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# **GENERALIZATION OF FUZZY SEMI OPEN SETS IN FUZZY BITOPOLOGICAL SPACES**

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## ABSTRACT

**F**ocus of this paper is to introduce the concept of fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi open sets  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi closed sets boundary subsets of a fuzzy bitopological space where  $\mathfrak{C} : [0,1] \rightarrow [0,1]$  is a complement function. Several examples are given to illustrate the concepts introduced in this paper.

**Keywords:** Fuzzy complement function  $\mathfrak{C}$ , fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  – semi closed sets, fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  – semi open sets, fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  – semi closure, fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  – semi interior subsets and fuzzy bitopological spaces.

## **1. INTRODUCTION**

The concept of fuzzy sets and fuzzy set operations were first introduced by L. A. Zadeh [8] in the year 1965. The theory of fuzzy topological space was introduced and developed by C. L. Chang [3]. A. Kandil [5] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological space. The concept of complement function  $\mathfrak{C}$ : [0, 1]  $\rightarrow$  [0,1] was introduced by K. Bageerathi and P. Thangavelu in [2]. The concept of fuzzy ( $\tau_i, \tau_j$ ) - semi open set and fuzzy ( $\tau_i, \tau_j$ ) - semi closed set was introduced and studied by S.S.Thakur,R.Malvia in [6] In this paper the concept of fuzzy  $\mathfrak{C}$ -( $\tau_i, \tau_j$ ) - semi open sets, fuzzy  $\mathfrak{C}$ -( $\tau_i, \tau_j$ ) - semi closed sets, fuzzy  $\mathfrak{C}$ -( $\tau_i, \tau_j$ ) - semi interior operators in fuzzy bitopological spaces is introduced and several examples are given to illustrate the concepts introduced in this paper.

## 2. PRELIMINARIES

In this section we list some definitions and results that are needed. Any function  $\mathfrak{C}: [0, 1] \rightarrow [0, 1]$  defined from the interval [0, 1] to itself is called a complement function. Throughout the paper  $\mathfrak{C}$  denotes an arbitrary complement function and  $(X, \tau_i, \tau_j)$  is a fuzzy bitopological space in the sense of A.Kandil [5].Throughout this paper, for fuzzy set  $\lambda$  of a fuzzy bitopological space  $(X, \tau_i, \tau_j)$ ,  $\tau_i - int\lambda$  and  $\tau_j - cl_{\mathfrak{C}}\lambda$  means, respectively, the interior and closure of  $\lambda$  with respect to fuzzy topologies  $\tau_i$  and  $\tau_j$ .

**Definition 2.1[2]:** If  $\lambda$  is a fuzzy subset of X then the complement  $\mathfrak{C}\lambda$  of a fuzzy set  $\lambda$  is a fuzzy subset with membership function defined by  $\mu_{\mathfrak{C}}\lambda(x) = \mathfrak{C}(\mu\lambda(x))$  for all  $x \in X$ .

A subset  $\lambda$  of a fuzzy topological space is fuzzy closed if its standard complement  $\lambda'$ , where  $\lambda'(x) = 1 - \lambda(x)$  is fuzzy open. Several fuzzy topologists used this type of complement while extending the concepts in general topological spaces to fuzzy topological spaces. But there are other complements available in the fuzzy literature.

The properties of fuzzy complement function  $\mathfrak{C}$  and  $\mathfrak{C}\lambda$  are given in George Klir [4] and Bageerathi *et al.* [2]. The following lemma will be useful in sequel. Some of the complement functions are given below.

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#### Examples 2.2[4]:

- (i) The standard complement function:  $\mathfrak{C}_1(x) = 1 x$ .
- (ii) The Threshold type complement function for any  $t \in [0, 1)$ :

$$\mathfrak{C}_{t}(x) = \begin{cases} 1 & \text{for } 0 \le x \le t \\ 0 & \text{for } t < x \le 1 \end{cases}$$

(iii) Sugeno class complement function for any  $\lambda \in (1, \infty)$ :

$$\mathfrak{C}_{S\lambda}(\mathbf{x}) = \frac{1-x}{1+\lambda x}$$
, for  $\mathbf{x} \in [0, 1]$ 

(iv) Yagor class of complement function for  $\mathcal{O} \in (0, \infty)$ :

$$\mathfrak{C}_{\mathbf{Y}\omega}(\mathbf{x}) = (1 - \mathbf{x}^{\omega})^{1/\omega}$$
, for  $\mathbf{x} \in [0, 1]$ 

The next lemma can be easily established.

**Lemma 2.3[4]:** The complement functions  $\mathfrak{G}_1, \mathfrak{G}_5, \mathfrak{G}_{S\lambda}$  and  $\mathfrak{G}_{Y, \omega}$  satisfy the following conditions.

- (i) Boundary condition:  $\mathfrak{C}(0) = 1$  and  $\mathfrak{C}(1) = 0$ ;
- (ii) Monotonicity: for all x,  $y \in [0, 1]$ ,  $x \le y \Rightarrow \mathfrak{C}(x) \ge \mathfrak{C}(y)$ ;
- (iii) C is continuous and
- (iv) Involutive:  $\mathfrak{C}(\mathfrak{C}(\mathbf{x})) = \mathbf{x}$  for all  $\mathbf{x} \in [0, 1]$ .

**Definition 2.4[1]:** For a family  $\{A_{\alpha}: \alpha \in \Delta\}$  of fuzzy sub sets of X, the union,  $A = \bigcup \{A_{\alpha}: \alpha \in \Delta\}$  and the intersection, B=  $\cap \{A_{\alpha}: \alpha \in \Delta\}$  are defined with membership functions respectively  $\mu_A(x) = \sup\{\mu_{A\alpha}(x): \alpha \in \Delta\}$  and  $\mu_B(x) = \inf\{\mu_{A\alpha}(x): \alpha \in \Delta\}$ ,  $x \in X$ .

**Lemma 2.5 [2]:** Let  $\mathfrak{C}$ :  $[0, 1] \rightarrow [0, 1]$  be a complement function that satisfies the involutive and monotonicity properties. Then for any family  $\{A_{\alpha} : \alpha \in \Delta\}$  of fuzzy subsets of X we have

- (i)  $\mathfrak{C}(\sup\{\mu_{A\alpha}(x): \alpha \in \Delta\} = \inf\{\mathfrak{C}(\mu_{A\alpha}(x)): \alpha \in \Delta\} = \inf\{(\mu_{\mathfrak{C}A\alpha}(x)): \alpha \in \Delta\}$  and
- (ii)  $\mathfrak{C}(\inf\{\mu_{A\alpha}(x): \alpha \in \Delta\} = \sup\{\mathfrak{C}(\mu_{A\alpha}(x)): \alpha \in \Delta\} = \sup\{(\mu_{\mathfrak{C}A\alpha}(x)): \alpha \in \Delta\}.$

**Lemma 2.6[4]:** Let  $\mathfrak{C}$ :  $[0, 1] \rightarrow [0, 1]$  be a complement function that satisfies involutive and monotonicity properties. Then for any family {A<sub> $\alpha$ </sub> :  $\alpha \in \Delta$  } of fuzzy subsets of X. we have

(i)  $\mathfrak{C}(\bigcup\{A_{\alpha}:\alpha\in\varDelta\}) = \bigcap\{\mathfrak{C}A_{\alpha}:\alpha\in\varDelta\} \text{ and } (\text{ii}) \mathfrak{C}(\bigcap\{A_{\alpha}:\alpha\in\varDelta\}) = \bigcup\{\mathfrak{C}A_{\alpha}:\alpha\in\varDelta\}.$ 

**Proposition 2.7[7]:** If the complement functions  $\mathfrak{C}$  satisfies the monotonicity and involutive properties, then for any fuzzy subset  $\lambda$  of a fuzzy bitopological space ,we have

(i)  $\mathfrak{C}(\tau_i - \operatorname{int}\lambda) = \tau_i - \operatorname{cl}_{\mathfrak{C}}(\mathfrak{C}\lambda)$  and (ii)  $\mathfrak{C}(\tau_i - \operatorname{cl}_{\mathfrak{C}}\lambda) = \tau_i - \operatorname{int}(\mathfrak{C}\lambda)$ .

