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REGULAR PRE-CLOSED MAPINGS

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ABSTRACT

The aim of this paper is to introduce and study the concept of rp-closed mappings and the interrelationship between other closed maps.

***KEYWORDS**: rp-closed mappings*

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1. INTRODUCTION:

Mappings place an important role in the study of modern mathematics, especially in Topology and Functional analysis. Closed mappings are one such mapping which is studied for different types of closed sets by various mathematicians for the past many years. In this paper we tried to study a new variety of closed maps called rp-closed maps. Throughout the paper X, Y means topological spaces (X, τ) and (Y, σ) on which no separation axioms are assured.

2. PRELIMINARIES:

Definition 2.1: $A \subseteq X$ is called

(1) pre open if $A \subseteq (\bar{A})^0$ and pre closed if $\overline{(A^0)} \subseteq A$.

(2) semiopen if $A \subseteq \overline{(A^0)}$ and semiclosed if $(\bar{A})^0 \subseteq A$.

(3) semi pre open if $A \subseteq \overline{((\bar{A})^0)}$ and semi pre closed if $\overline{(A^0)}^0 \subseteq A$.

(4) regular open if $A = (\bar{A})^0$ and regular closed if $A = \overline{(A^0)}$.

(5)regular pre open (or) simply rp-open if there exists a pre open set U such that $U \subseteq A \subseteq \bar{U}$, equivalently $A = pint(pcl(A))$.

(6) α- open if $A \subseteq (\overline{(A^0)})^0$ and α-closed if $\overline{((\bar{A})^0)} \subseteq A$.

(7) β-open (or semi pre open) if $A \subseteq cl(Int(cl(A))$ and β-closed if $Int(cl(Int(A)) \subseteq A$.

(8) regular α-closed (ie., rα-closed) if there exists a regular closed set U such that $\alpha(U)^0 \subseteq A \subseteq U$.

(9) generalized closed [resp. regular generalized closed] (ie. briefly g-closed, rg-closed) if $\bar{A} \subseteq U$ whenever $A \subseteq U$ and U is open [resp. regular open].

Definition 2.2: A function $f: X \rightarrow Y$ is said to be

(1) continuous [resp., semicontinuous, r-continuous, rp-continuous] if the inverse image of every open set is open. [resp., semi open, regular open, regular pre-open].

(2) irresolute [resp., r-irresolute, rp-irresolute] if the inverse image of every semi open [resp., regular open, rp-open] set is semi open. [resp., regular open, rp-open].

(3) closed [resp., semi-closed, r-closed] if the image of every closed set is closed [resp., semi closed, regular closed].

(4) g-continuous [resp., rg-continuous] if the inverse image of every closed set is g-closed. [resp., rg-closed].

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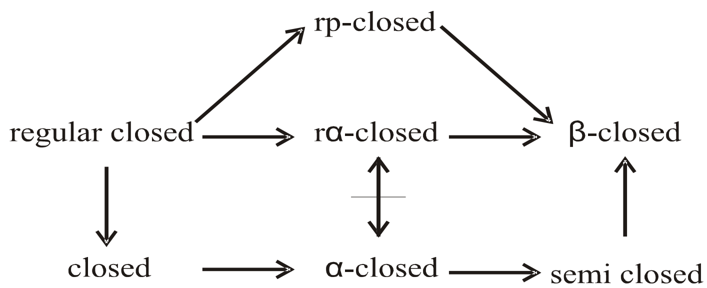
Definition 2.3: X is said to be $T_{1/2}$[r-$T_{1/2}$] if every (regular) generalized closed set is (regular) closed.

Note: From the above definitions we have the following implication diagram.



3. rp-CLOSED MAPPINGS:

Definition 3.1: A function $f: X\rightarrow Y$ is said to be rp-closed if the image of every closed set in X is rp-closed in Y.

Theorem 3.1:
(a) Every r-closed map is rp-closed but not conversely.
(b) Every rp-closed map is β-closed but not conversely.

Proof: (a) Let $A\subseteq X$ be closed $\Rightarrow f$ (A) is r-closed in Y since $f: X\rightarrow Y$ is r-closed $\Rightarrow f$ (A) is rp-closed in Y since every regular closed set is rp-closed. Hence f is rp-closed.

But the converse is not true as shown by the following example.

Let $X = \{a, b, c\} = Y$; $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a\}, \{b, c\}, Y\}$. Let $f: X\rightarrow Y$ be the identity map. Then f is rp-closed but not r-closed.

