International Journal of Mathematical Archive-8(4), 2017, 28-36
Available online through www.ijma.info ISSN 2229 – 5046

SOME NEW CONCEPTS OF CONTINUITY IN TOPOLOGICAL SPACES

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(Received On: 15-03-17; Revised & Accepted On: 10-04-17)

ABSTRACT

*The purpose of this paper is to introduce and investigate several continuous functions namely gs_α** -continuous functions and contra gs_α** -continuous functions along with their several characterizations. Further we introduce new types of graphs called gs_α**-closed graphs, contra gs_α** -closed graphs and investigated several characterizations of such notions.*

MSC: *54C08, 54C05.*

Keywords: *gs_α** -continuous functions, contra gs_α** -continuous functions, gs_α**-closed graph, contra gs_α**-closed graph, locally gs_α**-indiscrete space.*

1. INTRODUCTION

In recent literature, we find many topologists have focused their research in the direction of investigating types of generalized continuity. The notion of contra-continuity was first investigated by Dontchev[7]. A good number of researchers have initiated different types of contra-continuous functions which are found in the papers [4],[5],[6]. In 1970, Levine [10] discussed the notion of generalized closed sets in topological spaces. Extensive research on generalizing closedness was done in recent years. In 1963, Levine [11] introduced the concepts of semi-open sets in topological spaces. W. Dunham [9] introduced the concept of generalized closure and defined a new topology τ* and investigated some of their properties. Quite recently the authors Robert.A and Pious Missier.S introduced and studied semi−open [15] sets and semi*α−open [15] sets using the generalized closure operator. Recently Santhini *et.al* [16] introduced gs_α** -closed sets in topological spaces. In 1969, Long [12] introduced closed graphs in topological spaces. In this paper, by means of gs_α**-closed sets, we introduce namely, gs_α**-continuous functions and contra gs_α**-continuous functions along with their several properties, characterizations and mutual relationships. Further we introduce new types of graphs, called gs_α**-closed graphs, contra gs_α**-closed graphs via gs_α**-open sets. Several characterizations and properties of such notions are investigated.

2. PRELIMINARIES

In this section, we recall some basic definitions and properties used in our paper.

Definition 2.1: A subset A of a space (X, τ) is said to be
- (i) semi-open [11] if A $\subseteq$ cl(intA).
- (ii) semi-open if [15] A $\subseteq$ cl_(intA).
- (iii) semi*α-open [15] if A $\subseteq$ cl_(αintA).
- (iv) a g-closed set [2] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

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(v) a ω-closed set [17] if cl(A) $\subseteq$ U whenever A and U is semi-open in X.
(vi) a generalized-semi closed set(briefly gs-closed) [5] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
(vii) a g*s -closed set[14] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is gs-open in X.
(viii) a generalized semi pre-closed set(briefly gsp-closed)[8] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

Definition 2.2: A subset A of a space (X, τ) is called generalized gs_α**-closed set (briefly gs_α**-closed) [16] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi*α-open in (X, τ).

The class of all gs_α**-open subsets of X is denoted by gs_α**O(X, τ) and the class of all gs_α**-open subsets of X containing x is denoted by gs_α**O(X,x).

Definition 2.3: A function f: (X, τ) → (Y, σ) is called a
(1) semi-continuous [11] if f^{-1}(V) is semi-closed set in (X, τ) for every closed set V in (Y, σ).
(2) semi*-continuous [13] if f^{-1} (V) is semi*-closed set in (X, τ) for every closed set V in (Y, σ).
(3) semi*α-continuous [15] if f^{-1} (V) is semi*α−closed set in (X, τ) for every closed set V in (Y, σ).
(4) g-continuous [2] if f^{-1} (V) is g-closed set in (X, τ) for every closed set V in (Y, σ).
(5) generalized semi-continuous(briefly gs-continuous) [5] if f^{-1} (V) is gs-closed set in (X, τ) for every closed set V in (Y, σ).
(6) generalized semi-precontinuous (briefly gsp-continuous) [8] if f^{-1}(V) is gsp-closed set in (X, τ) for every closed set V in (Y, σ).
(7) ω-continuous [17] if f^{-1} (V) is ω-closed set in (X, τ) for every closed set V in (Y, σ).
(8) g*s-continuous [14] if f^{-1} (V) is g*s-closed set in (X, τ) for every closed set V in (Y, σ).

Definition 2.4: A function f: (X, τ) → (Y, σ) is said to be
(1) contra-continuous [7] if f^{-1} (V) is closed in (X, τ) for every open set V in (Y, σ).
(2) contra semi-continuous [6] if f^{-1} (V) is semi-closed in (X, τ) for every open set V in (Y, σ).
(3) contra semi*-continuous [13] if f^{-1}(V) is semi*-closed in (X, τ) for every open set V in (Y, σ).
(4) contra semi*α-continuous [15] if f^{-1}(V) is semi*α -closed in (X, τ) for every open set V in (Y, σ).
(5) contra gs-continuous [3] if f^{-1} (V) is gs-closed in (X, τ) for every open set V in (Y, σ).
(6) contra gsp-continuous [1] if f^{-1} (V) is gsp-closed in (X, τ) for every open set V in (Y, σ).
(7) contra g-continuous [4] if f^{-1} (V) is g-closed in (X, τ) for every open set V in (Y, σ).
(8) contra g*s-continuous [14] if f^{-1} (V) is g*s-closed in (X, τ) for every open set V in (Y, σ).

