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When m-Compact Sets Are m_x-Semi Closed

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ABSTRACT

This paper is devoted to introduce new concepts so called m-k(sc)-space several various theorems about these concepts are proved, Further properties are studied as well as the relationships between these concepts with another types of m-k(sc)-space are investigated.

***Key words**: m_x-open set, m-compact, m-kc-space.*

1. INTRODUCTION

It is known that there is no relation between m-compact space and m_x-closed sets, so this motivates the author [1] to introduce the concept of m-kc-spaces; these are the spaces in which every m-compact subset is m_x-closed.

In 2015 the authors [1] introduce new concepts namely m-k_2 (= A non empty set X with an m-space is said to be m-k_2 if m_x-cl(A) is m-compact in X). The aim of this paper is to continuous study m-kc-spaces.

2- PRELIMINARIES

The basic definitions that needed in this work are recalled. In this work ,a space (X, m_x) means an m-space where a sub family m_x of the power P(X) set, such that Φ and X belong to m_x [2] each member of m_x is said to be m_x-open set and the complement of an m_x-open set is said to be m_x-closed set, we denoted the (X, m_x) by m-space, for a subset A of an m-space X, the m_x-interior of A and the m_x-closure of A defined as follows:

$$mx - cl(A) = \cap \{F: A \subseteq F, X - F \ is \ mx - open\}$$
$$mx - int(A) = \cup\{U: U \subseteq A, U \in m\boldsymbol{X}\}$$

Note that $mx - cl(A)(mx - int(A))$ is not necessarily m_x-closed (m_x-open)

The m-space need not to be a topological space .And the union and the intersection of any two m_x-open sets are not necessarily to be m_x-open, as the following:

Example: Let X={1, 2, 3}, m_x={Φ,X{2,3},{1,2},{1},{3}}.

Then (X, m_x) is m-space but it is not topological space, since {2,3}∩{1,2}={2} ∉ m_x and {1}∪{3}={1,3}∉m_x. The authors [2] introduce the following definitions:

An m-space m_x on a nonempty set x is said to have the property (γ) if the intersection of finite number of m_x-open sets is m_x-open. An m-space m_x on a nonempty set X is said to have the property (β) if the union of any family of subsets of m_x belong to m_x, A nonempty set X with m-space is said to be m-compact if every cover of X with m_x-open sets has a finite sub cover(by [3]). An empty set X with m-space m_x is said to be $m - lindelof$ if every cover of X with m_x-open sets has countable sub cover (by [5]). Every m-compact set is $m - lindelof$ but the convers is not true. For example:

The m-discrete space (X,τ_D), where X is infinite countable set, and τ_D=discrete m-space, then (X, τ_D) is m-lindelof, which is not m-compact. The aim of the paper is to continuous study m-kc-space.

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3-On m-k(sc)-spaces

In this work we introduce a generalizations of m-kc-spaces namely m-k(sc)-space, where m-kc-space is the space in which every m-compact set is closed (by [1]).

Also we study the properties and facts about this concept and the relationships between these concepts.

First we introduce the following definitions:

Definition 1: A subset A of m-space X is said to be m_x-semi closed of X if $mx-int(mx-cl(A)) \subseteq A$.

For example: (R,τ_{cof}) is $m-k(sc)$-space, where X is co countable space.

Lemma 1: A sub set A is m_x- semi closed if and only if there is am m_x-closed set G, such that $mx-int(G) \subseteq A \subseteq G$, wherever an m-space, whenever X has the property β.

Proof: suppose that A is m_x-semi closed, to prove that there is am m_x-closed set G, such that $mx-int(G) \subseteq A \subseteq G, since\ A\ is\ mx-semi\ closed\ (i.e)mx-int\big(mx-cl(A)\big) \subseteq A, put\ G = mx-cl(A) \rightarrow mx-int(G) \subset A \ldots (1)\ and\ A \subset mx-cl(A) \ldots (2),\ then\ we\ get\ that\ A \subset G, then\ by\ (1)\ and\ (2)\ we\ get\ that\ mx-int(G) \subseteq A \subseteq mx-cl(A) = G\ \Rightarrow mx-int(G) \subseteq A \subseteq G, conversely: suppose\ that\ mx-int(G) \subseteq A \subseteq G,\ to\ prove\ that\ A\ is\ mx-semi\ closed, put\ G\ = mx-cl(A)\ then\ we\ get\ that\ mx-int\big(mx-cl(A)\big) \subseteq A \subseteq mx-cl(A)$.