Assume that $f: X\rightarrow Y$ is a rp-closed map. Let $A\subseteq X$ be closed. Then by assumption, f(A) is rp-closed. As every rp-closed set is β-closed, f (A) is β-closed. This implies f is β-closed. But the converse is not true.

Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a, c\}, X\}$; $\sigma = \{\varphi, \{a\}, \{a, b\}, Y\}$. Let $f: X\rightarrow Y$ be the identity map. Then f is β-closed but not rp-closed.

Example 3.1: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, X\}$; $\sigma = \{\varphi, \{a\}, \{a, b\}, Y\}$. Let $f: X\rightarrow Y$ be the identity map. Then f is not rp-closed and semi-closed and f is not r-closed.

Example 3.2: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a\}, \{b\}, \{a, b\}, Y\}$. Assume $f: X\rightarrow Y$ be the identity map. Then f is closed but not rp-closed.

Theorem 3.2:
(i) If $R\alpha C(Y)=RPC(Y)$ then f is rα-closed iff f is rp-closed.
(ii) If $RPC(Y)=RC(Y)$ then f is r-closed iff f is rp-closed.
(iii) If $RPC(Y)=\alpha C(Y)$ then f is α-closed iff f is rp-closed.

Note 2:
(i) Closed maps and rp-closed maps are independent of each other.
(ii) α-closed map and rp-closed map are independent of each other.
(iii) Semi closed map and rp-closed map are independent of each other.

Example 3.3: Let $f: X\rightarrow Y$ be the identity map. Where $X = Y = \{a, b, c\}$.
Let $\tau = \{\varphi, \{a\}, X\} = \sigma$. Then

(i) f is closed but not rp-closed.
(ii) f is α-closed but not rp-closed.
(iii) f is semi closed but not rp-closed.

Example 3.4: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a\}, \{b, c\}, Y\}$. Then f is rp-closed which is not closed, α-closed and semi closed.

Note 3: We have the following implication diagram among the closed maps.

$$
\begin{tikzcd}
 & \text{rpc-map} \arrow[d, leftrightarrow, "|" marking] \arrow[dr] & \\
\text{rc-map} \arrow[ur] \arrow[r] \arrow[d] & \text{r}\alpha\text{c-map} \arrow[r] \arrow[d, leftrightarrow, "|" marking] & \beta\text{-closed map} \\
\text{closed map} \arrow[r] & \alpha\text{-closed map} \arrow[r] & \text{semi closed map} \arrow[u]
\end{tikzcd}
$$

None is reversible.

Theorem 3.3:
(i) If f is closed and g is rp-closed then $g\ o\ f$ is rp-closed.
(ii) If f is closed and g is r-closed then $g\ o\ f$ is rp-closed.
(iii) If f and g are r-closed then $g\ o\ f$ is rp-closed.
(iv) If f is r-closed and g is rp-closed then $g\ o\ f$ is rp-closed.

Proof: (i) Let A$\subseteq X$ be closed $\Rightarrow f$ (A) is closed in $Y \Rightarrow g$ (f (A)) is rp-closed in $Z \Rightarrow g\ o\ f$(A) is rp-closed in Z. Hence $g\ o\ f$ is rp-closed.

Similarly we can prove the remaining results.

Corollary 3.1:
(i) If f is closed and g is rp-closed then $g\ o\ f$ is β-closed.
(ii) If f is r-closed and g is rp-closed then $g\ o\ f$ is β-closed.

Theorem 3.4: If $f: X\to Y$ is rp-closed then rp $\overline{(f(\mathrm{A}))} \subseteq f(\bar{\mathrm{A}})$

Proof: Let A$\subseteq X$ and $f: X\to Y$ be rp-closed. Then $f(\bar{\mathrm{A}})$ is rp-closed in Y and $f(\mathrm{A}) \subseteq f(\bar{\mathrm{A}})$. This implies

rp $\overline{(f(\mathrm{A}))} \subseteq$ rp $\overline{(f(\bar{A}))}$ (1)

Since $f(\bar{\mathrm{A}})$ is rp-closed in Y, rp $\overline{(f(\bar{A}))} = f(\bar{\mathrm{A}})$ (2)

Using (1) & (2) we have rp $\overline{(f(\mathrm{A}))} = f(\bar{\mathrm{A}})$ for every subset A of X.

Remark: Converse is not true in general as shown by the following example.

Example 3.5: Let $X = Y = \{a, b, c\}$ $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$ $\sigma = \{\varphi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f: X\to Y$ be the identity map. Then rp $\overline{(f(\mathrm{A}))} \subseteq f(\bar{\mathrm{A}})$ for every subset A of X. But f is not rp-closed since $f(\{c\}) = \{c\}$ is not rp-closed.

