved the Prime labeling for some helm related graphs.

In this paper I proved that the Desargues graph is a prime graph.

Definition 1.1: Let G = (V (G), E (G)) be a graph with p vertices .A bijection f: V (G)→ {1, 2,…p} is called a prime labeling if for each edge e $=uv$, $\gcd\{f(u), f(v)\} = 1$. A graph which admits prime labeling is called a prime graph.

Definition 1.2: Duplication of a vertex v of graph G produces a new graph G' by adding a new vertex v' such that $N(v') = N(v)$. In other words a vertex v' is said to be duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' *in* G'.

Definition 1.3: Duplication of a vertex v_k by a new edge e = $v_k' v_k''$ in a graph G produces a new graph G' such that $N(v_k') = \{v_{k,} v_k''\}$ *and* $N(v_k'') = \{v_{k,} v_k'\}$.

Definition 1.4: Duplication of an edge $e = uv$ by a new vertex w in a graph G produces a new graph G' such that $N(w) = \{u, v\}$.

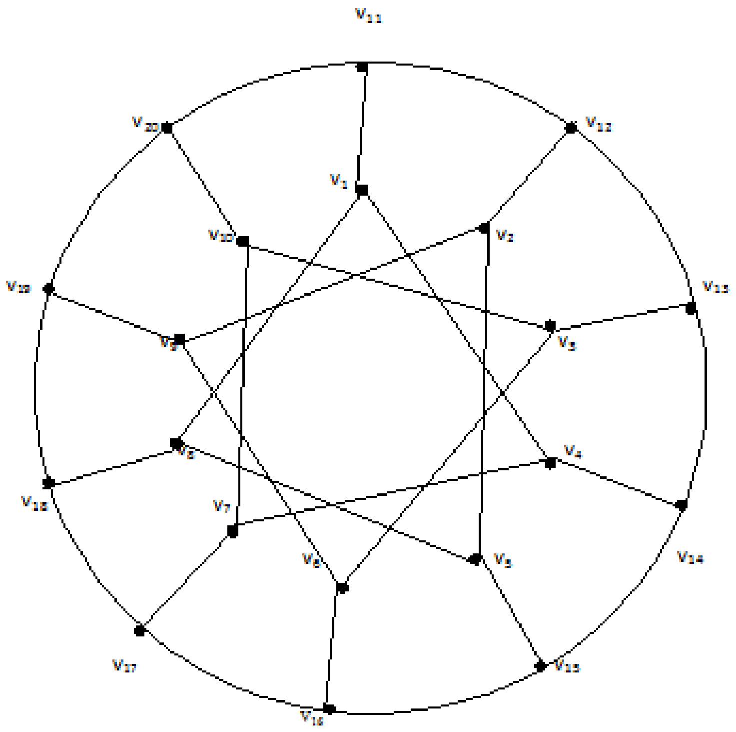
Definition 1.5: Duplication of an edge $e = uv$ of a graph G produces a new graph G' by adding an edge $e' = u'v'$ such that $N(u') = N(u) \cup \{v'\} - \{v\}$ and $N(v') = N(v) \cup \{u'\} - \{u\}$.

Corresponding Author: S. Chandrakala, Assistant professor of Mathematics,*
T. D. M. N. S College, T. Kallikulam, Tamilnadu, India.

Definition 1.6: A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 1.7: The Desargues Graph is a distance transitive Cubic graph with 20 vertices and 30 edges.

Example:



III. MAIN RESULTS

Theorem 3.1: The Desargues Graph is a prime graph.

Proof: Let G be a Desargues graph with 20 vertices and 30 edges.

Then | V (G) | = 20 and |E (G)| = 30 .

The edge set E (G) = $\{v_i v_{i+3} \ /1 \le i \le 7\}$
$\cup \{v_i v_{i+7} \ \ /1 \le i \le 3\} \cup \{v_i v_{i+1} \ \ /11 \le i \le 19\} \cup \{v_{11} v_{20}\}$
$\cup \{v_i v_{i+3} \ \ /i = 8,9,10\} \cup \{v_i v_{i+13} \ /1 \le i \le 7\}$.