Definition 2: A sub set A of an m-space X is said to be m_x-semi open set of x if $A \subseteq mx-cl(mx-int(A))$.

Lemma 2: A sub set A is m_x-semi open if and only if there is an m_x-open set U, such that U⊆ A ⊆m_x-cl(U)), whenever X has the property β.

Proof: suppose that A is m_x-semi open , to prove that there is an m_x-open set G, such that G ⊆ A ⊆m_x-cl(G), since A is m_x-semi open (i.e) $A\ \subseteq mx-cl(mx-int(A),\quad put\ G = mx-int(A) \Longrightarrow A \subset mx-cl(G) \ldots (1) and\ mx-int(a) \subseteq A \ldots (2),\ then\ by\ (1)\ and\ (2)\ we\ get\ that\ G \subseteq A \subseteq mx-cl(G),\ conversely: given\ G \subseteq A \subseteq mx-cl(G),\ but\ mx-int(A)\ is\ mx-open\ \big(by\ the\ property(\beta)\big)\ and\ mx-int(A) \subseteq A, if\ we\ put\ G = mx-int(A), then\ we\ get\ that\ mx-int(A) \subseteq A \subseteq mx-cl\big(mx-int(A)\big)$.

Definition 3: Let (X, m_x) be am m-space we say that (X, m_x) is an m-k(sc)-space if every m-compact sub set of X is m_x-semi-closed. For example: (R,τ_D), whenever τ_D= m-discrete space.

Remark 1: Every m_x-open (m_x-closed) set is m_x-semi open (m_x-semi closed) set, but the converse is not true.

Since A⊆m_x-cl(A) (1)
(by definition of m_x-closure set), but A is m_x-open in X , so A= m_x-int(A) (2)

Then by (1) and (2) we get that A⊆m_x-int(A).

Example 1: Let m-usual space (R, τ_u), then the set [0, 1) in R is m_x-semi open (m_x-semi closed), but not m_x-open (m_x-closed) set

Remark 2: An m-kc-space is m-k(sc)-space but the converse may be not true for Example(2). Let R be the real line, N be a sub set of R and m_x={U⊆ R /U=R or U∩N=Φ}

The finite sub sets of (R, m_x) which does not contain any members of N is m-compact and m_x-semi closed but not m_x-closed

Since if we take a sub set {1/2,1/3} it is m_x-open of (R,m_x) and it is m-compact, so ∃ F={1/2,3/4,1} is m_x-closed of (R, m_x), s.t {1/2,3/4}=m_x-int({1/2,3/4,1})⊆{1/2,3/4}⊆{1/2,3/4,1}, but {1/2,3/4} is not m_x-closed sub set of R.

Definition 4: Anon empty set X with m-space m_x is said to be m_x-semi compact if every cover of X with m_x-semi open sets has a finite sub cover

Proposition 1: m-semi compact is m- compact. but the converse may be not true.

Proof: Since $\{U\alpha\}\alpha \in \gamma$be an m_x-open cover of X, so $\{U\alpha\}\alpha \in \gamma$is m_x- semi open cover to X, but X is m_x-semi compact, so X=$\bigcup_{i=1}^{n} u_{\alpha i}$

That is, X is m-compact

Example 3: Let R be the real line, N be a sub set of That is of R
$Mx = \{U \subseteq R: U = R \text{ or } U \cap N = \Phi\}, \text{it is clear that } (R, mx) \text{ is an } m - \text{space put } Ui = Nc \cup \{i\} = \{R - N\} \cup \{i\}, i = 1,2, \ldots, Ui \text{ is not } mx - \text{ open subset of } R, \text{since if } i \in N \text{ since } Ui \cap N = \{i\}, i = 1,2, \ldots$

Now to show that
$Ui \text{ is } mx - \text{semi open subset of } R, \text{since the only } mx - \text{open of } R \text{ which is contained in } Ui \text{ is } Nc \text{ and so } Nc \subseteq Ui = Nc \cup \{i\} \subseteq mx - cl(Nc) = R, \text{ this implies that } Ui \text{ is } mx - \text{ semi open for each } i = 1,2, \ldots$

Hence the family$\{U\}_{i=1}^{\infty}$ *forms* m_x*-semi open cover of R is* $\bigcup_{i=1}^{\infty} ui$=$\bigcup_{i=1}^{\infty}(\{R - N\}\cup\{i\})$=R