**Theorem 2.8 [7]:** Let  $\mathfrak{C}$  be a complement function that satisfies the monotonicity and involutive properties. For any two fuzzy subsets  $\lambda$  and  $\mu$  of a fuzzy bitopological space we have

- (i)  $\lambda \leq \tau_i cl_{\mathfrak{C}}\lambda;$
- (ii)  $\lambda$  is fuzzy  $\mathfrak{C}$ - $\tau_i$ -closed  $\Leftrightarrow \tau_i cl_{\mathfrak{C}}\lambda = \lambda$ ;
- (iii)  $\tau_i$ -cl<sub>c</sub> ( $\tau_i$ -cl<sub>c</sub>  $\lambda$ )=  $\tau_i$  cl<sub>c</sub> $\lambda$ ;
- (iv) If  $\lambda \leq \mu$  then  $\tau_i cl_{\mathfrak{C}} \lambda \leq \tau_i cl_{\mathfrak{C}} \mu$
- (v)  $\tau_i$ -cl<sub>c</sub> ( $\lambda_{\sqrt{\mu}}$ ) =  $\tau_i$ -cl<sub>c</sub>  $\lambda_{\sqrt{\nu}}$  $\tau_i$  cl<sub>c</sub> $\mu$  and
- (vi)  $\tau_{i} cl_{\mathfrak{C}} (\lambda \wedge \mu) \geq \tau_{i} cl_{\mathfrak{C}} \lambda \wedge \tau_{i} cl_{\mathfrak{C}} \mu$

#### 3. Fuzzy $\mathfrak{C}$ - $(\tau_i, \tau_i)$ - semi open and Fuzzy $\mathfrak{C}$ - $(\tau_i, \tau_i)$ - semi closed sets

In this section we define the notion of fuzzy  $\mathfrak{C}$  -  $(\tau_i, \tau_j)$  - semi open set and  $\mathfrak{C}$  -  $(\tau_i, \tau_j)$  - semi closed set and discussed some of their properties.

**Definition 3.1:** A fuzzy set  $\lambda$  of a fuzzy bitopological space  $(X, \tau_1, \tau_2)$  is called (a) fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi open if there exists a  $\tau_i$  - fuzzy open set  $\mu$  such that  $\mu \leq \lambda \leq \tau_j$  - cl<sub>C</sub> ( $\mu$ )  $\tau_i$  - cl<sub> $\mathfrak{C}$ </sub> ( $\mu$ ) (b) fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi closed if there exists a fuzzy  $\mathfrak{C} - \tau_i$  - closed  $\mu$  such that  $\mathfrak{C} \mu \leq \mathfrak{C} \lambda \leq \tau_j$  - cl<sub> $\mathfrak{C}$ </sub> ( $\mathfrak{C} \mu$ ).

The concept of all fuzzy  $\mathfrak{C}$  -  $(\tau_i, \tau_j)$  - semi open sets and  $\mathfrak{C}$  -  $(\tau_i, \tau_j)$  - semi closed sets coincides with the concept of all fuzzy  $(\tau_i, \tau_j)$  - semi open sets and fuzzy  $(\tau_i, \tau_j)$  semi closed sets if the arbitrary complement coincides with the standard complement function.

**Theorem 3.2:** Let  $(X, \tau_1, \tau_2)$  be a fuzzy bitopological space and  $\mathfrak{C}$  be a complement function that satisfies the involutive properties. Then  $\lambda$  is fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$ - semi closed if and only if  $\mathfrak{C} \lambda$  is s fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$ - semi open.

**Proof:** Let  $\lambda$  be a fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi closed. Then by using Definition 3.1 there exists a fuzzy  $\mathfrak{C} - \tau_i$  - closed  $\mu$  such that  $\mathfrak{C} \ \mu \leq \mathfrak{C} \ \lambda \leq \tau_j$  - cl<sub> $\mathfrak{C}$ </sub> ( $\mathfrak{C} \mu$ ). By replacing  $\mathfrak{C} \ \mu = \delta$ ,  $\delta \leq \mathfrak{C} \ \lambda \leq \tau_j$  - cl<sub> $\mathfrak{C}$ </sub> ( $\delta$ ). By using Definition 3.1 (b),  $\mathfrak{C} \ \lambda$  is fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi open. Conversely let  $\lambda$  be a fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi open. Then by using Definition 3.1 (a), there exists a  $\eta \in \tau_i$  such that  $\eta \leq \lambda \leq \tau_j$  - cl<sub> $\mathfrak{C}$ </sub>  $\mathfrak{q}$ . Let  $\mu = \mathfrak{C} \ \eta$ . Since  $\mathfrak{C}$  satisfies the involutive condition,  $\eta = \mathfrak{C} \ (\mathfrak{C} \ \eta) = \mathfrak{C} \ \mu$ . That is  $\mathfrak{C} \ \mu \leq \mathfrak{C} \ (\mathfrak{C} \ \lambda) \leq \tau_j$  - cl<sub> $\mathfrak{C}$ </sub>  $(\mathfrak{C} \ \mu)$ . Thus  $\mathfrak{C} \ \lambda$  is fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi closed.

The next example shows that if the complement function does not satisfy the involutive property, then the conclusion of above theorem is false.

**Example 3.3:** Let  $X = \{a, b, c\}, \tau_1 = \{0, 1, \{a_{.8}, b_{.7}\}, \{a_{.8}, b_{.7}, c_{.3}\}, 1\}$  and  $\tau_2 = \{0, 1, \{c_{.9}\}, \{a_{.8}, b_{.9}\}, \{a_{.8}, b_{.9}, c_{.9}\}, \{a_{.9}, b_{.9}, c_{.9}\}\}$ . Let  $\mathfrak{C}(x) = \frac{1}{1+3x}, 0 \le x \le 1$ , be a complement function does not satisfies involute property. Then the family of all fuzzy  $\mathfrak{C}$ - $\tau_i$  - closed sets are  $\mathfrak{C}(\tau_1) = \{1, \{a_{.25}, b_{.25}, c_{.25}\}, \{a_{.294}, b_{.3225}, c_1\}, \{a_{.294}, b_{.3225}, c_{.526}\}\}$  and  $\mathfrak{C}(\tau_2) = \{1, \{a_{.25}, b_{.25}, c_{.25}\}, \{a_{.1}, b_{1}, c_{.27}\}, \{a_{.29}, b_{.27}, c_{.1}\}, \{a_{.29}, b_{.27}, c_{.27}\}\}$ . Let  $\lambda = \{a_{.2}, b_{.2}, c_{.8}\}$ . Then  $\lambda$  is not fuzzy  $\mathfrak{C}$  -  $(\tau_1, \tau_2)$  - semi open. Now  $\mathfrak{C}\lambda = \{a_{.625}, b_{.625}, c_{.29}\}$  and it is fuzzy  $\mathfrak{C} - (\tau_1, \tau_2)$  - semi closed.