Define a labeling $f: V(G) \to \{1,2, \dots \dots \dots \dots .20\}$ by

$$f(v_1) = 9$$
$$f(v_3) = 7$$
$$f(v_5) = 1$$
$$f(v_7) = 3$$
$$f(v_9) = 5$$
$$f(v_{2i}) = 2i\,, \ \ i = 1,2,3,4,5$$
$$f(v_i) = i\,, \ \ 11 \le i \le 20$$

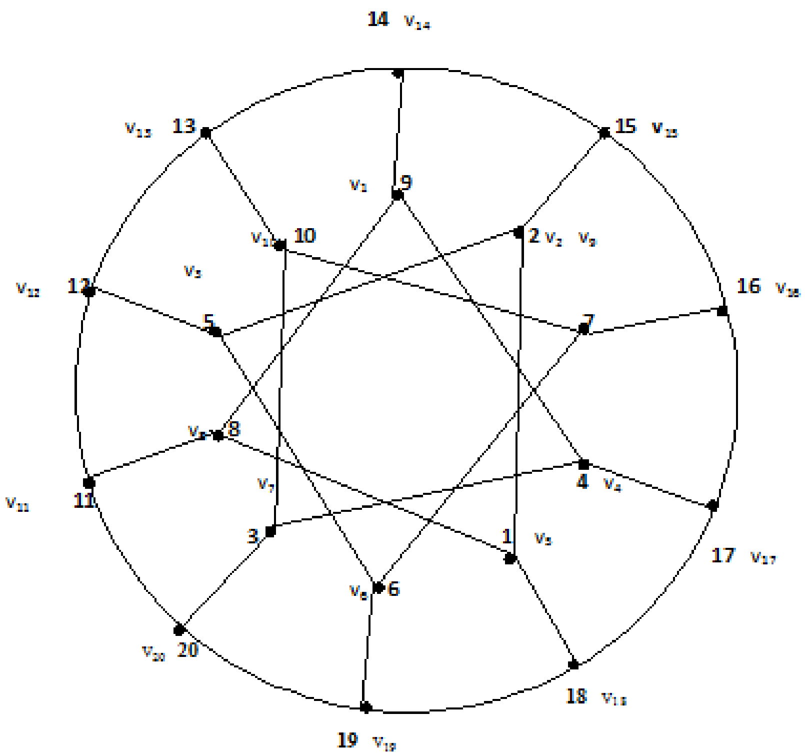
clearly ,$g.c.d \{f(v_i), f(v_{i+1})\} = 1$, $i = 11,12, \dots \dots \dots 19$

$$g.c.d \{f(v_i), f(v_{i+3})\} = 1, \ \ i = 1,2,3,4.5,6,7$$
$$g.c.d \{f(v_{11}), f(v_{20})\} = 1$$
$$g.c.d \{f(v_i), f(v_{13+i})\} = 1, \qquad i = 1,2, \dots \dots \dots 7$$
$$g.c.d \{f(v_i), f(v_{i+3})\} = 1, \qquad i = 8,9,10$$

Then f admits prime labeling.

Hence G is a prime graph.

Illustration of theorem 3.1:



Prime labeling of Desargues graph

Theorem 3.2: The graph obtained by duplication of a vertex in the Desargues graph is a prime graph.

Proof: Let G be the Desargues graph .Then | V (G)| =20 and |E (G)| = 30. Let G* be the graph obtained by duplication of the vertex v_{12} by a new vertex v_k .

Then $|V(G^*)| = 21$ and $|E(G^*)| = 33$.