But this cover can not reducible in to finite subcover, now if U is afinite subset of R which contains at least one point I of N, where i=1,2,3,… so U is not m_x-open but it is m_x-semi open, since m_x-U-{i}⊆m_x-U⊆cl(m_x-U-{i}=R, then m_x-U is not m_x-semi open set, also it is not has finite subcover since if we remove a one element of the m_x-semi open cover, then it will not cover to R therefore R is not m_x-semi compact to show that R is m-compact , since the only m_x-open set which is cover N is m_x-U=R and so every m_x-open cover to R must be contains m_x-U=R this means every m_x- open cover to R, we can choose finite subfamily {R} cover to R, therefore R is m-compact .

Remark 3: m-compact is m$-$ $lindel\ of$, but the converse is not true.

For example: the m$-$ $discreat$ space (Z, τ_D) be an m-lindelof, but not m-compact.

Definition 5: let (X, m_x) be an m- space we say that (X, m_x) is an m$-(sk)sc$-space if every m-semi compact is m_x-semi closed.

Definition 6: let (X, m_x) be an m-space we say that (X, m_x) is an m-(sk)c-space if every m-semi compact is m_x-closed, for example (R, τ_D), whenever$\tau_{D=}$ m-discrete space.

Remark 4: m-kc-space is m-(sk)c-space.

Since if A is be an m-semi compact to prove that A is m_x-closed

A is m-compact ((by m_x-semi compact is m-compact)), but X is m-kc-space, then we get that A is m_x- closed ((by every m_x-compact is m-kc-space is m_x-closed)).

Remark 5: $m - k(sc) - space \ is \ m - (sk)sc - space.$

Definition 7: A space X is said to be m_x-T_1-space if for every two distinct points x and y in X, ∃ two m_x-open sets u and v s.t x∈u and $y \in$ vbut y∉ u and x∉v **[1].**

Definition 8: An m-space X is called m-semi k_2-space or (m-sk_2) if m_x-semi cl(A) is m-compact, wherever A is m-compact, where m_x-semi cl(A)((= the intersection of all semi closed set which contain A)).

Definition 9: A space X is said to be m_x-semi T_1(m_x-sT_1) if for every two distinct points x and y in X ,∃ two m_x-semi open sets u and v s.t x∈u, but y∉u and y∈v, but x∉v.

Example 4: the m-cofinite space (R, τ_{cof}) is m_x-sT_1-space.

*****Every m_x-T_1-space is m_x-sT_1-space, but the converse is not true.

For example (*): Let X = {1, 2, 3} and, let m_x={Φ, X, {a}, {b}, {a, b}}, the semi open sets in X are Φ, X, {b, c}, {a, c}, {a}, {b},{a, b}, it is clear that X is sT_2, but not T_2-space.

Definition 10: Let (X, m_x) be an m-space, then X is m-semi T_2-space or (m-sT_2) if for every two distinct points x, y∈ X, there are disjoint m_x-semi open sets u and v s.t x∈u & y∈v, it is clear that m-sT_2-space is m-ST_1-space, but the converse is not true.

For example:(X, τ_{cof}), where X is infinite set and τ_{cof} is the m-cofinite space, τ_{cof} = {U⊆X/ U^c is finite}⋃{Φ}.

Lemma 3: Every m-T_2-space is s-T_2-space, but the converse is not true. For Example (*).

Remark 6: Every m-k(sc)-space X is m-sT_1-space.

Since {x} is an m-compact in X which is a m-k(sc)-space so , {x} is m_x-semi closed, there for X is m-ST_1-space.

Remark 7: m-sT_1-space is not m-k(sc)-space,

For example: The m-cofinite space (R, τ_{cof}) is m-ST_1-space, which is not a m-k(sc)-space, whereτ_{cof} is m-cofinite space R it is definition by τ_{cof}={U⊆X/U^c is finite}⋃{Φ}

Since $(R, \tau cof)$ is m-T_1-space and every m-T_1-space is m-sT_1-space, but it is not m-$k(sc)$-space, since(R,τ_{cof}) is m-compact, also if we take (Q, τ_{cof}) is m-compact, but it is not m_x-semi closed, because the only m_x-closed set which contains Q is just R, but $R = int(R) \subseteq Q \subseteq R$, so (Q, τ_{cof}) is not m-k(sc)-space.

From remark (1) and (2) we get that:
$m - k(sc)$ is m-ST_1-space iff {x} is m_x-semi closed.

