

**$(\sigma - \tau)$  - DERIVATIONS OF NEAR-FIELD SPACES OVER A NEAR-FIELD**

**Dr. N. V. NAGENDRAM\***

**Professor of Mathematics,  
 Kakinada Institute of Technology & Science (K.I.T.S.)  
 Department of Humanities & Science (Mathematics),  
 Tirupathi (vill) Peddapuram (M), Divili 533 433  
 East Godavari District. Andhra Pradesh. INDIA.**

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**ABSTRACT**

*Let  $N$  be a left-near-field space and let  $\sigma, \tau$  be automorphisms of  $N$ . An additive mapping  $d: N \rightarrow N$  is called a  $(\sigma, \tau)$  – derivation on  $N$  if  $d(xy) = \sigma(x) d(y) + d(x)\tau(y)$  for all  $x, y \in N$ . In this paper, Dr N V Nagendram as author obtain Leibnitz' formula for  $(\sigma, \tau)$  – derivations on near-field spaces over a near-field which facilitates the proof of the following result. Let  $n \geq 1$  be an integer,  $N$  be a  $n$ -torsion free and  $d$  a  $(\sigma, \tau)$  – derivation on  $N$  with  $d^n(N) = \{0\}$ . If both  $\sigma, \tau$  commute with  $d^n$  for all  $n \geq 1$ , then  $d(z) = \{0\}$ . Further, besides proving some more related results, we investigate commutativity of  $N$  satisfying either of the properties  $d([x, y]) = 0$ , or  $d(x\sigma y) = 0$ , for all  $x, y \in N$  a near-field space over a near-field.*

**Keywords:** prime near-field space, near-field space,  $(\sigma, \tau)$  – derivation, sub near-field space, sigma automorphism, and tow automorphism of near-field space.

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**SECTION 1: INTRODUCTION**

Throughout the paper,  $N$  will denote a zero-symmetric left (or right) prime near-field space over a near-field with multiplicative centre  $Z$ . For any  $x, y \in N$  as usual  $[x, y] = xy - yx$  and  $x\sigma y = xy + yx$  will denote the well known Lie and Jordan products respectively. While the symbol  $(x, y)$  will denote the additive Commutator  $x + y - x - y$ .

There are several results asserting that prime near-field spaces over a near-field with certain constrained derivations have near-field like behaviour over a near-ring. Recently many authors have studied commutativity of prime and semi prime near-fields with derivations. In view of these results it is natural to look for comparable results on near-field spaces. In order to facilitate our discussion we need to extend Leibnitz' theorem for derivations in near-field spaces to  $(\sigma, \tau)$  – derivation to prime near-field spaces over a near-field.

Proving Leibnitz' formula for  $(\sigma, \tau)$  – derivations in near-field spaces over a near-field Dr N V Nagendram extend some results due for  $(\sigma, \tau)$  – derivations on prime near-field spaces over a near-field. Some new results have also been obtained for prime near-field spaces. Finally, it is shown that under appropriate additional hypothesis a prime near-field space must be a commutative near-field space over a near-field.

**Definition 1.1: Prime near-field space over a near-field.** A near-field space  $N$  is said to be prime near-field space if  $aNb = \{0\} \Rightarrow a = 0$  or  $b = 0$ .

**Definition 1.2: Distributive element.** An element  $x$  of  $N$  is said to be distributive element if  $(y + z)x = yx + zx$  for all  $x, y, z \in N$ .

**Definition 1.3: zero symmetric.** A near-field space  $N$  is called zero-symmetric if  $ox = 0$  for all  $x \in N$ .

**Note 1.4:** recall that left distributivity yields  $x0 = 0$ .

**Corresponding Author: Dr. N. V. Nagendram\***  
**E-mail ID: [nvn220463@yahoo.co.in](mailto:nvn220463@yahoo.co.in)**

**Definition 1.5: derivation on N.** An additive endomorphism  $d$  of  $N$  is called a derivation on  $N$  if  $d(xy) = xd(y) + d(x)y$  for all  $x, y \in N$  or equivalently that  $d(xy) = d(x)y + xd(y)$  for all  $x, y \in N$ .