Definition 2.5: A space X is locally indiscrete [18] if every open set in X is closed.

Definition 2.6:
(i) A space (X, τ) is called a ${}_\alpha T_{s**}$ -space [16] if every gs_α**-closed set in it is closed.
(ii) A space (X, τ) is called a T^α_{s**}-space [16] if every gs-closed set in it is gs_α**-closed.

3. gs_α**-Continuous and gs_α**-Irresolute functions

In this section, the concepts of gs_α**-continuity and gs_α**-irresoluteness are introduced and studied.

Definition 3.1: A function f: (X, τ) → (Y, σ) is called gs_α**-continuous if f^{-1} (V) is gs_α**-closed set in (X, τ) for every closed set V in (Y, σ).

Example 3.2: Let X =Y = {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}} and σ = {φ, Y, {a, b}, {a}}. Then f: (X, τ) →(Y, σ) defined by f(a) = c, f(b) = a, f(c) = b is gs_α**-continuous.

Theorem 3.3:
(1) Every continuous function is gs_α**-continuous.
(2) Every ω-continuous function is gs_α** -continuous.
(3) Every g*s-continuous function is gs_α**-continuous.
(4) Every semi-continuous function is gs_α**-continuous.
(5) Every semi*α-continuous function is gs_α**-continuous.
(6) Every gs_α**-continuous function is gs-continuous.
(7) Every gs_α**-continuous function is gsp-continuous.

Proof:
(1) Let V be a closed set in Y. Since, f is continuous, f^{-1} (V) is closed in X. By theorem 3.2 [16], f^{-1} (V) is gs_α**-closed in X and so f is gs_α**-continuous.
(2)-(7). Similar to the proof of (1).

Remark 3.4: The converses of the above theorems are not be true as seen from the following examples.

Example 3.5: Let X = Y = {a, b, c, d}, τ = {φ, X, {a}, {d}, {a, d}, {c, d}, {a, c, d}} and σ = {φ, Y, {a}, {b}, {a, b}, {a, b, c}}. Then f: (X, τ) → (Y, σ) defined by f(a) = b, f(b) = c, f(c) = d, f(d) = a is gs_α**-continuous but not continuous.

Example 3.6:.Let X = Y = {a, b, c}, τ = {φ, X, {a, b}, {b}} and σ = {φ, Y, {a, b}}. Then f: (X, τ) → (Y, σ) defined by f(a) = c, f(b) = a, f(c) = b is gs_α**-continuous but not ω-continuous.

Example 3.7: Let X = Y = {a, b, c}, τ = {φ, X, {b, c}, {a}} and σ = {φ, Y, {a, b}}. Then f: (X, τ) → (Y, σ) defined by f(a) = b, f(b) = c, f(c) = a is gs_α**-continuous but not g*s continuous.

Example 3.8: Let X = Y = {a, b, c}, τ = {φ, X, {b, c}, {a}} and σ = {φ, Y, {a}, {a, b}}. Then f: (X, τ) → (Y, σ) defined by f(a) = c, f(b) = a, f(c) = b is gs_α**-continuous but not semi-continuous.

Example 3.9:. Let X = Y = {a, b, c, d}, τ = {φ, X, {b, c, d}, {a, d}} and σ = {φ, Y, {a, b}}. Then f: (X, τ) → (Y, σ) defined by f(a) = f(b) = a, f(c) = b, f(d) = c is gs_α**-continuous but not semi*-continuous.

Example 3.10: Let X = Y = {a, b, c}, τ = {φ, X, {a, b}, {a}} and σ = {φ, Y, {a}, {b, c}}. Then f: (X, τ) → (Y, σ) defined by f(a) = c, f(b) = a, f(c) = b is gs-continuous but not gs_α**-continuous.

Example 3.11: Let X = Y = {a, b, c}, τ = {φ, X, {a, b}} and σ = {φ, Y, {a}, {a, b}}. Then f: (X, τ) → (Y, σ) defined by f(a) = c, f(b) = a, f(c) = b is gsp-continuous but not gs_α**-continuous.

Remark 3.12:. gs_α** -continuous and g-continuous functions are independent of each other.

Example 3.13: Let X = {a, b, c, d}, Y = {a, b, c}, τ = {φ, X, {b}, {a, b}} and σ = {φ, Y, {a, b}}. Then f: (X, τ) → (Y,σ) defined by f(a) = b, f(b) = f(c) = c, f(d) = a is g-continuous but not gs_α**-continuous.