**Theorem 3.4:** Let  $(X, \tau_1, \tau_2)$  be a fuzzy bitopological space and  $\mathfrak{C}$  be a complement function that satisfies the monotonic and involutive properties. Then (i) fuzzy subset  $\lambda$  of a fuzzy bitopological space  $(X, \tau_1, \tau_2)$  is fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi open if and only if  $\lambda \leq \tau_j - cl_{\mathfrak{C}}(\tau_i - Int \lambda)$ . (ii) fuzzy subset  $\lambda$  of a fuzzy bitopological space  $(X, \tau_1, \tau_2)$  is fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi closed if and only if  $\tau_j - Int(\tau_i - cl_{\mathfrak{C}}(\lambda)) \leq \lambda$ .

#### **Proof:**

- (i) Let  $\lambda$  be a fuzzy  $\mathfrak{C} (\tau_i, \tau_j)$  semi open. Then by using Definition 3.1 (a), there exists a  $\mu \in \tau_i$  such that  $\mu \leq \lambda \leq \tau_j$  cl<sub> $\mathfrak{C}$ </sub>  $\mu$ . Since  $\mathfrak{C}$  satisfies the monotonic and involutive conditions, by using Proposition 2.7, we have  $\mu \leq \lambda \leq \mathfrak{C}$  ( $\tau_j$  Int  $\mathfrak{C} \mu$ ). Since  $\mathfrak{C}$  satisfies the monotonic and involutive conditions,  $\tau_j$  Int  $\mathfrak{C} \mu \leq \mathfrak{C} \lambda \leq \mathfrak{C} \mu$ . By applying Theorem [2.8],  $\mathfrak{C} \lambda \leq \mathfrak{C} \mu$  implies that  $\tau_i cl_{\mathfrak{C}} (\mathfrak{C} \lambda) \leq \tau_i cl_{\mathfrak{C}} (\mathfrak{C} \mu) = \mathfrak{C} \mu$ . This implies that  $\tau_j$  Int ( $\tau_i cl_{\mathfrak{C}} (\mathfrak{C} \lambda)$ )  $\leq \tau_i cl_{\mathfrak{C}} (\mathfrak{C} \lambda) \leq \mathfrak{C} \mu$ . Therefore  $\tau_j$  Int ( $\tau_i cl_{\mathfrak{C}} (\mathfrak{C} \lambda)$ )  $\leq \tau_j$  Int ( $\mathfrak{C} \mu \leq \mathfrak{C} \lambda$ . Therefore  $\tau_j$  Int ( $\tau_i cl_{\mathfrak{C}} (\mathfrak{C} \lambda)$ )  $\leq \tau_j$  Int ( $\mathfrak{C} \mu \leq \mathfrak{C} \lambda$ . Therefore  $\tau_j Int (\tau_i cl_{\mathfrak{C}} (\mathfrak{C} \lambda)) \leq \tau_j$  Int ( $\mathfrak{C} \mu \leq \mathfrak{C} \lambda$ . Therefore  $\tau_j cl_{\mathfrak{C}} (\mathfrak{C} \lambda)$ )  $\leq \tau_j$  Int ( $\mathfrak{C} \mu \leq \mathfrak{C} \lambda$ . Therefore  $\tau_j cl_{\mathfrak{C}} (\mathfrak{C} \lambda)$ )  $\leq \tau_j$  Int ( $\mathfrak{C} \mu \leq \mathfrak{C} \lambda$ . Therefore  $\tau_j cl_{\mathfrak{C}} (\mathfrak{C} \lambda)$ )  $\leq \tau_j$  Int ( $\mathfrak{C} \mu \leq \mathfrak{C} \lambda$ . Therefore  $\tau_j cl_{\mathfrak{C}} (\mathfrak{C} \lambda)$ )  $\leq \tau_j$  Int ( $\mathfrak{C} \mu \leq \mathfrak{C} \lambda$ . This implies that  $\lambda \leq \tau_j cl_{\mathfrak{C}} (\mathfrak{C} \lambda)$ . Conversely, we assume that  $\lambda \leq \tau_j cl_{\mathfrak{C}} (\tau_i Int \lambda)$ . Let  $\mu = \tau_i Int \lambda$ . Taking complement on both sides,  $\mathfrak{C} \mu = \mathfrak{C} (\tau_i Int \lambda)$ . By using Proposition 2.7,  $\mathfrak{C} \mu = \tau_i cl_{\mathfrak{C}} (\mathfrak{C} \lambda) \geq \mathfrak{C} \lambda$ . Since  $\mathfrak{C}$  satisfies the monotonic condition,  $\mu \leq \lambda$ . By our assumption  $\lambda \leq \tau_j cl_{\mathfrak{C}} (\tau_i Int (\lambda)) = \tau_j cl_{\mathfrak{C}} \mu$ . We have  $\mu \leq \lambda \leq \tau_j cl_{\mathfrak{C}} (\mu)$ . By using Definition 3.1,  $\lambda$  is fuzzy  $\mathfrak{C} (\tau_i, \tau_j) \mathfrak{C}$  are in open.
- (ii) Let λ be a fuzzy C (τ<sub>i</sub>, τ<sub>j</sub>) semi closed. Then by using Definition 3.1, there exists C μ ∈ τ<sub>i</sub> such that C μ ≤ C λ ≤ τ<sub>j</sub> cl<sub>C</sub> (C μ). Since C satisfies the monotonic and involutive conditions, by using applying proposition 2.7, we have C μ ≤ C λ ≤ C (τ<sub>j</sub> Int μ). In particular C satisfies the monotonic condition that implies τ<sub>j</sub> Int μ ≤ λ ≤ μ. Since λ ≤ τ<sub>i</sub> cl<sub>C</sub> λ that implies τ<sub>j</sub> Int μ ≤ λ ≤ μ. Therefore τ<sub>i</sub> cl<sub>C</sub> λ ≤ μ. Therefore τ<sub>i</sub> cl<sub>C</sub> λ ≤ μ. Therefore τ<sub>i</sub> cl<sub>C</sub> λ) ≤ τ<sub>i</sub> lnt μ ≤ λ. Therefore τ<sub>j</sub> Int μ ≤ λ ≤ C (τ<sub>j</sub> Int μ ≤ λ ≤ τ<sub>i</sub> cl<sub>C</sub> λ ≤ μ. Therefore τ<sub>j</sub> Int μ ≤ λ. Therefore τ<sub>j</sub> Int μ ≤ λ. Conversely, suppose τ<sub>j</sub> Int (τ<sub>i</sub> cl<sub>C</sub> λ) ≤ λ. Let μ = τ<sub>i</sub> cl<sub>C</sub> λ. Therefore τ<sub>j</sub> Int μ ≤ λ. Taking complement on both sides, C λ ≤ C (τ<sub>j</sub> Int μ). Since C satisfies the monotonic and involutive conditions, by applying proposition 2.7 C λ ≤ τ<sub>j</sub> cl<sub>C</sub> C μ). Since μ = τ<sub>i</sub> cl<sub>C</sub> λ. Taking complement on both sides, we have C μ = C (τ<sub>i</sub> cl<sub>C</sub> λ) = τ<sub>i</sub> Int C λ ≤ C λ. So C μ ≤ C λ. Thus we have C μ ≤ C λ ≤ τ<sub>i</sub> cl<sub>C</sub> C μ). Again by using Definition 3.1, λ is fuzzy C (τ<sub>i</sub>, τ<sub>j</sub>) semi closed.