**Definition 1.6: constant.** An element  $x \in N$  for which  $d(x) = 0$  is called a constant.

**Definition 1.7:  $(\sigma, \tau)$  - derivation.** Let  $\sigma, \tau$  be two automorphisms on a near-field space  $N$  over a near-field. Define an additive endomorphism  $d : N \rightarrow N$  is called a  $(\sigma, \tau)$  - derivation if

$$\exists \text{ automorphism } \sigma, \tau : N \rightarrow N \ni d(xy) = \sigma(x) d(y) + d(x) \tau(y) \text{ for all } x, y \in N.$$

**Definition 1.8:  $\tau$ -derivation.** If  $\sigma = 1$ , the identity mapping  $d$  is called a  $\tau$ -derivation.

**Definition 1.9:  $\sigma$ -derivation.** If  $\tau = 1$ , the identity mapping  $d$  is called a  $\sigma$ -derivation.

## SECTION 2: PRELIMINARY RESULTS

In this section begin the following known results.

**Lemma 2.1:** An additive endomorphism  $d$  on a near-field space  $N$  is a  $(\sigma, \tau)$  - derivation if and only if  $d(xy) = d(x) \tau(y) + \sigma(x) d(y)$  for all  $x, y \in N$ .

**Lemma 2.2:** Let  $d$  be a  $(\sigma, \tau)$  - derivation on a near-field space  $N$  over a near-field. Then  $N$  satisfies the following partial distributive laws:

- (a)  $(\sigma(x) d(y) + d(x) \tau(y))z = \sigma(x) d(y)z + (d(x) \tau(y))z, \forall x, y, z \in N.$
- (b)  $(d(x) \tau(y) + \sigma(x) d(y))z = d(x) \tau(y)z + \sigma(x) d(y)z, \forall x, y, z \in N.$

**Lemma 2.3:** Let  $N$  be a prime near-field space admitting a non-trivial  $(\sigma, \tau)$  derivation  $d$  for which  $d(N) \subseteq Z$ . Then  $(N, +)$  is abelian. Moreover, if  $N$  is 2-torsion free and  $\sigma, \tau$  commute with  $d$ . Then  $N$  is a commutative near-field space over a near-field.

**Lemma 2.4:** Let  $N$  be a prime near-field space over a near-field. If  $d$  is a  $(\sigma, \sigma)$  - derivation on  $N$ , the  $d(Z) \subseteq Z$ .

## SECTION 3: MAIN RESULTS

The following theorem has its independent interest in the study of  $(\sigma, \tau)$  - derivations in near-field spaces. In fact Leibnitz' formula has already been obtained by Dr N V Nagendram for derivations in near-field spaces over a near-field. Now, we shall extend this result for  $(\sigma, \tau)$  - derivations in prime near-field spaces over a near-field.

**Theorem 3.1:** Let  $N$  be a near-field space and  $d$  a  $(\sigma, \tau)$  - derivation on  $N$ . If both  $\sigma, \tau$  commute with  $d^n$ , for all positive integer  $n \geq 1$ , then for all  $x, y \in N$   $d^n(xy) = \sum_{r=0}^n {}^nC_r a^{n-r}(a^r(x))d^n(r^{n-r}(y)).$

**Proof:** By An additive endomorphism  $d$  on a near-field space  $N$  is a  $(\sigma, \tau)$  - derivation we have

$$d(xy) = d(x) \tau(y) + \sigma(x) d(y) \quad \forall x, y \in N. \quad (a)$$

$$\Rightarrow n d(x) \tau(y) + n \sigma(x) d(y) = n (d(x) \tau(y) + \sigma(x) d(y)) \quad \forall x, y \in N. \quad (b)$$