Example 3.14: Let X = Y = {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}} and σ = {φ, Y, {b}, {a, b}}. Then f: (X, τ) →(Y, σ) defined by f(a) = b, f(b) = c, f(c) = d, f(d) = a is gs_α**–continuous but not g-continuous.

Remark 3.15: gs_α**-continuous and semi*α-continuous functions are independent of each other.

Example 3.16: Let X = {a, b, c, d}, Y = {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}, {a, b, c} and σ = {φ, Y, {b, c}, {a}}.Then f: (X, τ) → (Y, σ) defined by f(a) = f(d) = b, f(b) = a, f(c) = c is gs_α**-continuous but not semi*α-continuous.

Example 3.17: Let X = {a, b, c, d},Y = {a, b, c}, τ = {φ, X, {b}, {a, b}} and σ = {φ, Y, {a, b}}. Then f: (X, τ) → (Y,σ) defined by f(b) = f(c) = a, f(a) = c, f(d) = b is semi*α-continuous but not gs_α** -continuous.

4. Characterizations of gs_α**-continuous functions

Theorem 4.1: The following are equivalent for a function f: (X, τ) → (Y, σ). Assume that gs_α**O(X, τ) is closed under any union.

(i) f is gs_α**-continuous.
(ii) For each x ∈ X and each open set F in Y containing f(x), there exists a gs_α**-open set U in X containing x such that f(U) ⊆ F.

Proof:
(i)⇒(ii): Let x ∈ X and F be an open set in Y containing f(x). Since f is gs_α**-continuous, f^{-1} (F) is gs_α**-open in X containing x. Take U = f^{-1}(F) then U is a gs_α**-open set in X containing x such that f(U) ⊆ F.

(ii) ⇒ (i): Let F be an open set in Y such that x ∈ f^{-1} (F). Then F is an open set containing f(x). By (i), there exists a gs_α**-open set U_x in X containing x such that f(U) ⊆ F which implies U ⊆ f^{-1} (F). Therefore f^{-1} (F) = ∪ {U_x : x ∈ f^{-1} (F)}. Since U_x is gs_α**-open and gs_α**O(X,τ) is closed under any union. Hence f^{-1} (F) is open and so f is gs_α**-continuous.

Theorem 4.2: A function f: (X, τ) → (Y, σ) is gs_α**-continuous if and only if f^{-1} (V) is gs_α**-open in X for every open set V in Y.

Proof: Since f^{-1} (V^c) = (f^{-1} (V))^c, proof follows.

Remark 4.3: The composition of two gs_α**-continuous functions is not gs_α**-continuous.

Example 4.4: Let X = Y = Z = {a, b, c}, τ = {φ, X, {a}, {a, b}}, σ = {φ, Y, {a, b}} and μ = {φ, Z, {a}}. Then f: (X, τ) → (Y, σ) defined by f(a) = a, f(b) = c, f(c) = b and g: (Y, σ) → (Z, μ) defined by g(a) = b, g(b) = a, g(c)= c. Then f and g are gs_α**-continuous but g ∘ f: (X, τ) → (Z, μ) is not gs_α**-continuous.

Theorem 4.5: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, μ) be any functions. Then

(i) g ∘ f: (X, τ) → (Z, μ) is gs_α**-continuous if g is continuous and f is gs_α**-continuous.
(ii) g ∘ f: (X, τ) → (Z, μ) is gsp-continuous if g is continuous and f is gs_α**-continuous.

Proof:

(i) Let V be any closed set in Z. Since g is continuous, $g^{-1}(V)$ is closed in Y. Since f is gs_α**-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is gs_α**-closed set in X. Hence g ∘ f is gs_α**-continuous.
(ii) Similar to the proof of (i).

Theorem 4.6: Let X and Z be any topological spaces and Y be a ${}_\alpha T_{s^{**}}$ -space then the following hold.

(i) g ∘ f : (X, τ) → (Z, μ) is gs_α**-continuous if g is gs_α**-continuous and f is gs_α**-continuous.
(ii) g ∘ f : (X, τ) → (Z, μ) is semi-continuous if g is gs_α**-continuous and f is semicontinuous.
(iii) g ∘ f : (X, τ) → (Z, μ) is g*s-continuous if g is gs_α**-continuous and f is g*s-continuous.

Proof: (i) Let U be any closed set in Z. Since g is gs_α**-continuous, $g^{-1}(U)$ is gs_α**-closed in Y. But Y is a ${}_\alpha T_{s^{**}}$-space implies $g^{-1}(U)$ is closed in Y. Since f is gs_α** -continuous, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is gs_α**-closed in X and hence g ∘ f is gs_α**-continuous.
(ii)-(iii) similar to the proof of (i).

Theorem 4.7: If a function f: X →Y is gs_α**-continuous where X is a ${}_\alpha T_{s^{**}}$-space then f is continuous.(resp.semi-continuous)

Proof: Let V be a closed set in Y. Since f is gs_α**-continuous, $f^{-1}(V)$ is gs_α**-closed in X. Since X is a ${}_\alpha T_{s^{**}}$-space, $f^{-1}(V)$ is closed in X and so f is continuous.

