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# APPLICATION OF PADE APPROXIMATION TO POWER SERIES SOLUTION OF NONLINEAR COUPLED DIFFERENTIAL EQUATIONS

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*ABSTRACT*

*Exact solutions for a class of nonlinear coupled differential equations of Boussinesq system are obtained by power series method. Singularities are identified by using Pade approximations.*

***Key words:*** *Power Series, Pade approximation, Coupled equations, Bousinesq fluid, Homogeneous balance method, Singularity.*

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## 1. INTRODUCTION

Different methods have been proposed for obtaining solitary wave solutions [1- 5] to non linear coupled differential equations, but there is no particular rule to obtain exact solution for nonlinear coupled differential equations. Yan [5], Wang [6, 7] and Fan [8], Yan [10] have solved coupled nonlinear differential equations using the idea of homogeneous balance method. Rajaraman [1] studied coupled nonlinear differential equations of quantum field theory which are given by,

$$\sigma_{xx} = -\sigma + \sigma^3 + d\rho^2\sigma, \tag{1}$$

$$\rho_{xx} = (f - d)\rho + \lambda\rho^3 + d\rho\sigma^2. \tag{2}$$

Where $\sigma$ and $\rho$ are real scalar fields and $d, f, \lambda$ are parameters. By using an orbit function, Rajaraman [1] found some soliton solutions for certain special values of $\lambda$. Wang [7] and Lou [3] have found two kinds of soliton solutions for equations (1) and (2) by ansatz method. Fan [4] and Yan [5] have obtained some special solutions for special values of $d, f, \lambda$.

Robert [9] have constructed an infinite family of rational solutions of the completely integrable variant of Boussinesq system [9] given by,

$$u_t + \rho_x + uu_x = 0, \tag{3}$$

$$\rho_t + u_x + u_{xxx} + (\rho u)_x = 0. \tag{4}$$

Using variable $H = 1 + \rho$ in equations (3) and (4) we get,

$$u_t + H_x + uu_x = 0, \tag{5}$$

$$H_t + u_{xxx} + (uH)_x = 0. \tag{6}$$

Where u is velocity & H is total depth.

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Dong [11] have obtained more general soliton solutions of equations (5) and (6) using a trigonometric functions transformation method by using homogeneous balance method.

Equations (5) and (6) are coupled nonlinear differential equations solved by Dong [11] and has obtained solution as,

$$H_1 = \frac{1}{2}\lambda^2 \sec h^2\left[\frac{1}{2}\lambda\left(x-\lambda t\right)+x_0\right], \tag{7}$$

$$u_1 = \lambda \pm \lambda \tanh\left[\frac{1}{2}\lambda\left(x-\lambda t\right)+x_0\right], \tag{8}$$

$$H_2 = \frac{1}{2}\lambda^2 - \lambda^2 \sec^2\left[\sqrt{\frac{1}{2}}\lambda\left(x-\lambda t\right)+x_0\right], \tag{9}$$

$$u_2 = \lambda \pm \sqrt{2}\lambda \sec\left[\sqrt{\frac{1}{2}}\lambda\left(x-\lambda t\right)+x_0\right]. \tag{10}$$

Equations (7) and (8) are soliton solutions and equations (9) and (10) are trigonometric function solutions.

Inspired by these solutions, we thought of a simple series solution will exist for equation (5) and (6). Thus in the following section, we have solved equations (5) and (6) by using series solution method and we have discussed the convergence and singularities by Pade approximation.

## 2. POWER SERIES SOLUTION

Let us introduce $\xi = x - t, H = H\left(\xi\right), u = u\left(\xi\right)$.

Using above transformation in equations (5) and (6) and integrating once we get

$$-H + uH + u_{\xi\xi} = 0, \tag{11}$$

$$-u + H + \frac{1}{2}u^2 = 0. \tag{12}$$

Solving (12) for H we get,

$$H = u - \frac{1}{2}u^2. \tag{13}$$

Using (13) in (11) we get,

$$2u_{\xi\xi} - u^3 + 3u^2 - 2u = 0. \tag{14}$$

Let us consider series solution of (14) as follows

$$u = \sum_{n=0}^{\infty} a_n \xi^n. \tag{15}$$

Substituting (15) in (14) and comparing the terms of same power we get recurrence relation as,

$$a_{n+2} = \frac{1}{2\left(n+1\right)\left(n+2\right)}\left\{\sum_{i=0}^{n} b_i a_{n-i} - 3b_n + 2a_n\right\}, \tag{16}$$

where

$$b_n = \sum_{i=0}^{n} a_i a_{n-i}. \tag{17}$$

The solution (10) obtained by Dong [11] suggests that we can choose initial condition for equations (11) and (12) as

$$\left.\begin{aligned} u\left(0\right) &= 3 \\ u'\left(0\right) &= 0 \end{aligned}\right\} \tag{18}$$