Remark 8: Every m-k(sc)-space is m-sk_2, Since if M is a m-compact subset of X, which $is\ m - k(sc)$ −space, thenM is a m_x-semi closed in X, so M=m_x-semi cl(M), therefore m_x-semi cl(M) is m-compact in X, so X is a m-sk_2-space.

Proposition 2: Every m_x-closed subset of m-compact space is m-compact [4].

Proposition 3: The m-continuous image of m-compact set is also m-compact [4].

Theorem 1: Every m-continuous function f from m-compact space X in to a m-kc-space Y is m_x-semi closed function .

Proof: Since if F is a m_x-closed subset of X , which is m-compact space, then F is a m-compact in X, so f(F) is m-compact in Y, which is m-kc-space, then f(F) is m_x-closed subset in a space ((by theorem of [1]" every m-continuous function from m-compact space in to m-kc-space is m-closed function)), therefore f is m_x-closed function , also f is m_x-semi closed function ((since every m_x-closed is m_x-semi closed)).

Lemma 4: If W is m_x-semi closed in X, and Y is a subspace of X, then W∩Y is m_x-semi closed in Y.

Proof: Since W is m_x-semi closed in X, then ∃ m_x-closed set F of X, such that m_x-$int(FinX) \subseteq W \subseteq F, so\ mx - int(FinY) = mx - int(FinX) \cap Y \subseteq W \cap Y \subseteq F \cap Y \subseteq F\ in\ Y, therefor\ W \cap Y$ is m_x-semi closed in Y.

Proposition 4: Every subspace of m-k(sc)-space is m-k(sc)-space.

Proof: Let Y be a subspace of m-m-k(sc)-space X and A be any m-compact subset of Y, then A is m-compact in X which is m-k(sc)-space, then A is m_x-semi closed in X, but A∩Y=A, then A is m_x-semi closed in Y, therefore Y is also m-k(sc)-space.

Remark 9: The intersection of any family of m_x-semi open set is not m_x-semi open set.

Example 5: X={1,2,3}, m_x={Φ,X,{1,2},{2,3}} it is m_x-semi open, let A ={1,2}& B={2,3}, then A∩B={2} ∉ m_x, but {2} is not m_x-open set

Definition 11: An m-space m_x on a nonempty set X is said to have the property (sβ) if the union of any family of m_x-semi subsets of m_x belong to m_x.

Theorem 2: An m-space X which has (sβ) property is m_x-sT_1-space iff every singleton set is m_x-semi closed set.

Proof: Suppose that X is m_x –sT_1-space, let x∈X T.P {x} is m_x-semi closed subset of X.
First {x}⊆m_x-semi cl ({x}) by remark (*)
$(i.e)$ T.P m_x-semi cl ({x}) = {x}, now T.P m_x-semi cl({x})⊆{x}

Suppose not $(i.e)$ m_x-semi cl({x})⊈{x}, then ∃ y ∈ m_x-semi cl({x}), but y∉{x} s.t x≠y, since X is an m $_x$-sT_1-space $(i.e)$ Ux, Vy are m_x-semi open subsets of X s.t x∈U_x&$y \in Vy$, this implies, then y∉cl(A^s) C! y is not semi $aldhreant$ point, so m_x-semi cl({x})⊆{x}…(**), then by (*) and (**) we get that m_x-cl({x})={x}, then {x} is m_x-semi closed subset of X. the converse direction suppose that for all x∈X, {x} is m_x-semi closed in X T.P X is m_x-sT_1-space, Let x, y ∈X s.t x≠y, then by hypothis {x}, {y} are m_x-semi closed in X, X-{x} and X-{y} are m_x-semi open subsets of X s.t x∈{x}, x∉X-{x} &y∉{x}, y∈X-{x}&x∉{y}, x∈X-{y}, then (X, m_x) is m_x-ST_1-space.

Definition 12: An m- space (X, τ) is said to be an m- locally kc-space if and only if each point has a neighborhood which is an m-kc-subspace.

Proposition 5: Every m-T_2-space is m-locally kc-space.

Proof: Let X be an m-T_2-space and $x \in X$to show that x has a neighborhood, let N_x be neighborhood to x and k be an m-compact subset of a subspace X is m-T_2, and the property of a space being m- T_2 is $a\ hereditrary$ property so N is an m-T_2-subspace, which implies that k is $mx - closed$ (since X has the property (β)), then X is m-locally kc-space.

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