Theorem 4.8: If a function f: X →Y is gs_α**-continuous where X is a ${}_\alpha T_{s^{**}}$-space then f is gs-continuous.

Proof: Let V be a closed set in Y. Since f is gs_α**-continuous, $f^{-1}(V)$ is gs_α**-closed in X. Since X is a ${}_\alpha T_{s^{**}}$-space, $f^{-1}(V)$ is closed in X By theorem 3.2[16], $f^{-1}(V)$ is gs-closed in X and so f is gs-continuous.

Definition 4.9: A function f: (X, τ) → (Y, σ) is called a gs_α**-irresolute if $f^{-1}(V)$ is gs_α**- closed set in (X,τ) for every gs_α**-closed set V in (Y, σ).

Example 4.10: Let X = Y = {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}, {b, c}} and σ = {φ, Y, {a, b}}. Then f: (X, τ) → (Y, σ) defined by f(a) = b, f(b) = a, f(c) = c is g gs_α** -irresolute.

Theorem 4.11:

(1) Every gs_α** -irresolute function is gs_α**-continuous.
(2) Every gs_α** -irresolute function is gs-continuous.
(3) Every gs_α**-irresolute function is gsp-continuous.

Proof:
(1) Let V be a closed set in Y. By theorem 3.2[16], V is gs_α**-closed in Y. Since f is gs_α**-irresolute, $f^{-1}(V)$ is gs_α**-closed set in X and so f is gs_α**-continuous.
(2)-(3) similar to the proof of (1).

Remark 4.12: The converses of the above theorems are not true as seen from the following example.

Example 4.13: Let X = Y = {a, b, c}, τ = {φ, X, {a, b}} and σ = {φ, Y, {a}, {a, b}}.
Then f: (X, τ) → (Y, σ) defined by f(a) = b, f(b) = a, f(c) = c, is gs_α**-continuous but not gs_α**-irresolute.

Example 4.14: Let X = Y = {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}, {b, c}} and σ = {φ, Y, {a}, {b, c}}. Then f: (X, τ) → (Y, σ) defined by f(a) = a, f(b) = c, f(c) = b, is gs-continuous but not gs_α**-irresolute.

Example 4.15: Let X = Y = {a, b, c}, τ = {φ, X, {a, b}} and σ = {φ, Y, {a}, {b, c}}.
Then f: (X, τ) → (Y, σ) defined by f(a) = a, f(b) = c, f(c) = b, is gsp-continuous but not gs_α**-irresolute.

Theorem 4.16: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, μ) be any functions. Then the following holds.

(i) g ∘ f: (X, τ) → (Z, μ) is gs_α**-irresolute if g is gs_α**-irresolute and f is gs_α**-irresolute.

(ii) g ∘ f: (X, τ) → (Z, μ) is gs_α**-continuous if g is gs_α**-continuous and f is gs_α**-irresolute.

Proof:

(i) Let V be gs_α**-irresolute in Z . Then g^{-1} (V) is gs_α**-closed in Y. Also f is gs_α**-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is gs_α**-closed set in X. Hence g ∘ f is gs_α**-irresolute.

(ii) Similar to the proof of (i).

Theorem 4.17: A function f: (X, τ) → (Y, σ) is gs_α**-irresolute if and only if $f^{-1}(V)$ is gs_α**-open in X for every gs_α**-open set V in Y.

Proof: Since $f^{-1}(V^c) = (f^{-1}(V))^c$, the proof follows.

Theorem 4.18: If a function f: X →Y is gs_α**-continuous where X is a ${}_\alpha T_{s^{**}}$ -space then f is gs_α**-irresolute.

Proof: Let U be a gs_α**-closed set in Y. Since Y is a ${}_\alpha T_{s^{**}}$-space, then U is closed in Y. By theorem 3.2 [16], U is gs_α**-closed set in Y. Since f is gs_α**-irresolute, $f^{-1}(U)$ is gs_α**-closed in X and so f is gs_α**-irresolute.

Theorem 4.19: Let X and Z be any topological spaces and Y be a ${}_\alpha T_{s^{**}}$-space then g ∘ f : (X, τ) → (Z, μ) is gs_α**-continuous if g is gs_α**-irresolute and f is gs_α**-continuous.

Proof: Let U be any closed set in Z. Since g is gs_α**-irresolute, $g^{-1}(U)$ is gs_α**-closed in Y. But X is a ${}_\alpha T_{s^{**}}$-space which implies $g^{-1}(U)$ is closed in Y. Since f is gs_α**-continuous, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is gs_α**-closed in X and hence g ∘ f is gs_α**-continuous.

Theorem 4.20: Let X and Z be any topological spaces and Y be a ${}^\alpha T_{s^{**}}$-space then g ∘ f: (X, τ) → (Z, μ) is gs_α**-continuous if g is gs-continuous and f is gs_α**-irresolute.