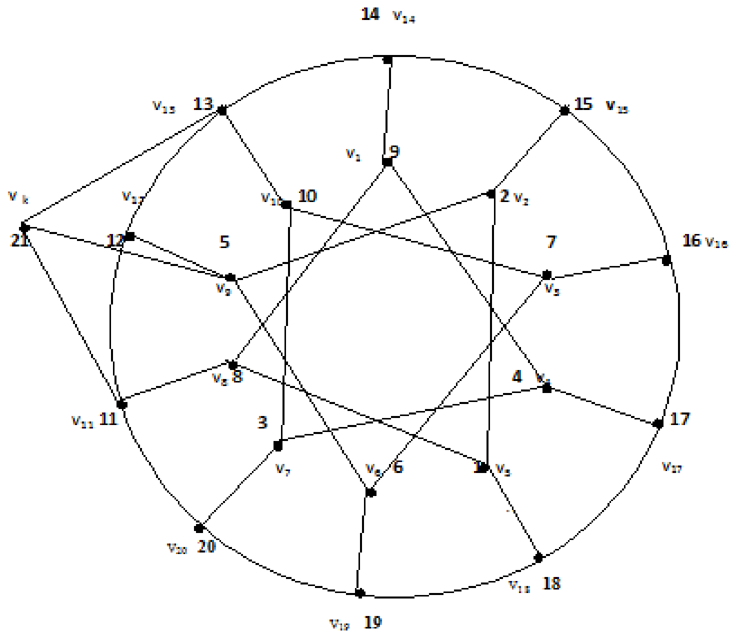
Then the edge set $E(G^*) = E(G) \cup \{v_k v_{13}, v_k v_{11}, v_k v_5\}$.

Define a labeling $f: V(G^*) \to \{1,2, \ldots \ldots \ldots \ldots .21\}$ by

$$f(v_1) = 9$$
$$f(v_3) = 7$$
$$f(v_5) = 1$$
$$f(v_7) = 3$$
$$f(v_9) = 5$$
$$f(v_{2i}) = 2i\,, \quad i = 1,2,3,4,5$$
$$f(v_i) = i\,, \quad 11 \leq i \leq 20$$
$$f(v_k) = 21$$

Then f admits a prime labeling for G* .Hence G* is a prime graph.

Illustration of theorem 3.2:



Prime labeling of duplication of a vertex v_{12} by a new vertex v_k in the Desargues graph.

Theorem 3.3: The graph obtained by duplication of a vertex by an edge in the Desargues graph is a prime graph.

Proof: Let G be the Desargues graph .Then |v (G)| = 20 and |E (G)|=30. Let G* be the graph obtained by duplication of a vertex v_{13} by an edge $v_k^{'} v_k^{''}$ in the Desargues graph.

Then $|V(G^*)| = 22$ and $|E(G^*)| = 33$.The edge set $E(G^*) = E(G) \cup \{v_{13} v_k^{'}\ , v_{13} v_k^{''}, v_k^{'} v_k^{''}\}$.

Define $f: V(G^*) \to \{1,2, \ldots \ldots \ldots \ldots \ldots \ldots \ldots 22\}$ by

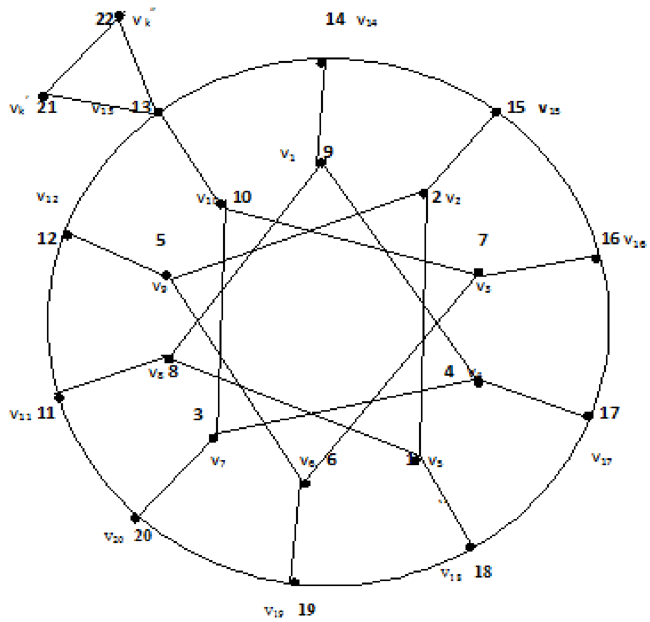
$f(v_1) = 9;\ f(v_3) = 7;\ f(v_5) = 1;\ f(v_7) = 3;\ f(v_9) = 5;$

$f(v_{2i}) = 2i,\ i = 1,2,3,4,5$

$f(v_i) = i,\ 11 \le i \le 20;\ f(v_k^{'}) = 21;\ f(v_k^{''}) = 22.$

Then f admits a prime labeling for G*. Hence G* is a prime graph.