Now we apply induction on  $n$ . when  $n = 2$ , we get

$$\begin{aligned} d^2(xy) &= d(d(xy)) = d(d(x) \tau(y) + \sigma(x) d(y)) \\ &= d^2(x) \tau^2(y) + \sigma(d(x)) d(\tau(y)) + d(\sigma(x) d(y)) + \sigma^2(x) d^2(y) \end{aligned} \quad (c)$$

Since  $\sigma$  and  $\tau$  commute with  $d$ , equation (c) reduces to

$$d^2(xy) = d^2(x) \tau^2(y) + 2 \sigma(d(x)) d(\tau(y)) + \sigma^2(x) d^2(y) \quad \forall x, y \in N.$$

$$\Rightarrow d^2(xy) = \sum_{r=0}^2 {}^2C_r d^{2-r}(\sigma^r(x))d^r(\tau^{2-r}(y)) \quad \forall x, y \in N. \quad (d)$$

Assume that Leibnitz' rule holds good for  $n-1$ , then

$$d^{n-1}(xy) = \sum_{r=0}^{n-1} {}^{n-1}C_r d^{n-r-1}(\sigma^r(x))d^r(\tau^{n-r-1}(y)) \quad \forall x, y \in N. \quad (e)$$

i.e.

$$d^{n-1}(xy) = d^{n-1}(x)\tau^{n-1}(y) + \dots + {}^{n-1}C_{i-1}d^{m-i}(\sigma^{i-1}(x)d^{n-1}(\tau^{n-i}(y))) + {}^{n-1}C_i d^{m-i-1}((\sigma^i(x))d^i(\tau^{n-i-1}(y))) \\ + \dots + \sigma^{n-1}(x)d^{m-1}(y) \quad \text{for all } x, y \text{ in } N. \quad (f)$$

$$d^m(xy) = d(d^{m-1}(xy)) \\ = d(d^{m-1}(x)\tau^{n-1}(y) + \dots + {}^{n-1}C_{i-1}d^{m-i}(\sigma^{i-1}(x)d^{n-1}(\tau^{n-i}(y))) + {}^{n-1}C_i d^{m-i-1}((\sigma^i(x))d^i(\tau^{n-i-1}(y))) \\ + \dots + \sigma^{n-1}(x)d^{m-1}(y)) \quad \text{for all } x, y \text{ in } N. \\ = d(d^{m-1}(x)\tau^{n-1}(y)) + \dots + {}^{n-1}C_{i-1}d(d^{m-i}(\sigma^{i-1}(x)d^{n-1}(\tau^{n-i}(y)))) + {}^{n-1}C_i d(d^{m-i-1}((\sigma^i(x))d^i(\tau^{n-i-1}(y)))) \\ + \dots + \sigma^{n-1}(x)d(d^{m-1}(y))) \quad \text{for all } x, y \text{ in } N.$$

$$\Rightarrow d^n(xy) = \sum_{r=0}^n {}^nC_r a^{n-r}(a^r(x))d^n(r^{n-r}(y)) \quad \text{for all } x, y \in N.$$

This completes the proof of the theorem.

**Corollary 3.2:** Let  $N$  be a near-field space. If  $N$  admits a derivation  $d$ , then for any integer  $n \geq 1$  and for all  $x, y \in N$ ,

$$\text{we have } a^m(xy) = \sum_{r=0}^n {}^nC_r a^{m-r}(x) a^r(y), \text{ where } 0 \leq r \leq n.$$

**Theorem 3.3:** Let  $n \geq 1$  be a fixed positive integer and let  $N$  be an  $n$ -torsion free near-field space. Suppose that  $\sigma, \tau$  are automorphisms of  $N$  and  $d$  a  $(\sigma, \tau)$  - derivation on  $N$  such that  $\sigma, \tau$  commute with  $d^k$  for all integers  $k \geq 1$ . If  $d^m(N) = \{0\}$ , then for each  $x \in N$ , either  $d(x) = 0$  or there exists an integer  $i, 0 < i < n$  such that  $a^n(x)$  is a non-zero divisor of zero.