The next example shows that if the complement function does not satisfy the monotonic and involutive conditions, then the conclusion of above theorem is false.

**Example 3.5:** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{0, \{a_{.2}, b_{.1}\}$ ,  $\{a_{.5}, b_{0}, c_{.6}\}$ ,  $\{a_{.2}, b_{0}, c_{.6}\}$ ,  $\{a_{.5}, b_{.1}, c_{.6}\}$ ,  $\{a_{.6}, b_{.4}, c_{.8}\}$ , 1} and  $\tau_2 = \{0, \{a_{.5}, c_{.1}\}$ ,  $\{a_{.6}, b_{.2}, c_{.3}\}$ ,  $\{a_{.5}, b_{.2}, c_{.1}\}$ , 1}. Let  $\mathfrak{C}(x) = \frac{1}{1+x}$ ,  $0 \le x \le 1$ , be a complement function. We see that the complement function  $\mathfrak{C}$  does not satisfy the involutive condition. Then the family of all fuzzy  $\mathfrak{C} - \tau_i$  - closed sets are  $\mathfrak{C}(\tau_1) = \{1, \{a_{.8}, b_{.9}, c_1\}$ ,  $\{a_{.7}, b_1, c_{.625}\}$ ,  $\{a_{.8}, b_1, c_{.625}\}$ ,  $\{a_{.7}, b_{.9}, c_{.625}\}$ ,  $\{a_{.625}, b_{.7}, c_{.55}\}$ ,  $\{a_{.5}, b_{.5}, c_{.5}\}$  and  $\mathfrak{C}(\tau_2) = \{1, \{a_{.7}, b_1, c_{.9}\}$ ,  $\{a_{.625}, b_{.8}, c_{.77}\}$ ,  $\{a_{.7}, b_{.8}, c_{.9}\}$ ,  $\{a_{.5}, b_{.5}, c_{.5}\}$ . Let  $\lambda = \{a_{.7}, b_{.5}, c_{.8}\}$ . Then we can see that  $\tau_2 - cl_{\mathfrak{C}}(\tau_1 - Int(\lambda)) = \tau_2 - cl_{\mathfrak{C}}\{a_{.5}, b_{.1}, c_{.6}\} = \{a_{.625}, b_{.8}, c_{.7}\}$ . Therefore  $\lambda \not\leq \tau_2 - cl_{\mathfrak{C}}(\tau_1 - Int \lambda)$ . But  $\lambda$  is fuzzy  $\mathfrak{C} - (\tau_1, \tau_2) - semi$  open. Also let  $\lambda = \{a_{.5}, b_{.4}, c_{.1}\}$ . Then it can be computed that  $\tau_2 - Int(\tau_1 - cl_{\mathfrak{C}}(\lambda)) = \tau_2 - Int\{a_{.5}, b_{.5}, c_{.5}\} = \{a_{.5}, b_{.2}, c_{.1}\}$ . This implies that  $\lambda \ge \tau_2 - Int(\tau_1 - cl_{\mathfrak{C}}(\lambda))$ . But  $\lambda$  is not fuzzy  $\mathfrak{C} - (\tau_1, \tau_2) - semi$  closed.

**Remark 3.6:** It is clear from Definition 3.1 that every  $\tau_i$  - fuzzy open (respectively  $\tau_i$  - fuzzy closed) set is fuzzy  $\mathfrak{C}$  -  $(\tau_i, \tau_j)$  - semi open (respectively fuzzy  $\mathfrak{C}$  -  $(\tau_i, \tau_j)$  - semi closed). The following example shows that the converse may not be true.

**Example 3.7:** Let  $X = \{a, b, c\}, \tau_1 = \{0, \{a.2, b.1\}, \{a.1, b.4, c.8\}, \{a.1, b.1\}, \{a.2, b.4, c.8\}, 1\}$  and  $\tau_2 = \{0, \{a.3, c.6\}, \{b.4\}, \{a.3, b.4, c.6\}, 1\}$ . Let  $\mathfrak{C}(x) = \frac{1-x}{1+3x}, 0 \le x \le 1$ , be a complement function. Then the family of all fuzzy  $\mathfrak{C} - \tau_i - closed$  sets are  $\mathfrak{C}(\tau_1) = \{1, \{a.5, b.7, c_1\}, \{a.7, b.3, c.1\}, \{a.7, b.7, c_1\}, \{a.5, b.3, c.1\}, 0\}$  and  $\mathfrak{C}(\tau_2) = \{1, \{a.4, b.1, c.14\}, \{a.1, b.3, c_1\}, \{a.4, b.3, c.14\}, 0\}$ . Let  $\lambda = \{a.1, b.1, c.1\}$ . Then it can be computed that  $\tau_1 - Int(\lambda) = \{a.1, b.1\}$  and  $\tau_2 - cl_{\mathfrak{C}}(\tau_1 - Int(\lambda)) = \{a.4, b.3, c.14\}$ . Then  $\lambda \le \tau_2 - cl_{\mathfrak{C}}(\tau_1 - Int(\lambda))$ . Therefore  $\lambda$  is fuzzy  $\mathfrak{C} - (\tau_1, \tau_2)$  - semi open but  $\lambda$  is not fuzzy  $\mathfrak{C} - \tau_i$  - open. Also let  $\lambda = \{a.2, b.1, c.1\}$ . Then it can be calculated that  $\tau_1 - cl_{\mathfrak{C}}(\lambda) = \{a.5, b.3, c.1\}$  and  $\tau_2 - Int(\tau_1 - cl_{\mathfrak{C}}\lambda) = \{0\}$ . Therefore  $\lambda \ge \tau_2$  - Int ( $\tau_1 - cl_{\mathfrak{C}}(\lambda)$ ). This implies that  $\lambda$  is fuzzy  $\mathfrak{C} - (\tau_1, \tau_2)$  - semi closed but  $\lambda$  is not fuzzy  $\mathfrak{C} - \tau_i$  - closed.

**Remark 3.8:** The concepts of fuzzy  $\mathfrak{C} - (\tau_1, \tau_2)$  - semi open (respectively fuzzy  $\mathfrak{C} - (\tau_1, \tau_2)$  - semi closed) and fuzzy  $\mathfrak{C} - (\tau_2, \tau_1)$  - semi open (respectively fuzzy  $\mathfrak{C} - (\tau_2, \tau_1)$  - semi closed) sets are independent.