 

Using initial condition (18) in (15) we get series solution as follows

$$
\begin{aligned}
u = 3 &+ \frac{3\xi^2}{2} + \frac{11\xi^4}{16} + \frac{337\xi^6}{960} + \frac{18827\xi^8}{107520} + \frac{1690369\xi^{10}}{19353600} + \frac{222666539\xi^{12}}{5109350400} \\
&+ \frac{3676160723\xi^{14}}{169073049600} + \frac{9684188812523\xi^{16}}{892705701888000} + \frac{2956987291071361\xi^{18}}{546335889555456000} \\
&+ \frac{1121237821745822411\xi^{20}}{415215276062146560000} + \frac{5168974858087786870577\xi^{22}}{383658915081423421440000} \\
&+ \frac{1125350165842910937959\xi^{24}}{16741479930825749299200000} + \ldots\ldots\ldots \qquad (19)
\end{aligned}
$$

Fig1: shows exact power series solution of equation of u and fig2 shows exact solution of equation H.

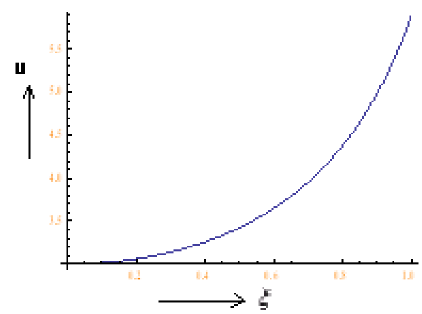


Fig. 1

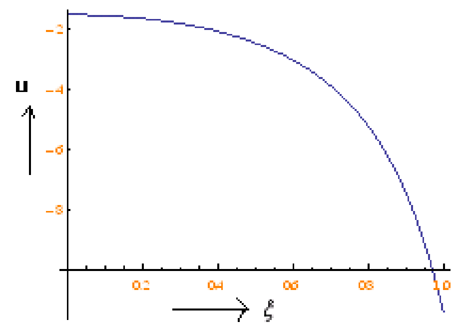


Fig. 2

## 3. NUMERICAL SOLUTION

We solve equations (11) and (12) numerically by using Runge-Kutta IV order method and graphs are drawn by using software MATHEMATICA as shown in fig 3 and fig 4.

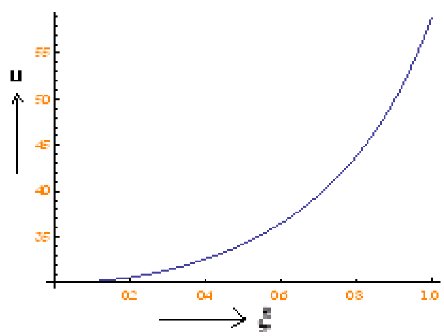


Fig. 3

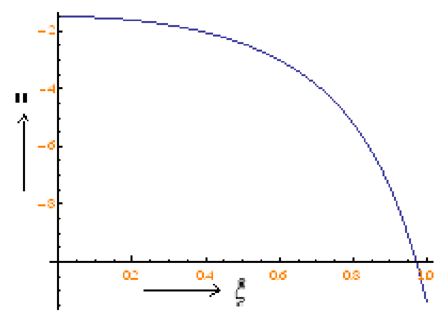


Fig 4

## 4. PADE APPROXIMATION

While solving equations (11) and (12) by using software MATHEMATICA we observed that there exist singularities for large values of $\xi$. To identify the singularity and reflect it in the graph we apply Pade. The Pade approximation for equation (19) is given in equation (20). In fig 5 and fig 6 there is no singularity in the range $0 \prec \xi \prec 1$ for u and H. Fig 7 and fig 8 shows that there are singularities at $\xi = 1.41$ for u and H respectively. Fig 9 and fig 10 shows that there are singularities at $\xi = 2.72$ for u and H respectively.

 

$$\left(3-\frac{342951674702421392287990993059\,\xi^{2}}{1562030604710470014357530841128}-\frac{13135693281079086719867819911385\xi^{4}}{159327121680467941464468145795056}\right.$$
$$+\frac{12552218571006990003636474857267\xi^{6}}{1108362585603255244970213188139520}-\frac{16673914537157602492799843527054379\xi^{8}}{167025808200068152396031246599873105920}$$
$$\left.-\frac{660791984731861875783879071120289971\,\xi^{10}}{12969062754358233009574190912460735283200}+\ldots\right)\Bigg/$$
$$\left(1-\frac{895332527256042137941429084917\xi^{2}}{1562030604710470014357530841128}+\frac{14312787233773650628348979846815\xi^{4}}{477981365041403824393404437385168}\right.$$
$$+\frac{10453724776425165011489594692103\xi^{6}}{3325087756809765734910639564418560}+\frac{118117595908738536201153128110378021\xi^{8}}{501077424600204457188093739799619317760}$$
$$\left.+\frac{11673822162059158331447638638197274703\,\xi^{10}}{661422200472269883488283736535497499443200}+\ldots\ldots\right) \tag{20}$$