**Proof:** The result is obvious for  $n = 1$ . By hypothesis, we have  $d^n(N) = \{0\}$ . We may assume that  $d^{m-1}(N) \neq \{0\}$  with  $d^{m-1}(x_0) \neq \{0\}$ , for some  $x_0 \in N$ . further, suppose that  $d(x) \neq 0$ . Then there exists  $i$  with  $0 < i < n$  for which  $d^n(x) \neq 0$  and  $d^{n+1}(x) = 0$ . This gives rise to  $d^m(x_0 d^{n-1}(x)) = 0$  for all  $x \in N$ . we find that  $nd^{m-1}(\sigma(x_0)d(\tau^{n-1}(d^{n-1}(x)))) = 0$  for all  $x \in N$ .

$$\Rightarrow nd^{m-1}(\sigma(x_0))d(\tau^{n-1}(d^n(x))) = 0 \text{ for all } x \in N.$$

Since  $\tau$  is an automorphism of  $N$  and  $N$  is a  $n$ -torsion free, the above expression yields that  $\sigma(d^{m-1}(x_0))\tau^{n-1}(d^n(x)) = 0$  for all  $x \in N$ . thus it follows that  $(\tau^{n-1})^{-1}(\sigma(d^{m-1}(x_0))d^n(x)) = 0$  for all  $x \in N$ . Since  $\sigma$  and  $\tau$  are automorphisms of  $N$  and  $d^{m-1}(x_0) \neq \{0\}$ , it follows that  $d^n(x)$  is a non-zero divisor of zero. Hence This completes the proof of the theorem.

**Theorem 3.4:** Let  $n \geq 1$  be a fixed positive integer and let  $N$  be an  $n$ -torsion free prime near-field space over a near-field. Suppose that  $\sigma$  is an automorphisms of  $N$  and  $d$  a  $(\sigma, \tau)$  - derivation on  $N$  such that  $\sigma$  commute with  $d^k$  for all integers  $k \geq 1$ . If  $d^m(N) = \{0\}$ , then  $d(Z) = \{0\}$ .

**Proof:** The result is obvious for  $n = 1$ . Now let  $n \geq 2$  and  $d(Z) \neq \{0\}$ . Choose  $z \in Z$  such that  $d(z) \neq 0$ . By Let  $N$  be a prime near-field space over a near-field. If  $d$  is a  $(\sigma, \sigma)$  - derivation on  $N$ , the  $d(Z) \subseteq Z$  and let  $n \geq 1$  be a fixed positive integer and let  $N$  be an  $n$ -torsion free near-field space. Suppose that  $\sigma, \tau$  are automorphisms of  $N$  and  $d$  a  $(\sigma, \tau)$  - derivation on  $N$  such that  $\sigma, \tau$  commute with  $d^k$  for all integers  $k \geq 1$ . If  $d^m(N) = \{0\}$ , then for each  $x \in N$ , either  $d(x) = 0$  or there exists an integer  $i, 0 < i < n$  such that  $a^n(x)$  is a non-zero divisor of zero, there exists a positive integer  $i, 0 < i < n$  such that  $d^n(z)$  is a non-zero divisor of zero contained in the center  $Z$ . Since  $N$  is a prime near-field space over a near-field  $d^n(z)$  cannot be a zero divisor. This contradiction shows that  $d(Z) = \{0\}$ . This completes the proof of the theorem.

**Counter example 3.5:** The conclusion of the above result need not be true even for arbitrary near-field spaces over a near-field with  $\sigma = 1$  the identity mapping on near-field space  $N$  over a near-field.

$$\text{Let } T \text{ be any near-field space. Let } N = \left\{ \begin{pmatrix} 0 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} / a, b \in T \right\}. \text{ Define a mapping } d : N \rightarrow N \text{ such that}$$

$d \begin{pmatrix} 0 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ . Then it can be easily seen that  $d$  is a derivation on near-field space  $N$  such that  $d^2(N) = \{0\}$ , but  $d(Z) \neq \{0\}$ .