**Example 3.9:** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{0, \{a.2, b.1\}$ ,  $\{a.1, b.4, c.8\}$ ,  $\{a.1, b.1\}$ ,  $\{a.2, b.4, c.8\}$ ,  $1\}$  and  $\tau_2 = \{0, \{a.3, c.6\}$ ,  $\{b.4\}$ ,  $\{a.3, b.4, c.6\}$ ,  $1\}$ . Let  $\mathfrak{C}(x) = \frac{1-x}{1+3x}$ ,  $0 \le x \le 1$ , be arbitrary complement function. Then the family of all fuzzy  $\mathfrak{C} - \tau_i$  - closed sets are  $\mathfrak{C}(\tau_1) = \{1, \{a.5, b.7, c_1\}, \{a.7, b.3, c.1\}, \{a.7, b.7, c_1\}, \{a.5, b.3, c.1\}, 0\}$  and  $\mathfrak{C}(\tau_2) = \{1, \{a.4, b.1, c.14\}, \{a_1, b.3, c_1\}, \{a.4, b.3, c.14\}, 0\}$ . Let  $\lambda = \{a.3, b.2, c.1\}$ . Then it can be computed that  $\tau_2 - cl_{\mathfrak{C}}(\tau_1 - Int(\lambda)) = \tau_2 - cl_{\mathfrak{C}}\{a.2, b.1\} = \{a.4, b.3, c.1\}$ . Therefore  $\lambda \le \tau_2 - cl_{\mathfrak{C}}(\tau_1 - Int \lambda)$ . This implies that  $\lambda$  is fuzzy  $\mathfrak{C} - (\tau_1, \tau_2)$  - semi open. Now  $\tau_1 - cl_{\mathfrak{C}}(\tau_2 - Int(\lambda)) = \{0\}$ . Therefore  $\lambda \le \tau_1 - cl_{\mathfrak{C}}(\tau_2 - Int \lambda)$ . This implies that  $\lambda$  is fuzzy  $\mathfrak{C} - (\tau_1, \tau_2)$  - semi open but  $\lambda$  is not fuzzy  $\mathfrak{C} - (\tau_2, \tau_1)$  - semi open. Let  $\lambda = \{a.1, b.2, c.1\}$ . Then it can be computed that  $\tau_2 - Int(\tau_1 - cl_{\mathfrak{C}}(\lambda)) = \tau_2 - Int\{a.5, b.3, c.1\} = \{0\}$ . Therefore  $\lambda \ge \tau_2$  - Int  $(\tau_1 - cl_{\mathfrak{C}}(\lambda))$ . This implies that  $\lambda$  is fuzzy  $\mathfrak{C} - (\tau_1, \tau_2)$  - semi open but  $\lambda$  is not fuzzy  $\mathfrak{C} - (\tau_2, \tau_1)$  - semi open. Let  $\lambda = \{a.1, b.2, c.1\}$ . Then it can be computed that  $\tau_2 - Int(\tau_1 - cl_{\mathfrak{C}}(\lambda)) = \tau_2 - Int\{a.5, b.3, c.1\} = \{0\}$ . Therefore  $\lambda \ge \tau_2$  - Int  $(\tau_1 - cl_{\mathfrak{C}}(\lambda))$ . This implies that  $\lambda$  is fuzzy  $\mathfrak{C} - (\tau_1, \tau_2)$  - semi closed. Also  $\tau_1$  - Int  $(\tau_2 - cl_{\mathfrak{C}}(\lambda))$ . Therefore  $\lambda \ge \tau_2$  - Int  $(\tau_2 - cl_{\mathfrak{C}}(\lambda))$ . Therefore  $\lambda \ge \tau_2$  - Int  $(\tau_2 - cl_{\mathfrak{C}}(\lambda))$ . Therefore  $\lambda \ge \tau_2$  - Int  $(\tau_2 - cl_{\mathfrak{C}}(\lambda))$ . Therefore  $\lambda \ge \tau_1$  - Int  $(\tau_2 - cl_{\mathfrak{C}}(\lambda))$ . Therefore  $\lambda$  is not fuzzy  $\mathfrak{C} - (\tau_2, \tau_1)$  - semi closed.

**Theorem 3.10:** Let  $(X, \tau_1, \tau_2)$  be a fuzzy bitopological space and  $\mathfrak{C}$  be a complement function that satisfies the monotonic and involutive properties. Then (i) Arbitrary union of fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi open sets is fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi open. (ii) Arbitrary intersection of fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi closed sets is fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi closed.

#### Proof

- (i) Let {λ<sub>α</sub>} be a collection of all fuzzy 𝔅 (τ<sub>i</sub>, τ<sub>j</sub>) semi open sets of a fuzzy bitopological space (X, τ<sub>1</sub>, τ<sub>2</sub>). Then for each α, there exists a μ<sub>α</sub> ∈ τ<sub>i</sub> such that μ<sub>α</sub> ≤ λ<sub>α</sub> ≤ τ<sub>j</sub> cl<sub>𝔅</sub> (μ<sub>α</sub>). Thus νμ<sub>α</sub> ≤ νλ<sub>α</sub> ≤ ν τ<sub>j</sub> cl<sub>𝔅</sub> (μ<sub>α</sub>). Since 𝔅 satisfies the monotonic and involutive properties by applying Theorem 2.8, we have τ<sub>j</sub> cl<sub>𝔅</sub> (μ<sub>α</sub>) ≤ τ<sub>j</sub> cl<sub>𝔅</sub> (νμ<sub>α</sub>), that implies ν τ<sub>j</sub> cl<sub>𝔅</sub> (μ<sub>α</sub>) ≤ τ<sub>j</sub> cl<sub>𝔅</sub> (νμ<sub>α</sub>). Hence νμ<sub>α</sub> ≤ νλ<sub>α</sub> ≤ τ<sub>j</sub> cl<sub>𝔅</sub> (νμ<sub>α</sub>) and νμ<sub>α</sub> ∈ τ<sub>i</sub>. By using Definition 3.1, we have νλ<sub>α</sub> is fuzzy 𝔅 (τ<sub>i</sub>, τ<sub>j</sub>) semi open.
- (ii) Let  $\{\lambda_{\alpha}\}$  be a collection of all fuzzy  $\mathfrak{C} (\tau_i, \tau_j)$  semi closed sets of a fuzzy bitopological space  $(X, \tau_1, \tau_2)$ . Then for each  $\alpha$ , there exists a fuzzy  $\mathfrak{C} - \tau_i$  - closed set  $\mu_{\alpha}$  such that  $\mathfrak{C} \ \mu_{\alpha} \leq \mathfrak{C} \ \lambda_{\alpha} \leq \tau_j$  - cl<sub> $\mathfrak{C}$ </sub> ( $\mathfrak{C} \ \mu_{\alpha}$ ). Thus  $\vee \mathfrak{C} \ \mu_{\alpha} \leq \vee \mathfrak{C} \ \lambda_{\alpha} \leq \vee \tau_j$  - cl<sub> $\mathfrak{C}$ </sub> ( $\mathfrak{C} \ \mu_{\alpha}$ ). Since  $\mathfrak{C}$  satisfies the monotonic and involutive properties by applying Theorem 2.8, we have  $\vee \tau_j$  - cl<sub> $\mathfrak{C}$ </sub> ( $\mathfrak{C} \ \mu_{\alpha}$ )  $\leq \tau_j$  - cl<sub> $\mathfrak{C}$ </sub> ( $\mathfrak{C} \ \mu_{\alpha}$ ). This implies that  $\vee \mathfrak{C} \ \mu_{\alpha} \leq \vee \mathfrak{C} \ \lambda_{\alpha} \leq \tau_j$  - cl<sub> $\mathfrak{C}$ </sub> ( $\mathfrak{C} \ \mu_{\alpha}$ ). By using Lemma 2.6  $\mathfrak{C} \ (\land \mu_{\alpha}) \leq \mathfrak{T}_j$  - cl<sub> $\mathfrak{C}$ </sub> ( $\mathfrak{C}(\land \mu_{\alpha})$ ). Since arbitrary intersection of fuzzy  $\mathfrak{C} - \tau_i$  - closed sets is fuzzy  $\mathfrak{C} - \tau_i$  - closed. That implies that  $\land \mu_{\alpha} = \mu$  is fuzzy  $\mathfrak{C} - \tau_i$  - closed set. Therefore  $\mathfrak{C} \ \mu \leq \mathfrak{C} \ (\land \lambda_{\alpha}) \leq \tau_j$  - cl<sub> $\mathfrak{C}$ </sub> ( $\mathfrak{C} \ \mu$ ). By using Definition 3.1, we have  $\land \lambda_{\alpha}$  is fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi closed.