Illustration of theorem 3.3:



Prime labeling of duplication of a vertex v_{13} by an edge $v_k^{'} v_k^{''}$ in Desargues graph

Theorem 3.4: The graph obtained by duplication of an edge by a vertex in Desargues graph is a prime graph.

Proof: Let G be the Desargues graph. Then |V (G)| =20 and $E(G) = 30$.

Let G* be the graph obtained by Duplication of an edge $v_{11}v_{20}$ by a vertex w in Desargues graph

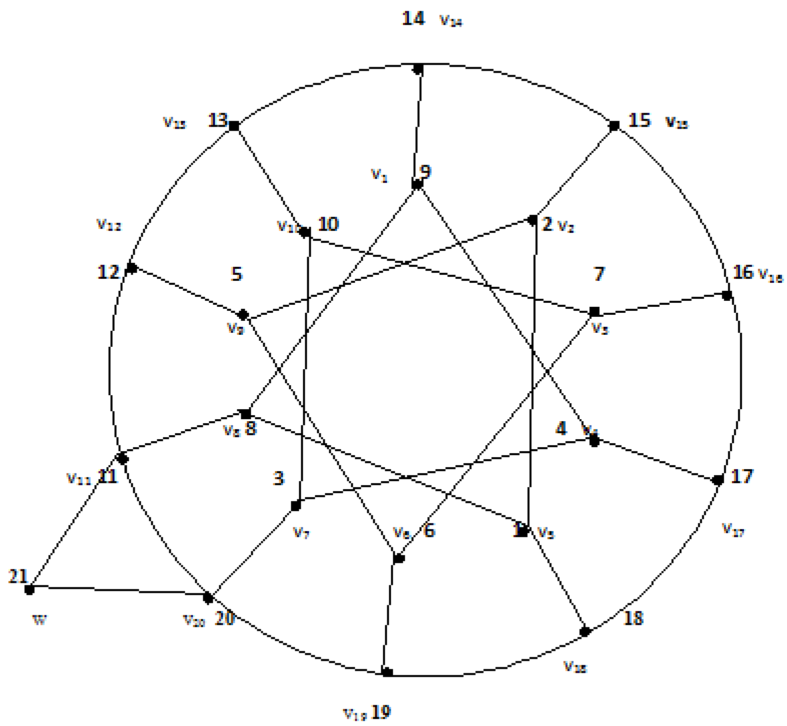
Then |V (G^*)| = 21 and |$E(G^*)$| = 32.The edge set $E(G^*) = E(G) \cup \{v_{11}v_w, v_{20}v_w\}$.

Define a labeling $f: V(G) \to f$ {1,2, … … … ….21} by

$$f(v_1) = 9; f(v_3) = 7;\ f(v_5) = 1;\ f(v_7) = 3;\ \ f(v_9) = 5;$$
$$f(v_{2i}) = 2i,\ \ i = 1,2,3,4,5;$$
$$f(v_i) = i,\ \ 11 \le i \le 20;\quad f(w) = 21.$$

Then G*admits a prime labeling .Hence G^* is a prime graph.

Illustration of theorem 3.4:



Prime labeling of duplication of an edge $v_{11}v_{20}$ by a vertex w in Desargues graph.

Theorem 3.5: The graph obtained by duplication of an edge in the Desargues graph is a prime graph.

Proof: Let G be the Desargues graph .Then |V (G)| =20 and $E(G) = 30$.