Proof: Let U be any closed set in Z. Since g is gs-continuous, $g^{-1}(U)$ is gs-closed in Y. But Y is a ${}^\alpha T_{s^{**}}$-space implies $g^{-1}(U)$ is gs_α**-closed in Y. Since f is gs_α** -irresolute, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is gs_α**-closed in X . Consequently g ∘ f is gs_α**-continuous.

5. Contra gs_α** -continuous functions

In this section, we define contra gs_α**-continuous functions and derives some of their properties.

Definition 5.1: A function f: X → Y is said to be contra gs_α** -continuous if $f^{-1}(V)$ is gs_α**-closed in X for every open set V in Y.

Example 5.2: Let X = Y = {a, b, c}, τ = {φ, X, {a}, {b, c}} and σ = {φ, Y, {a}}. Then f: (X, τ) → (Y, σ) defined by f(a) = c, f(b) = a, f(c) = b is a contra gs_α** -continuous.

Theorem 5.3: The following are equivalent for a function f: (X, τ) → (Y, σ).
Assume that gs_α**O(X, τ) is closed under any union.

(1) f is contra gs_α**-continuous.

(2) For every closed set F of Y, $f^{-1}(F)$ is gs_α**-open in X.

(3) For each x ∈ X and each closed set F of Y containing f(x), there exists gs_α**-open set U containing x in X such that f(U) ⊂ F.

Proof:

(1) ⟹(2): Let F be a closed set in Y. Then Y–F is an open set in Y. By (1), $f^{-1}(Y - F) = X - f^{-1}(F)$ is gs_α**-closed in X. which implies $f^{-1}(F)$ is gs_α**-open in X.

(2) ⟹ (1): Similar to the proof of (1).

(2) ⟹ (3): Let F be a closed set in Y containing f(x). Then x ∈ $f^{-1}(F)$. By (2), $f^{-1}(F)$ is gs_α**-open in X containing x.

Let U = $f^{-1}(F)$. Then U is gs_α**-open in X containing x and $f(U) = f(f^{-1}(F)) \subset F$.

(3) ⟹ (2): Let F be a closed set in Y containing f(x) which implies x ∈ $f^{-1}(F)$. From (3), there exists gs_α**-open set U_x in X containing x such that $f(U_x) \subset F$ which implies $U_x \subset f^{-1}(F)$. Therefore $f^{-1}(F) = \cup \{U_x : x \in f^{-1}(F)\}$, Since U_x is gs_α**-open and gs_α**O(X,τ) is closed under any union, $f^{-1}(F)$ is gs_α**-open in X.

Remark 5.4: Composition of two contra gs_α**-continuous function is not contra gs_α**-continuous.

Example 5.5: X = {a, b, c, d}, Y = Z = {a, b, c}, τ = {φ, X, {a, b}, {a, b, c}, {a, b, d}}, and σ = {φ, Y, {a}, {a, b}} and μ = {φ, Z, {a}}. Then f: (X, τ) → (Y, σ) defined by f(a) = f(b)= c, f(c) = b, f(d) = a and g: (Y, σ) → (Z, μ) defined by g(a) = b, g(b) = c, g(c) = a are gs_α**-continuous but g ∘ f: (X, τ) → (Z, μ) is not gs_α**-continuous.

Theorem 5.6:
(i) Every contra-continuous function is contra gs_α**-continuous.
(ii) Every contra semi-continuous function is contra gs_α**-continuous.
(iii) Every contra semi*-continuous function is contra gs_α**-continuous.
(iv) Every contra gs_α**-continuous function is contra gs-continuous.
(v) Every contra gs_α**-continuous function is contra gsp-continuous.

Proof:
(i) Let V be any open set in Y. Since f is contra-continuous, $f^{-1}(V)$ is closed in X. By theorem 3.2[16], $f^{-1}(V)$ is gs_α**-closed in X. Hence f is contra gs_α**irresolute.

(ii) - (v). Similar to the proof of (i).

Remark 5.7: The converses of the above theorems are not true as seen from the following examples.

Example 5.8: Let X = Y = {a, b, c}, τ = {φ, X, {a}, {a, b}} and σ = {φ, Y, {a}}. Then f: (X, τ) → (Y, σ) defined by f(a) = c, f(b) = a, f(c) = b is contra gs_α**-continuous but not contra-continuous.

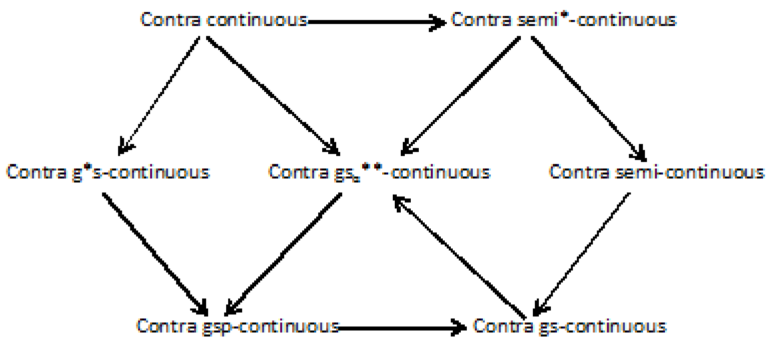
Example 5.9: Let X = {a, b, c, d}, Y = {a, b, c}, τ = {φ, X, {a, b}, {a, b, c}, {a, b, d}} and σ = {φ, Y, {a}, {a, b}}.Then f: (X, τ) → (Y, σ) defined by f(a) = f(d) = b, f(c) = a, f(b) =c is contra gs_α**-continuous but not contra semi-continuous.