**Remark 3.11:** The following example shows that Intersection of two fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi open sets need not be fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi open.

**Example 3.12:** Let  $X = \{a, b\}$ ,  $\tau_1 = \{0, \{a_{.2}\}, \{b_{.6}\}, \{a_{.2}, b_{.6}\}, 1\}$  and  $\tau_2 = \{0, \{a_{.3}\}, \{b_{.4}\}, \{a_{.3}, b_{.4}\}, 1\}$ . Let  $\mathfrak{C}(x) = \frac{1-x}{1+4x}$ ,  $0 \le x \le 1$ , be a complement function. Then the family of all fuzzy  $\mathfrak{C} - \tau_i$  - closed sets are  $\mathfrak{C}(\tau_1) = \{1, \{a_{.4}, b_1\}, \{a_{.4}, b_{.1}\}, \{a_{.4}, b_{.1}\}, 0\}$  and  $\mathfrak{C}(\tau_2) = \{1, \{a_{.3}, b_1\}, \{a_{.1}, b_{.2}\}, \{a_{.3}, b_{.2}\}, 0\}$ . Let  $\lambda = \{a_{.3}, b_{.1}\}$  and  $\mu = \{a_{.1}, b_{.7}\}$ . Then it can be calculated that  $\tau_2 - cl_{\mathfrak{C}}(\tau_1 - \operatorname{Int}(\lambda)) = \{a_{.3}, b_{.2}\}$ . Therefore  $\lambda \le \tau_2 - cl_{\mathfrak{C}}(\tau_1 - \operatorname{Int} \lambda)$ . Also  $\tau_2 - cl_{\mathfrak{C}}(\tau_1 - \operatorname{Int}(\mu)) = \{a_{.3}, b_1\}$ . Therefore  $\mu \le \tau_2 - cl_{\mathfrak{C}}(\tau_1 - \operatorname{Int} \lambda)$ . This shows that  $\lambda$  and  $\mu$  are fuzzy  $\mathfrak{C} - (\tau_1, \tau_2)$  - semi open. But  $\lambda \land \mu = \{a_{.1}, b_{.1}\}$ . Now  $\tau_2 - cl_{\mathfrak{C}}(\tau_1 - \operatorname{Int}(\lambda \land \mu)) = \{0\}$ . This shows that  $\lambda \land \mu \not\le \tau_2 - cl_{\mathfrak{C}}(\tau_1 - \operatorname{Int}(\lambda \land \mu))$ . This implies that  $\lambda \land \mu$  need not be fuzzy  $\mathfrak{C} - (\tau_1, \tau_2)$  - semi open even if  $\mathfrak{C}$  satisfies the monotonic and involutive conditions.

**Remark 3.13:** The following example shows that Union of two fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi closed sets need not be fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi closed set even though the complement function satisfies the monotonic and involutive condition.

**Example 3.14:** Let  $X = \{a, b\}$ ,  $\tau_1 = \{0, \{a_{.4}, b_1\}$ ,  $\{a_1, b_{.3}\}$ ,  $\{a_{.4}, b_{.3}\}$ ,  $1\}$  and  $\tau_2 = \{0, \{a_{.1}, b_{.1}\}$ ,  $\{a_{.3}, b_{.1}\}$ ,  $\{a_{.1}, b_{.5}\}$ ,  $\{a_{.3}, b_{.5}\}$ ,  $1\}$ . Let  $\mathfrak{C}(x) = \frac{1-x}{1+6x}$ ,  $0 \le x \le 1$ , be a complement function. Then the family of all fuzzy  $\mathfrak{C} - \tau_i$  - closed sets are  $\mathfrak{C}(\tau_1) = \{1, \{a_{.2}, b_0\}, \{a_{0}, b_{.3}\}$ ,  $\{a_{.2}, b_{.3}\}$ ,  $0\}$  and  $\mathfrak{C}(\tau_2) = \{1, \{a_{.6}, b_{.6}\}, \{a_{.3}, b_{.6}\}, \{a_{.6}, b_{.1}\}, \{a_{.3}, b_{.1}\}, 0\}$ . Let  $\lambda = \{a_{.05}, b_0\}$  and  $\mu = \{a_{.0}, b_{.05}\}$ . Then it can be calculated that  $\tau_2$  - Int<sub> $\mathfrak{C}$ </sub> ( $\tau_1$  - cl<sub> $\mathfrak{C}$ </sub> ( $\lambda$ )) =  $\{0\}$  and  $\tau_2$  - Int<sub> $\mathfrak{C}$ </sub> ( $\tau_1$  - cl<sub> $\mathfrak{C}$ </sub> ( $\lambda \lor \mu$ )) =  $\{a_{.1}, b_{.1}\} \not\leq \lambda \lor \mu$ . This implies that  $\lambda \lor \mu$  is not fuzzy  $\mathfrak{C} - (\tau_1, \tau_2)$  - semi closed.

**Remark 3.15:** The following example shows that the intersection of fuzzy  $\tau_i$  - open set, i=1,2 and fuzzy  $\mathfrak{C}$  - ( $\tau_i$ ,  $\tau_j$ ) - semi open sets need not be fuzzy  $\mathfrak{C}$  - ( $\tau_i$ ,  $\tau_j$ ) - semi open even if the complement function satisfies the monotonic and involutive properties.