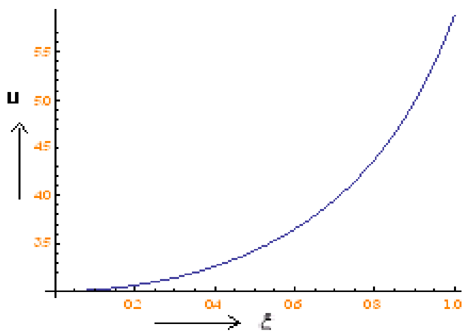


Fig. 5

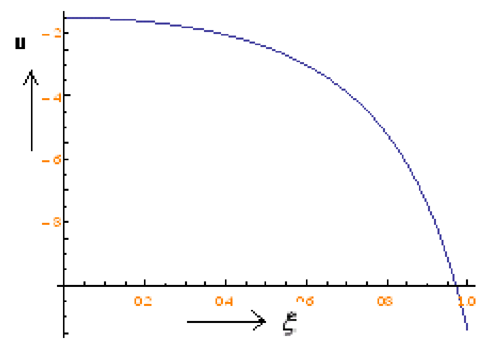


Fig. 6

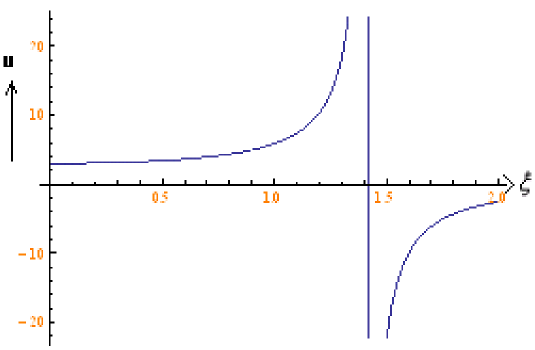


Fig. 7

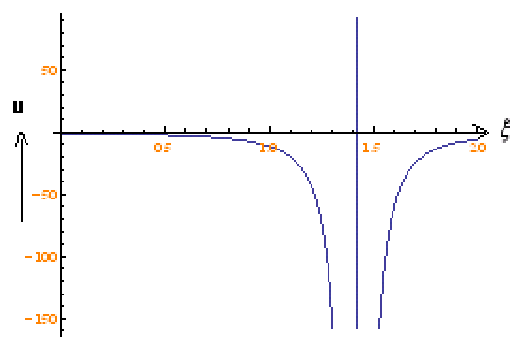


Fig. 8

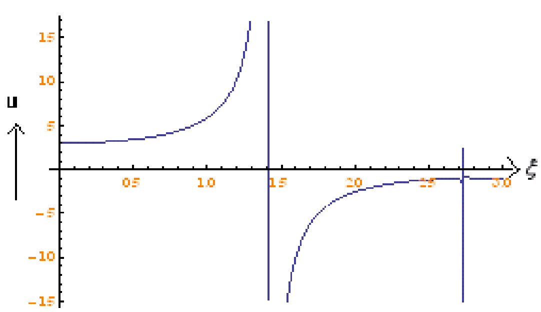


Fig. 9

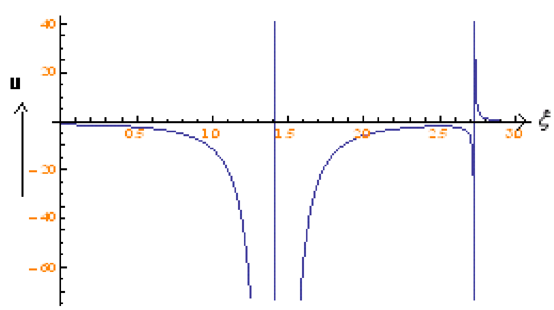


Fig. 10

 

## 5. DISCUSSION

Power series solution is a powerful and easiest tool to solve complicated nonlinear coupled differential equations. In this paper exact solutions for nonlinear coupled differential equations of Boussinesq system are obtained by power series method and applied Pade to find singularities and show the convergence. Fig 1 and 2 depict exact power series solution up to 25 terms and fig 3 and 4 are the graphs by numerical method which shows exact matching. In fig 5, 7 and 9 Singularities are identified by using Pade approximations for u graph. In fig 6, 8 and 10 we have shown that again same singularities are seen for H graph. In our future work we would like analyse these solutions about the singular points.

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