#### SECTION 4: COMMUTATIVITY OF PRIME NEAR-FIELD SPACES OVER A NEAR-FIELD.

In this section, Dr N V Nagendram studied commutativity in prime near-field spaces over a near-field with a non-zero derivation  $d$  for which  $d(xy) = d(yx)$  for all  $x, y$  in some non-zero one sided sub-prime near-field space over a near-field. We continue this study and obtain some more general results for  $(\sigma, \tau)$  – derivations in prime near-field spaces over a near-field.

**Theorem 4.1:** Let  $N$  be 2-torsion free prime near-field space over a near-field. Suppose  $d$  is a non-zero  $(\sigma, \sigma)$  – derivation on  $N$  such that  $d(x, y) = 0$  for all  $x, y \in N$ . Then  $N$  is a commutative near-field space.

**Proof:** By hypothesis we have,  $d(xy) = d(yx)$ , for all  $x, y \in N$ . (i)

Equation (i) can be written as

$$\sigma(x) d(y) + d(x) \sigma(y) = \sigma(y) d(x) + d(y) \sigma(x), \text{ for all } x, y \in N. \quad (\text{ii})$$

Replacing  $x$  by  $yx$  in equation (ii) and using equation (i) we obtain,

$$\begin{aligned} \sigma(yx) d(y) + d(yx) \sigma(y) &= \sigma(y) d(xy) + d(y) \sigma(yx) \text{ for all } x, y \in N. \\ \Rightarrow \sigma(yx) d(y) + (\sigma(y) d(x) + d(y) \sigma(x)) \sigma(y) &= \sigma(yx) d(y) + \sigma(y) d(x) \sigma(y) + d(y) \sigma(x) \text{ for all } x, y \in N. \end{aligned} \quad (\text{iii})$$

Let  $d$  be a  $(\sigma, \tau)$  – derivation on a near-field space  $N$  over a near-field. Then  $N$  satisfies the above said partial distributive laws yields that

$$d(y) \sigma(x) \sigma(y) = d(y) \sigma(y) \sigma(x), \text{ for all } x, y \in N. \quad (\text{iv})$$

Again replace  $x$  by  $xz$  in equation (iv) and use equation (iv) to get

$$d(y) \sigma(x) \sigma(z) \sigma(y) = d(y) \sigma(xy) \sigma(z), \text{ for all } x, y, z \in N. \quad (\text{v})$$

$$\Rightarrow d(y) \sigma(x) \sigma(z) \sigma(y, z) = 0 \text{ for all } x, y, z \in N. \quad (\text{vi})$$

Since  $\sigma$  is an automorphism of  $N$ , we get

$$\sigma^{-1}(d(y))N[y, z] = \{0\}, \text{ for all } y, z \in N. \quad (\text{vii})$$

This yields that for each fixed point  $y \in N$  either  $d(y) = 0$  or  $[y, z] = 0$ , for all  $z \in N$  i.e. for each fixed  $y \in N$ , we have either  $d(y) = 0$  or  $y \in Z$ . But  $y \in Z$  also implies that  $d(y) \in Z$ , for all  $y \in N$ .

$\therefore$  in both the cases we find that  $d(y) \in Z$ , for all  $y \in N$  and hence  $d(N) \subseteq Z$ .

Thus by, a prime near-field space admitting a non-trivial  $(\sigma, \tau)$  – derivation  $d$  for which  $d(N) \subseteq Z$ . Then  $(N, +)$  is abelian. Moreover, if  $N$  is 2-torsion free and  $\sigma, \tau$  commute with  $d$ . Then  $N$  is a commutative near-field space over a near-field. This completes the proof of the theorem.

**Theorem 4.2:** Let  $N$  be a 2-torsion free prime near-field space. Suppose  $d$  is a non-zero  $(\sigma, \sigma)$  – derivation on  $N$  such that  $d(xoy) = 0 \forall x, y \in N$ . Then  $N$  is a commutative near-field space.