Let G* be the graph obtained by duplication of an edge $v_{17}v_{18}$ by a new edge uv in Desargues graph

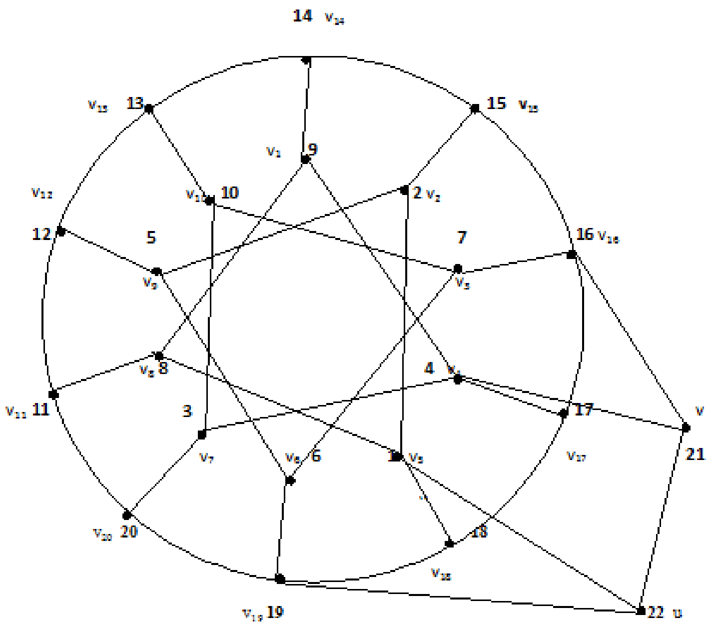
Then |V (G^*)| = 22 and |$E(G^*)$| = 35. The edge set$E(G^*) = E(G) \cup \{v_{19}u, v_{16}v\ , v_4v, v_5u, uv\}$.

Define a labeling $f: V(G) \to f$ {1,2, … … … ….22} by

$$f(v_1) = 9; f(v_3) = 7;\ f(v_5) = 1;\ f(v_7) = 3;\ \ f(v_9) = 5;$$
$$f(v_{2i}) = 2i,\ \ i = 1,2,3,4,5;$$
$$f(v_i) = i,\ \ 11 \le i \le 20;\quad f(v) = 21;\ f(u) = 22.$$

Then G*admits a prime labeling .Hence G^* is a prime graph.

Illustration of theorem 3.5:



Prime labeling of duplication of an edge $v_{17}v_{18}$ by an edge uv in Desargues graph.

Theorem 3.6: The graph G_k obtained by switching a vertex in the Desargues graph is a prime graph.

Proof: Let G be the Desargues graph .Then $|V(G)| = 20$ and $|E(G)| = 30$.

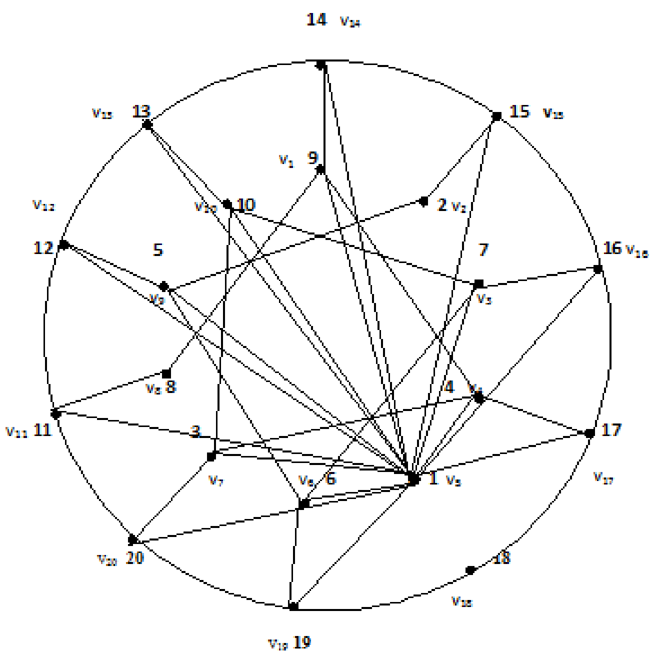
Let G_k be the graph obtained by switching the vertex v_5 in the Desargues graph.