Corollary 3.2: If $f: X\to Y$ is r-closed then rp $\overline{(f(\mathrm{A}))} \subseteq f(\bar{\mathrm{A}})$.

Theorem 3.5: If $f: X\to Y$ is rp-closed and A$\subseteq X$ is closed, f(A) is τ_{rp}-closed in Y.

Proof: Let A$\subseteq X$ and $f{:}X\to Y$ be rp-closed $\Rightarrow$ rp $\overline{(f(\mathrm{A}))} \subseteq f(\bar{\mathrm{A}})$ (by theorem 3.4.) $\Rightarrow$ rp $\overline{(f(\mathrm{A}))} \subseteq f(\mathrm{A})$ since $f(\mathrm{A}) = f(\bar{\mathrm{A}})$ as A is closed. But $f(\mathrm{A}) \subseteq$ rp $\overline{(f(\mathrm{A}))}$. Therefore we have $f(\mathrm{A}) =$ rp $\overline{(f(\mathrm{A}))}$. Hence f(A) is τ_{rp}-closed in Y.

Corollary 3.3: If $f: X\to Y$ is r-closed, then f (A) is τ_{rp} closed in Y if A is r-closed set in X.

Theorem 3.6: If rp$((\bar{\mathrm{A}}))$= r$((\bar{\mathrm{A}}))$ for every A$\subseteq Y$ then the following are equivalent.
(i) $f{:}X\to Y$ is rp-closed map.
(ii) rp $\overline{(f(\mathrm{A}))} \subseteq f(\bar{\mathrm{A}})$.

Proof: (i) $\Rightarrow$(ii) follows from theorem 3.4.
(ii) $\Rightarrow$(i) Let A by any closed set in X. Then $f(\mathrm{A}) = f(\bar{\mathrm{A}}) \supset$ rp $\overline{(f(\mathrm{A}))}$ [by hypothesis] We have $f(\mathrm{A}) \subseteq$ rp $\overline{(f(\mathrm{A}))}$. Combining these two we have, $f(\mathrm{A}) =$ rp $\overline{(f(\mathrm{A}))}$ = r$\overline{(f(\mathrm{A}))}$ (by given condition) which implies f(A) is r-closed and hence f(A) is rp-closed. Thus for every closed set A in X, we have f(A) is rp-closed in Y. This implies f is rp-closed.

Theorem 3.7: $f{:}X\to Y$ is rp-closed iff for each subset S of Y and each open set U containing f^{-1}(S), there is an rp-open set V of Y such that S$\subseteq$V and f^{-1}(V) $\subseteq$ U.

Proof: Assume $f: X \to Y$ is rp-closed. Let $S \subseteq Y$ and U be an open set of X containing $f^{-1}(S)$. Then X-U is closed in X and $f(X\text{-U})$ is rp-closed in Y as f is rp-closed and $V = Y - f(X\text{-U})$ is rp-open in Y. $f^{-1}(S) \subseteq U \Rightarrow S \subseteq f(U) \Rightarrow S \subseteq V$ and $f^{-1}(V) = f^{-1}(Y - f(X\text{-U})) = f^{-1}(Y) - f^{-1}(f(X\text{-U})) = f^{-1}(Y) - (X\text{-U}) = X - (X\text{-U}) = U$

Conversely Let F be closed in $X \Rightarrow F^c$ is open. Then $f^{-1}(f(F^c)) \subseteq F^c$.

By hypothesis there exists an rp-open set V of Y, such that $f(F^c) \subseteq V$ and $f^{-1}(V) \supset F^c$ and so $F \subseteq [f^{-1}(V)]^c$. Hence $V^c \subseteq f(F) \subseteq f[f^{-1}(V)^c] \subseteq V^c \Rightarrow f(F) \subseteq V^c \Rightarrow f(F) = V^c$. Thus $f(F)$ is rp-closed in Y. Therefore f is rp-closed.

Remark: Composition of two rp-closed maps is not rp-closed in general.

Theorem 3.8: Let X, Y, Z be topological spaces and every rp-closed set is closed [r-closed] in Y. Then the composition of two rp-closed maps is rp-closed.

Proof: Let $f: X \to Y$ and $g: Y \to Z$ be rp-closed maps. Let A be any closed set in $X \Rightarrow f(A)$ is closed in Y (by assumption)] $\Rightarrow g(f(A))$ is rp-closed in $Z \Rightarrow g\ o\ f(A)$ is rp-closed in Z. Therefore $g\ o\ f$ is rp-closed.