**Example 3.16:** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{0, \{b_{.1}\}, \{a_{.2}\}, \{a_{.2}, b_{.1}\}, \{a_{.2}, b_{.8}, c_{.9}\}, 1\}$  and  $\tau_2 = \{0, \{b_{.1}\}, \{a_{.2}, b_{.05}, c_{.3}\}, \{a_{.2}, b_{.1}, c_{.3}\}, \{b_{.05}\}, 1\}$ . Let  $\mathfrak{C}(x) = \frac{1-x}{1+x}$ , be a complement function that satisfies the monotonic and involutive properties. Then the family of all fuzzy  $\mathfrak{C} - \tau_i$  - closed sets are  $\mathfrak{C}(\tau_1) = \{0, \{a_1, b_8, c_1\}, \{a_{.7}, b_1, c_1\}, \{a_{.7}, b_{.8}, c_1\}, \{a_{.7}, b_{.9}, c_{.5}\}, \{a_{.7}, b_{.8}, c_{.5}\}, \{a_{.1}, b_{.9}, c_{.1}\}, 1\}$ . Let  $\lambda = \{a_{.2}, b_{.05}, c_{.1}\}$ . Then it can be calculated that  $\tau_2 - cl_{\mathfrak{C}}(\tau_1 - Int(\lambda)) = \{a_{.7}, b_{.8}, c_{.5}\}$ . This shows that  $\lambda \leq \tau_2 - cl_{\mathfrak{C}}(\tau_1 - Int \lambda)$  and  $\lambda$  is fuzzy  $\mathfrak{C} - (\tau_1, \tau_2)$  - semi open. But  $\mu = \{b_{.1}\}$  is fuzzy  $\tau_i$  - open, i=1,2. Now  $\mu \wedge \lambda = \{b_{.05}\}$ . It can be evaluated that  $\tau_2 - cl_{\mathfrak{C}}(\tau_1 - Int(\mu \wedge \lambda)) = \tau_2 - cl_{\mathfrak{C}}\{0\} = \{0\}$ . Therefore  $\mu \wedge \lambda \leq \tau_2 - cl_{\mathfrak{C}}(\tau_1 - Int(\mu \wedge \lambda))$ . This implies that  $\mu \wedge \lambda$  is not fuzzy  $\mathfrak{C} - (\tau_1, \tau_2)$  - semi open.

**Remark 3.17:** The following example shows that the union of fuzzy  $\mathfrak{C} - \tau_i$  - closed set, i=1,2 and fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi closed sets need not be fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi closed even if the complement function satisfies the monotonic and involutive properties.

**Example 3.18:** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{0, \{a_0, b_{.2}, c_0\}$ ,  $\{a_{.3}, b_0, c_0\}$ ,  $\{a_{.3}, b_{.2}, c_0\}$ ,  $\{a_{.3}, b_{.7}, c_{.8}\}$ , 1} and  $\tau_2 = \{0, \{b_{.2}\}$ ,  $\{a_{.3}, b_{.05}, c_{.4}\}$ ,  $\{a_{3}, b_{.2}, c_{.4}\}$ ,  $\{a_{0}, b_{.05}, c_{.4}\}$ , 1}. Let  $\mathfrak{C}(x) = \sqrt{1 - x^2}$ , be a complement function that satisfies the monotonic and involutive properties. Then the family of all fuzzy  $\mathfrak{C} - \tau_i$  - closed sets are  $\mathfrak{C}(\tau_1) = \{1, \{a_1, b_{.98}, c_1\}$ ,  $\{a_{.95}, b_{.98}, c_1\}$ ,  $\{a_{.95}, b_{.98}, c_1\}$ ,  $\{a_{.95}, b_{.7}, c_{.6}\}$ , 0} and  $\mathfrak{C}(\tau_2) = \{0, \{a_1, b_{.98}, c_1\}$ ,  $\{a_{.95}, b_{.05}, c_{.9}\}$ ,  $\{a_{.95}, b_{.98}, c_{.9}\}$ ,  $\{a_{.95}, b_{.99}, c_{.99}\}$ . Then it can be evaluated that  $\tau_2$  - Int ( $\tau_1 - cl_{\mathfrak{C}}(\lambda)$ ) =  $\{a_{.3}, b_{.2}, c_{.4}\}$ . Therefore  $\lambda \ge \tau_2$  - Int ( $\tau_1 - cl_{\mathfrak{C}} \lambda$ ). That is  $\lambda$  is fuzzy  $\mathfrak{C} - (\tau_1, \tau_2)$  - semi closed. But  $\mu = \{a_1, b_{.98}, c_1\}$  is fuzzy  $\mathfrak{C} - \tau_i$  - closed set, i=1, 2. Also  $\mu \lor \lambda = \{a_1, b_{.99}, c_1\}$ . Now  $\tau_2$  - Int ( $\tau_1 - cl_{\mathfrak{C}}(\lambda \lor \mu)$ ) =  $\tau_2$  - Int ( $1\} = \{1\}$ . This shows that  $\mu \lor \lambda \not\ge \tau_2$  - Int ( $\tau_1 - cl_{\mathfrak{C}}(\lambda \lor \mu)$ ). Therefore  $\mu \lor \lambda$  is not fuzzy  $\mathfrak{C} - (\tau_1, \tau_2)$  - semi closed.

**Theorem 3.19:** Let  $(X, \tau_1, \tau_2)$  be a fuzzy bitopological space and  $\mathfrak{C}$  be a complement function satisfies the monotonic and involutive properties, then (i) If  $\lambda$  is fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi open and  $\lambda \leq \lambda_1 \leq \tau_j$  - cl<sub> $\mathfrak{C}$ </sub> ( $\lambda$ ), then  $\lambda_1$  is fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi open and  $\lambda \leq \lambda_1 \leq \lambda_1 \leq \tau_j$  - cl<sub> $\mathfrak{C}$ </sub> ( $\lambda$ ), then  $\lambda_1$  is fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi closed and  $\tau_j$  - Int  $\lambda \leq \lambda_1 \leq \lambda$ , then  $\lambda_1$  is fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi closed set.

#### **Proof:**

- (i) Let λ be a fuzzy 𝔅 (τ<sub>i</sub>, τ<sub>j</sub>) semi open set. Then by Definition 3.1, there exists μ ∈ τ<sub>i</sub> such that μ ≤ λ ≤ τ<sub>j</sub> cl<sub>𝔅</sub> (μ). Since 𝔅 satisfies the monotonic and involutive properties by Theorem 2.8, we have τ<sub>j</sub> cl<sub>𝔅</sub> (λ) ≤ τ<sub>j</sub> cl<sub>𝔅</sub> (μ). Also λ ≤ λ<sub>1</sub> ≤ τ<sub>j</sub> cl<sub>𝔅</sub> (λ) implies that μ ≤ λ ≤ λ<sub>1</sub> ≤ τ<sub>j</sub> cl<sub>𝔅</sub> (λ) ≤ τ<sub>j</sub> cl<sub>𝔅</sub> (μ). Therefore μ ≤ λ<sub>1</sub> ≤ τ<sub>j</sub> cl<sub>𝔅</sub> (μ). Hence λ is fuzzy 𝔅 (τ<sub>i</sub>, τ<sub>j</sub>) semi open set.
- (ii) Let  $\lambda$  be a fuzzy  $\mathfrak{C} (\tau_i, \tau_j)$  semi closed set. Then by Definition 3.1, there exists fuzzy  $\mathfrak{C} \tau_i$  closed set  $\mu$  such that  $\mathfrak{C} \ \mu \leq \mathfrak{C} \ \lambda \leq \tau_j$  cl<sub> $\mathfrak{C}$ </sub> ( $\mathfrak{C} \ \mu$ ). Since  $\mathfrak{C}$  satisfies the monotonic and involutive conditions by proposition 2.7, we have  $\mu \geq \lambda \geq \tau_j$  lnt  $\mu$ . Since  $\tau_j$  -Int  $(\lambda) \leq \lambda_1 \leq \lambda$ ,  $\mathfrak{C} \ \lambda \leq \mathfrak{C} \ \lambda_1 \leq \tau_j$  cl<sub> $\mathfrak{C}$ </sub> ( $\mathfrak{C} \ \lambda$ ). Therefore  $\mathfrak{C} \ \mu \leq \mathfrak{C} \ \lambda \leq \mathfrak{C} \ \lambda_1 \leq \tau_j$  cl<sub> $\mathfrak{C}$ </sub> ( $\mathfrak{C} \ \lambda$ ). Therefore  $\mathfrak{C} \ \mu \leq \mathfrak{C} \ \lambda \leq \mathfrak{C} \ \lambda_1 \leq \tau_j$  cl<sub> $\mathfrak{C}$ </sub> ( $\mathfrak{C} \ \lambda$ ) =  $\tau_j$  cl<sub> $\mathfrak{C}$ </sub> ( $\mathfrak{C} \ \lambda$ ). Therefore  $\mathfrak{C} \ \mu \leq \mathfrak{C} \ \lambda_1 \leq \tau_j$  cl<sub> $\mathfrak{C}$ </sub> ( $\mathfrak{C} \ \lambda$ ). Therefore  $\mathfrak{C} \ \mu \leq \mathfrak{C} \ \lambda_1 \leq \tau_j$  cl<sub> $\mathfrak{C}$ </sub> ( $\mathfrak{C} \ \mu$ ). Hence  $\lambda_1$  is fuzzy  $\mathfrak{C} (\tau_i, \tau_j)$  semi closed set.