Example 5.10: Let X = {a, b, c, d}, Y = {a, b, c}, τ = {φ, X, {c}, {a, d}, {a, c, d}} and σ = {φ, Y, {a}, {a, b}}. Then f: (X, τ) → (Y, σ) defined by f(a) = b, f(b) =a, f(c) = d, f(d) = c is contra gs_α**-continuous but not contra semi*-continuous.

Example 5.11: Let X = Y = {a, b, c}, τ = {φ, X, {a}, {a, b}} and σ = {φ, Y, {a, b}}. Then f: (X, τ) →(Y, σ) defined by f(a) = b, f(b) = c, f(c) = a is contra gs-continuous but not contra gs_α**-continuous.

Example 5.12: Let X = Y = {a, b, c}, τ = {φ, X, {a, b}} and σ = {φ, Y, {a}, {a, b}}. Then f: (X, τ) →(Y, σ) defined by f(a) = c, f(b) = a, f(c) = b is contra gsp-continuous but not contra gs_α**-continuous.

Remark 5.13: From the above results we have the following diagram.



In the above diagram A → B denotes A implies B but not conversely.

Remark 5.14: Contra g-continuous function and contra gs_α**-continuous functions are independent of each other.

Example 5.15: Let X = {a, b, c, d}, Y = {a, b, c}, τ = {φ, X, {a}, {d}, {a, d}, {c, d}, {a, c, d}} and σ = {φ, Y, {a, b}}.Then f: (X, τ) → (Y, σ) defined by f(a) = f(b) = a, f(c) = d, f(d) = b is contra g-continuous but not contra gs_α**-continuous.

Example 5.16: Let X = Y = {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}, {b, c}} and σ = {φ, Y, {a, b}}. Then f: (X, τ) → (Y, σ) defined by f(a) = c, f(b) = a, f(c) = b is contra gs_α**-continuous but not contra g-continuous.

Remark 5.17: Contra gs_α**-continuous function and contra semi*α-continuous functions are independent of each other.

Example 5.18: Let X = {a, b, c, d}, Y = {a, b, c}, τ = {φ, X, {a}, {b},{a, b}, {a, b, c}} and σ = {φ, Y, {a},{b, c}}.Then f: (X, τ) → (Y, σ) defined by f(a) = f(d) = b, f(b) = a, f(c) = c is contra gs_α**-continuous but not contra semi*α-continuous.

Example 5.19: Let X = {a, b, c, d}, Y = {a, b, c}, τ = {φ, X, {b}, {a, b}} and σ ={φ, Y, {a, b}}. Then f: (X, τ)→ (Y, σ) defined by f(a) = f(c) = a, f(a) = c, f(d) = b is contra semi*α-continuous but not contra gs_α**-continuous.

Theorem 5.20:

(i) If f: X → Y is gs_α**-continuous and h: Y → Z is contra-continuous then h ∘ f: X → Z is contra gs_α**-continuous.

(ii) If f: X → Y is contra gs_α**-continuous and h: Y → Z is continuous then h ∘ f: X → Z is contra gs_α**-continuous.

(iii) If f: X → Y is contra gs_α**-continuous and h: Y → Z is conrta-continuous then h ∘ f: X → Z is gs_α**-continuous.

Proof:

(i) Let V be an open set in Z. Since h is contra-continuous, $h^{-1}(V)$ is closed in Y. Since f is gs_α**-continuous, $f^{-1}(h^{-1}(V)) = (h \circ f)^{-1}(V)$ is gs_α**-closed in X and hence h ∘ f is gs_α**-continuous.

(ii) - (iii) Similar to the proof of (i).

Remark 5.21: The concept of gs_α**-continuity and contra gs_α**-continuity are independent.

Example 5.22: Let X = Y = {a, b, c}, τ = {φ, X, {a, b}, {a}} and σ = {φ, Y, {a, b}}. Then f: (X, τ) → (Y, σ) defined by f(a) = c, f(b) = a, f(c) = b is contra gs_α**-continuous but not gs_α**-continuous.

Example 5.23: Let X = Y = {a, b, c}, τ = {φ, X, {a, b}} and σ = {φ, Y, {a}, {a, b}}. Then f: (X, τ) → (Y, σ) defined by f(a) = b, f(b) = a, f(c) = c is gs_α**-continuous but not contra gs_α**-continuous.