Theorem 3.9: If $f: X \to Y$ is g-closed, g: $Y \to Z$ is rp-closed [r-closed] and Y is $T_{1/2}$ [r-$T_{1/2}$] then $g\ o\ f$ is rp-closed.

Proof: Let A be a closed set in X. Then $f(A)$ is g-closed set in $Y \Rightarrow f(A)$ is closed in Y as Y is $T_{1/2} \Rightarrow g(f(A))$ is rp-closed in Z since g is rp-closed $\Rightarrow g\ o\ f(A)$ is rp-closed in Z. Hence $g\ o\ f$ is rp-closed.

Corollary 3.4: If $f: X \to Y$ is g-closed, g: $Y \to Z$ is rp-closed [r-closed] and Y is $T_{1/2}$ [r-$T_{1/2}$] then $g\ o\ f$ is β-closed.

Theorem 3.10: If $f: X \to Y$ is rg-closed, g: $Y \to Z$ is rp-closed [r-closed] and Y is r-$T_{1/2}$, then $g\ o\ f$ is rp-closed.

Proof: Let A be a closed set in X. Then $f(A)$ is rg-closed in Y f (A) is r-closed in Y since Y is r-$T_{1/2} \Rightarrow f(A)$ is closed in Y since every r-closed set is closed $\Rightarrow g(f(A))$ is rp-closed in $Z \Rightarrow$ g o $f(A)$ is rp-closed in Z. Hence g o f is rp-closed.

Corollary 3.5: If $f: X \to Y$ is rg-closed, g: $Y \to Z$ is rp-closed [r-closed] and Y is r-$T_{1/2}$, then g o f is β-closed.

Theorem 3.11: If $f: X \to Y$, g: $Y \to Z$ be two mappings such that g o f is rp-closed [r-closed] then the following statements are true.

(i) If f is continuous [r-continuous] and surjective then g is rp-closed.
(ii) If f is g continuous, surjective and X is $T_{1/2}$ then g is rp-closed.
(iii) If f is g-continuous [rg-continuous], surjective and X is r-$T_{1/2}$ then g is rp-closed.

Proof: Let A be a closed set in Y. Then $f^{-1}(A)$ is closed in $X \Rightarrow (g\ o\ f)(f^{-1}(A))$ is rp-closed in $Z \Rightarrow g(A)$ is rp-closed in Z. Hence g is rp-closed.

Similarly we can prove the statements (ii) & (iii).

Corollary 3.6: If $f: X \to Y$, $g: Y \to Z$ be two mappings such that g o f is rp-closed [r-closed] then the following statements are true.

(i) If f is continuous [r-continuous] and surjective then g is β-closed.
(ii) If f is g continuous, surjective and X is $T_{1/2}$ then g is β-closed.
(iii) If f is g-continuous [rg-continuous], surjective and X is r-$T_{1/2}$ then g is β-closed.

Definition 3.2: X is said to be rp-regular space (or rp-T_3 space) if for a closed set F and a point $x \notin F$, there exists disjoint rp-open sets G and H such that $F \subseteq G$ and $x \in H$.

Theorem 3.12: If X is rp-regular, $f: X \to Y$ is r-open, r-continuous, rp-closed surjective and $\bar{A} = A$ for every rp-closed set in Y then Y is rp-regular.

Proof: Let $p \in U \in RPO(Y)$. Then there exists a point $x \in X$ such that $f(x) = p$ as f is surjective. Since X is rp-regular and f is r-continuous there exists $V \in RO(X)$ such that $x \in V \subseteq \overline{V} \subseteq f^{-1}(U)$ which implies $p \in f(V) \subseteq f(\overline{V}) \subseteq f(f^{-1}(U)) = U \to (1)$
Since f is rp-closed, $f(\overline{V}) \subseteq U$, By hypothesis $\overline{f(\overline{V})} = f(\bar{v})$ *and* $\overline{f(\bar{v})} = \overline{f(V)} \quad \to \quad (2)$

By (1) & (2) we have $p \in f(V) \subseteq f(\overline{V}) \subseteq U$ and $f(V)$ is rp-open. Hence Y is rp-regular.

Corollary 3.7: If X is rp-regular, $f: X\rightarrow Y$ is r-open, r-continuous, rp-closed, surjective and $\bar{A}=A$ for every r-closed set in Y then Y is rp-regular.

Theorem 3.13: If $f: X\rightarrow Y$ is rp-closed [r-closed] and A is a closed set of X then $f_A:(X, \tau(A))\rightarrow(Y, \sigma)$ is rp-closed.