Then $|V(G_k)|$ =20 and the edge set $E(G_k) = E(G) \cup \{v_1 v_i \ /3 \le i \le 20;\ i \ne 8{,}18\}$.

Define a labeling $f : V(G_k) \to \{1,2, \dots \dots \dots \dots \dots 20\}$ $as\ follows.$
$f(v_1)$ =9; $f(v_3)$ =7; $f(v_5)$ =1; $f(v_7)$ =3; $f(v_9)$ =5;
$f(v_i) = i,\ i = 2,4,6,8,10,11,12,13,14,15,16,17,18,19,20.$

Then f admits a prime labeling. Hence G_k is a prime graph.

Illustration of theorem 3.6:



Prime labeling of switching a vertex v_5 in Desargues graph.

Theorem 3.7: The graph obtained by adjoining a vertex into each vertex in the outer cycle of the Desargues graph is a prime graph.

Proof: Let G be the Desargues graph .Then $|V(G)| = 20\ and\ |E(G)| = 30.$

Let G* be a graph obtained by adjoining vertices $\{w_1, w_2, \dots \dots \dots .. w_{10}\}$ into vertices$\{v_{20}, v_{19}, \dots \dots .., v_{11}\}$ in the outer cycle of Desargues graph respectively.

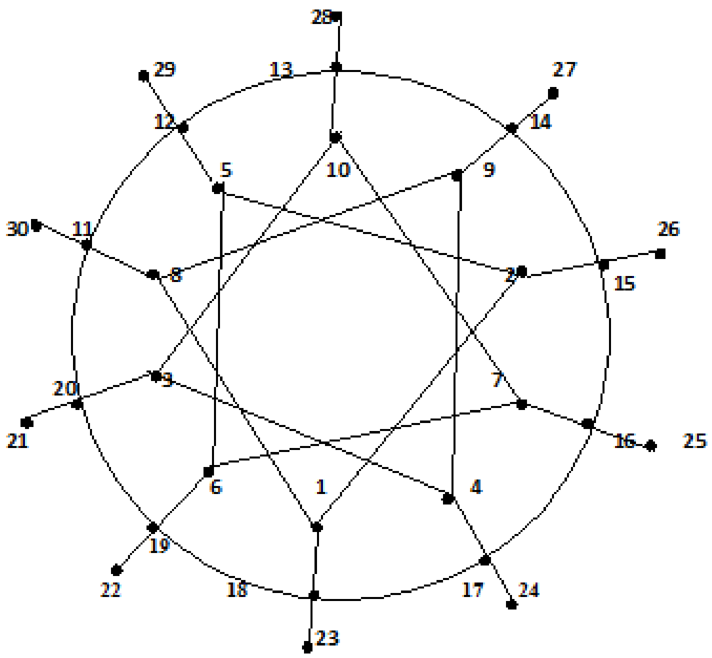
Then $|V(G^*)| = 30\ \ and\ \ |E(G^*)| = 40$.The edge set$E(G^*) = E(G) \cup \{v_i w_j \quad /11 \leq i \leq 20; 1 \leq j \leq 10\}$.

Define a labeling $f : V(G^*) \to \{1,2, \dots \dots \dots \dots \dots 30\}$ by

$$f(v_1) = 9; f(v_3) = 7;\ f(v_5) = 1;\ f(v_7) = 3;\ \ f(v_9) = 5;$$
$$f(v_{2i}) = 2i,\ \ i = 1,2,3,4,5;\ f(v_i) = i,\ \ 11 \leq i \leq 20;$$
$$f(\omega_i) = 20 + i, 1 \leq i \leq 10.$$

Then f admits a prime labeling. Hence G^* is a prime graph.

Illustration of theorem 3.7:



IV.CONCLUSION

I have investigated some results on prime labeling for the special graphs namely Desargues graph .Extending the study to other families of graph is an open area of research.

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