Theorem 5.24: If f: (X, τ) → (Y, σ) is gs_α**-irresolute and g: (Y, σ) → (Z, μ) is a contra gs_α**-continuous function then g ∘ f: X → Y is contra gs_α**-continuous.

Proof: Let V be an open set in Z. Since g is contra gs_α**-continuous, $g^{-1}(V)$ is gs_α –closed in Y. Since f is contra gs_α**-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is gs_α**-closed in X and hence g ∘ f is contra gs_α**-continuous.

Theorem 5.25: If a function f: X → Y is contra gs_α**-continuous and Y is regular, then f is gs_α**-continuous.

Proof: Let x ∈ X and V be an open set in Y containing f(x). Since Y is regular there exists an open set W in Y containing f(x) such that cl(W) ⊂ V. Since f is contra gs_α**-continuous. By theorem 4.1, there exists gs_α**-open set V in X containing x such that f(U) ⊂ cl(W). Then f(U) ⊂ cl(W) ⊂ V. Therefore f is gs_α**-continuous.

Theorem 5.26: Let f: (X, τ) → (Y, σ) be a function and X is a ${}_\alpha T_{s**}$-space. Then the following are equivalent.

(i) f is contra semi-continuous.

(ii) f is contra gs_α**-continuous.

Proof:

(i) ⟹ (ii): By theorem 5.6, proof follows.

(ii) ⟹ (i): Let V be any open set in Y. Since f is contra gs_α**-continuous, $f^{-1}(V)$ is gs_α**-closed in X. Since X is ${}_\alpha T_{s**}$-space, $f^{-1}(V)$ is closed in X and hence $f^{-1}(V)$ is semi-closed in X f is contra semi-continuous.

Theorem 5.27: Let f: (X, τ) → (Y, σ) be a function and X is a ${}^\alpha T_{s**}$-space. Then the following are equivalent.

(i) f is contra gs_α**-continuous.

(ii) f is contra gs-continuous.

Proof: Similar to the proof of theorem 5.26.

Theorem 5.28: If f is gs_α**- continuous and if Y is locally indiscrete then f is contra gs_α** - continuous.

Proof: Let V be an open set in Y. Since Y is locally indiscrete, V is closed in X. Since f is gs_α**-continuous , $f^{-1}(V)$ is gs_α**- closed in X hence f is contra gs_α** -continuous.

Theorem 5.29: If a function f: $(X, \tau) \to (Y, \sigma)$ is continuous and X is locally indiscrete then f is contra gs_α** -continuous.

Proof: Let V be an open set in (Y, σ). Since f is continuous, $f^{-1}(V)$ is open in X. Since X is locally indiscrete, $f^{-1}(V)$ is closed set in X. By theorem 3.2[16], $f^{-1}(V)$ is gs_α** -closed in X and hence f is contra gs_α** -continuous.

Theorem 5.30: If a function f: $(X, \tau) \to (Y, \sigma)$ is contra gs_α** -continuous and X is a ${}_\alpha T_{s**}$ - space then f: $(X, \tau) \to (Y,\sigma)$ is contra gs-continuous.

Proof: Let V be an open set in Y. Since f is contra gs_α** -continuous, $f^{-1}(V)$ is gs_α** -closed in X. Since X is ${}_\alpha T_{s**}$-space, $f^{-1}(V)$ is closed in X and so gs-closed in X and hence f is contra gs_α** -continuous.

Definition 5.31: A space X is called locally gs_α** -indiscrete if every gs_α** -open set is closed in X.

Theorem 5.32: If a function f: $(X, \tau) \to (Y, \sigma)$ is gs_α** -continuous and the space X is locally gs_α**-indiscrete then f is contra continuous.

Proof: Let V be an open set in Y. Since f is gs_α** -continuous, $f^{-1}(V)$ is gs_α** -open in X. Since X is locally gs_α** -indiscrete, $f^{-1}(V)$ is closed in X and by theorem 3.2[16] ,$f^{-1}(V)$ is gs_α** -closed in X. Consequently f is contra gs_α** -continuous.

Theorem 5.33: If a function f: $(X, \tau) \to (Y, \sigma)$ is gs_α**-irresolute where Y is a locally gs_α**-indiscrete space and g: $(Y, \sigma) \to (Z, \mu)$ is contra gs_α** -continuous function then $g \circ f$ is gs_α** -continuous.

Proof: Let V be any closed set in Z. Since g is contra gs_α**-continuous, $g^{-1}(V)$ is gs_α**-open in Y. But Y is locally gs_α**-indiscrete implies $g^{-1}(V)$ is closed in Y. By theorem 3.2[16], $g^{-1}(V)$ is gs_α**-closed in Y. Since f is gs_α**-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is gs_α**-closed in X and hence $g \circ f$ is gs_α**-continuous.

Theorem 5.34: If a function f: $(X, \tau) \to (Y, \sigma)$ is gs_α**-continuous and the space (X, τ) is locally gs_α**-indiscrete space then f is contra gs_α**-continuous.