Proof: Let F be a closed set in Y. Then $F = A \cap E$ for some closed set E of X and so F is closed in $X \Rightarrow f(A)$ is rp-closed in Y. But $f(F) = f_A(F)$ and therefore f_A is rp-closed.

Corollary 3.8: If $f: X\rightarrow Y$ is rp-closed [r-closed] and A is a closed set of X then $f_A:(X, \tau(A))\rightarrow(Y, \sigma)$ is β-closed.

Theorem 3.14: If $f: X\rightarrow Y$ is rp-closed [r-closed], X is $T_{1/2}$ and A is g-closed set of X then $f_A:(X, \tau(A))\rightarrow(Y, \sigma)$ is rp-closed.

Corollary 3.9: If $f: X\rightarrow Y$ is rp-closed [r-closed], X is $T_{1/2}$, A is g-closed set of X then $f_A:(X,\tau(A))\rightarrow(Y, \sigma)$ is β-closed.

Theorem 3.15: If $f_i:X_i\rightarrow Y_i$ be rp-closed [r-closed] for i =1, 2. Let $f: X_1xX_2\rightarrow Y_1xY_2$ be defined as $f(x_1,x_2)=(f_1(x_1),f_2(x_2))$. Then $f: X_1xX_2\rightarrow Y_1xY_2$ is rp-closed.

Proof: Let $U_1xU_2 \subseteq X_1xX_2$ where U_i is closed in X_i for i = 1,2. Then $f(U_1xU_2) = f_1(U_1)\, xf_2(U_2)$ is rp-closed set in Y_1xY_2. Then $f(U_1xU_2)$ is rp-closed set in Y_1xY_2. Hence f is rp-closed.

Corollary 3.10: If f_i: $X_i\rightarrow Y_i$ be rp-closed [r-closed] for i =1, 2. Let $f:X_1xX_2\rightarrow Y_1xY_2$ be defined as $f(x_1, x_2) = (f_1(x_1),f_2(x_2))$, then $f:X_1xX_2\rightarrow Y_1xY_2$ is β-closed.

Theorem 3.16: Let h: $X\rightarrow X_1xX_2$ be rp-closed [r-closed]. Let f_i: $X\rightarrow X_i$ be defined as $h(x) =(x_1, x2)$ and $f_i(x) =x_i$. Then f_i: $X\rightarrow X_i$ is rp- closed for i =1, 2.

Corollary 3.11: Let h: $X\rightarrow X_1xX_2$ be rp-closed [r-closed]. Let f_i: $X\rightarrow X_i$ be defined as $h(x) =(x_1, x2)$ and $f_i(x)=x_i$. Then f_i: $X\rightarrow X_i$ is β- closed for i =1, 2.

CONCLUSION: We studied some properties and inter relations of rp-closed mappings.

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REFERENCES:

[1] M. E. Abd. El. Monsef. S.N. El. Deep and R. A. Mohmoud, β-open sets and β-continuous mappings, Bull. Fac. Sci. Assiut Univ. A12 (1983) No.1, 77-90.

[2] D. Andrijevic, Semi pre-open sets, Mat. Vesnik 38 (1986) 24-32.

[3] K. Balachandran, P. Sundaram and H. Maki, On generalized continuous maps in topological spaces, Mem. Fac. Sci. Kochi Univ. Ser. A. Math 12 (1991); 5-13.

[4] W. Dunham, $T_{1/2}$- spaces, Kyungpook math. J. 1977; 17, 161-169.

[5] Erdal Ekici, Almost Contra-Pre continuous functions, Bull Malaysian Math. Sc. Soc 27 (2004) 53-65.

[6] N. Levine, Semi open sets and semi continuity in topological spaces, Amer. math. monthly 70 (1963). 36-41.

[7] N. Levine, Generalized closed sets in topological spaces, Rend. Civc. Mat. Palermo 19 (1970). 89-96.

[8] H. Maki, J. Umehara and K. Yamamura, Characterizations of $T_{1/2}$ spaces using generalized V-sets, Indian J. Pure and appl. math(1988) 634-640.

[9] A. S. Mashhour M.E. Abd. El- Monsef and S.N. El. Deep, On pre continuous and Weak pre continuous mappings, Proc. Math. Phy. Soc. Egypt 53 (1982) 47-53.

[10] O. Njastad, On some classes of nearly open sets, Pacific J. math. 109(1984) (2) 118-126.

[11] N. Palaniappan and K. Rao, Regular generalized closed sets, Kyungpook math. J. 33 (1993). 211-219.

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