**Proof:** we have  $d(xy) = -d(yx) \forall x, y \in N$ . (viii)  
i.e.

$$\sigma(x) d(y) + d(x) \sigma(y) = -(\sigma(y) d(x) + d(y) \sigma(x)), \forall x, y \in N. \quad (\text{ix})$$

Replacing  $x$  by  $yx$  in equation (ix) we get,

$$\sigma(yx) d(y) + d(yx) \sigma(y) = -(\sigma(y) d(yx) + d(y) \sigma(yx)), \forall x, y \in N. \quad (\text{x})$$

By using equation (viii) and  $N$  satisfies partial distributive laws we find that

$$\begin{aligned} \sigma(yx) d(y) + (\sigma(y) d(x) + d(y) \sigma(x)) \sigma(y) &= -(\sigma(y) (-d(xy)) + d(y) \sigma(yx)) \\ &= -(\sigma(-y) d(xy) + d(y) \sigma(yx)) \end{aligned}$$

$$\begin{aligned} &= -(\sigma(y) d(xy) + d(y) \sigma(yx)) \\ &= (\sigma(y)(\sigma(x) d(y) + d(x) \sigma(y)) - d(y) \sigma(yx)) \\ &= \sigma(y)(\sigma(x) d(y) + \sigma(y) d(x) \sigma(y) - d(y) \sigma(yx)). \end{aligned}$$

$$\Rightarrow d(y) \sigma(x) \sigma(y) = -d(y) \sigma(yx), \text{ for all } x, y \in N. \quad (\text{xi})$$

Again replace  $x$  by  $xz$  in equation (xi) and use (xi) we obtain

$$d(y) \sigma(x) \sigma(z) \sigma(y) = d(y) \sigma(x) \sigma(y) \sigma(z), \text{ for all } x, y, z \in N. \quad (\text{xii})$$

i.e.

$$d(y) \sigma(x) \sigma([y, z]) = 0, \text{ for all } x, y, z \in N. \quad (\text{xiii})$$

This yields that for each fixed point  $y \in N$  either  $d(y) = 0$  or  $[y, z] = 0$ , for all  $z \in N$  i.e. for each fixed  $y \in N$ , we have either  $d(y) = 0$  or  $y \in Z$ . But  $y \in Z$  also implies that  $d(y) \in Z$ , for all  $y \in N$ .

$\therefore$  in both the cases we find that  $d(y) \in Z$ , for all  $y \in N$  and hence  $d(N) \subseteq Z$ .

Thus by, a prime near-field space admitting a non-trivial  $(\sigma, \tau)$  - derivation  $d$  for which  $d(N) \subseteq Z$ . Then  $(N, +)$  is abelian. Moreover, if  $N$  is 2-torsion free and  $\sigma, \tau$  commute with  $d$ . Then  $N$  is a commutative near-field space over a near-field. We get the required result and this completes the proof of the theorem.

**Corollary 4.3:** Let  $N$  be a 2 - torsion free prime near-field space over a near-field. Suppose that  $d$  is a non-zero derivation on  $N$  such that  $d((x, y)) = 0$  for all  $x, y \in N$  or  $d(xoy) = 0$  for all  $x, y \in N$ . Then  $N$  is a commutative near-field space over a near-field.

**Note 4.4:** the following example demonstrates that the conclusion of the above results need not be true even in the case of arbitrary near-fields over near-rings.

**Example 4.5:** Let  $N = N_1 \oplus N_2$  where  $N_1$  is a non-commutative near-field space and  $N_2$  is a commutative near-field space of characteristic 2 with identity admitting a non-zero derivation  $\delta$ . Define a map  $d: N \rightarrow N$  such that  $d((a, b)) = (0, \delta(b))$ . Then it is easy to see that  $d$  satisfies the properties:  $d(x, y) = 0$  and  $d(xoy) = 0$  for all  $x, y \in N$ . However,  $N$  is not a commutative near-field space.

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