Proof: Let V be any open set in (Y, σ). Since f is gs_α**-continuous, $f^{-1}(V)$ is gs_α**-open in X. Since X is locally gs_α** -indiscrete, $f^{-1}(V)$ is closed in X. By theorem 3.2 [16], $f^{-1}(V)$ is gs_α** -closed set in X and hence f is contra gs_α**-continuous.

6. Contra gs_α** -closed graph

Definition 6.1: The graph G(f) of a function f: $X \to Y$ is said to be gs_α** -closed (resp.contra gs_α** -closed) if for each $(x, y) \in (X \times Y) - G(f)$, there exist an $U \in gs_\alpha$**O(X,x) and an open (resp.closed) set V in Y such that $(U \times V) \cap G(f) = \varphi$.

Lemma 6.2: A function f: $X \to Y$ is gs_α**-closed (resp.contra gs_α**-closed) if for each $(x, y) \in (X \times Y)-G(f)$ there exists $U \in gs_\alpha$**O(X,x) and an open set (resp.closed set) V in Y containing y such that $f(U) \cap V = \varphi$.

Proof: We shall prove that $f(U) \cap V = \varphi$ iff $(U \times V) \cap G(f) = \varphi$. Let $(U \times V) \cap G(f) \neq \varphi$. Then there exists $(x, y) \in (U \times V)$ and $(x, y) \in G(f)$ which implies $x \in U$, $y \in V$ and $y = f(x) \in V$. Therefore $f(U) \cap V \neq \varphi$.

Theorem 6.3: If a function f: $X \to Y$ is gs_α** -continuous and Y is a T_1-space then G(f) is contra gs_α**-closed in $X \times Y$.

Proof: Let $(x, y) \in (X \times Y)-G(f)$. Then $y \neq f(x)$. Since Y is T_1, there exists an open set V of Y such that $f(x) \in V$, $y \notin V$. Since f is gs_α**-continuous, by theorem 4.1 there exists a gs_α**-open set U of X containing x such that $f(U) \subset V$. Therefore $f(U) \cap (Y - V) = \varphi$ where $Y - V$ is closed in Y containing y. By lemma 6.2, G(f) is a gs_α**-closed graph in $X \times Y$.

Theorem 6.4: Let f: $X \to Y$ be a function and g: $X \times Y$ be the graph of f defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is contra gs_α**-continuous, then f is contra gs_α**- Continuous.

Proof: Let U be an open set in Y, then $X \times U$ is an open set in $X \times Y$. Since g is contra gs_α**-continuous,
$f^{-1}(U) = g^{-1}(X \times U)$ is gs_α**-closed in X. Thus f is contra gs_α**-continuous.

Definition 6.5:

(i) gs_α^{**} - T_0 if for every pair of distinct points x, y in X there exists a gs_α^{**} -open set U containing one of the points but not the other.

(ii) gs_α^{**} -T_1 if for every pair of distinct points x, y in X there exists a gs_α^{**} -open set U containing x not y and a gs_α^{**} -open set V containing y but not x.

(iii) gs_α^{**} -T_2 if for every pair of distinct points x, y in X there exists disjoint gs_α^{**} -open sets U and V containing x and y respectively.

Theorem 6.6: If f: $(X, \tau) \rightarrow (Y, \sigma)$ is an injective function with the gs_α^{**}-closed graph G(f) then X is gs_α^{**} -T_1.

Proof: Let x and y be two distinct points of X, then $f(x) \neq f(y)$. Thus $(x, f(y)) \in X \times Y\text{-}G(f)$. Since G(f) is gs_α^{**}-closed, there exists a gs_α^{**}-open set U containing x and an open set V containing f(y) such that $f(U) \cap V = \varphi$. By theorem 3.2 [16], U and V are gs_α^{**}-open sets containing x and f(y) such that $f(U) \cap V = \varphi$. Hence y $\in$ U. Similarly there exist gs_α^{**}-open sets M and N containing y and f(x) such that $f(M) \cap N = \varphi$. Hence $x \notin$ M. It follows that X is gs_α^{**}-T_1.

Theorem: 6.7: If f: $(X, \tau) \rightarrow (Y, \sigma)$ is an surjective function with the gs_α^{**}-closed graph G(f) then Y is gs_α^{**}-T_1.

Proof: Let y and z be two distinct points of Y. Since f is surjective there exist a point x in X such that f(x) = z. Therefore $(x, y) \notin G(f)$, by lemma 6.2, there exists a gs_α^{**}-open set U containing x and an open set V containing y such that $f(U) \cap V = \varphi$. By theorem 3.2[16], U and V are gs_α^{**}-open sets containing x and y such that $f(U) \cap V = \varphi$. It follows that $z \notin V$. Similarly there exist $w \in X$ such that f(w) = y. Hence $(w,z) \notin G(f)$. Similarly there exist gs_α^{**}-open sets M and N containing w and z respectively such that $f(M) \cap N = \varphi$. Thus $y \notin$ N. Hence the space Y is gs_α^{**}-T_1.

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Source of support: Nil, Conflict of interest: None Declared.