## **4.** Fuzzy $\mathfrak{C} - (\tau_i, \tau_j)$ - semi interior and Fuzzy $\mathfrak{C} - (\tau_i, \tau_j)$ - semi closure

In this section we define the notion of fuzzy  $\mathfrak{C}$ -  $(\tau_i, \tau_j)$  - semi interior and fuzzy  $\mathfrak{C}$ -  $(\tau_i, \tau_j)$  - semi closure operators and discussed some of their properties.

**Definition 4.1:** Let  $(X, \tau_i, \tau_j)$  be a fuzzy bitopological space and  $\mathfrak{C}$  be a complement function. Then for a fuzzy subset  $\lambda$  of X, the fuzzy  $\mathfrak{C}$  -  $(\tau_i, \tau_j)$  - semi interior of  $\lambda$  (briefly  $(\tau_i, \tau_j)$  - SInt  $\mathfrak{c} \lambda$ ), is the union of all fuzzy  $\mathfrak{C}$  -  $(\tau_i, \tau_j)$  - semi open sets of X contained in  $\lambda$ .

That is  $(\tau_i, \tau_j)$  - SInt  $\mathfrak{c}(\lambda) = \bigvee \{\mu : \mu \leq \lambda, \mu \text{ is fuzzy } \mathfrak{C} - (\tau_i, \tau_j) \text{ - semi open} \}.$ 

**Definition 4.2:** Let  $(X, \tau_i, \tau_j)$  be a fuzzy bitopological space and  $\mathfrak{C}$  be a complement function. Then for a fuzzy subset  $\lambda$  of X, the fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi closure of  $\lambda$  (briefly  $(\tau_i, \tau_j)$  - S cl<sub> $\mathfrak{C}$ </sub> $\lambda$ ), is the intersection of all fuzzy  $\mathfrak{C} - (\tau_i, \tau_j)$  - semi closed sets of X containing  $\lambda$ .

That is  $(\tau_i, \tau_j)$  - S cl<sub>C</sub>  $(\lambda) = \land \{\mu : \mu \ge \lambda, \mu \text{ is fuzzy } \mathbb{C} - (\tau_i, \tau_j) \text{ - semi closed} \}.$ 

The concepts of fuzzy  $\mathfrak{C}$  -  $(\tau_i, \tau_j)$  - semi closure and fuzzy  $(\tau_i, \tau_j)$  - semi closure are identical if  $\mathfrak{C}$  is the standard complement function.

**Theorem 4.3:** Let  $(X, \tau_i, \tau_j)$  be a fuzzy bitopological space and  $\mathfrak{C}$  be any complement function. Then for a fuzzy subset  $\lambda$  and  $\mu$  of a fuzzy bitopological space X, we have

- (i)  $(\tau_i, \tau_j)$ -SInt<sub>c</sub> $(\lambda) \leq \lambda$ .
- (ii)  $\lambda$  is fuzzy  $\mathfrak{C} (\tau_i, \tau_j)$  semi open  $\Leftrightarrow (\tau_i, \tau_j)$  SInt<sub> $\mathfrak{C}$ </sub> $(\lambda) = \lambda$ .
- (iii)  $(\tau_i, \tau_j)$  SInt<sub>c</sub>  $((\tau_i, \tau_j)$  SInt<sub>c</sub>  $(\lambda)) = (\tau_i, \tau_j)$  SInt<sub>c</sub>  $(\lambda)$ .
- (iv) If  $\lambda \leq \mu$  then  $(\tau_i, \tau_j)$  SInt<sub>C</sub>  $(\lambda) \leq (\tau_i, \tau_j)$  SInt<sub>C</sub>  $(\mu)$ .

#### **Proof:**

- (i) follows from Definition 4.1.
- (ii) Let  $\lambda$  be a fuzzy  $\mathfrak{C} (\tau_i, \tau_j)$  semi open. Since  $\lambda \leq \lambda$ , by Definition 4.1,  $\lambda \leq (\tau_i, \tau_j)$  SInt<sub> $\mathfrak{C}$ </sub> $(\lambda)$ . By (i),  $(\tau_i, \tau_j)$  S Int<sub> $\mathfrak{C}$ </sub> $(\lambda) = \lambda$ . Conversely we assume that  $(\tau_i, \tau_j)$  SInt<sub> $\mathfrak{C}$ </sub> $(\lambda) = \lambda$ , By Definition 4.1,  $\lambda$  is fuzzy  $\mathfrak{C} (\tau_i, \tau_j)$  semi open.
- (iii) By using (ii), we get  $(\tau_i, \tau_j)$  SInt  $\mathfrak{c}((\tau_i, \tau_j)$  SInt  $\mathfrak{c}(\lambda)) = (\tau_i, \tau_j)$  SInt  $\mathfrak{c}(\lambda)$ .
- (iv) Since  $\lambda \le \mu$ , by using (i),  $(\tau_i, \tau_j)$  SInt  $\mathfrak{c}(\lambda) \le \lambda \le \mu$ . This implies that  $(\tau_i, \tau_j)$  SInt  $\mathfrak{c}((\tau_i, \tau_j) S \operatorname{Int} \mathfrak{c}(\lambda)) \le (\tau_i, \tau_j)$  S Int  $\mathfrak{c}(\mu)$ . By using (iii),  $(\tau_i, \tau_j)$  SInt  $\mathfrak{c}(\lambda) \le (\tau_i, \tau_j)$  SInt  $\mathfrak{c}(\mu)$ .

**Theorem 4.4:** Let  $\mathfrak{C}$  be a complement function that satisfies the monotonic and involutive properties. Then for a fuzzy subset  $\lambda$  of a fuzzy bitopological space X, we have

- (i)  $\mathfrak{C}((\tau_i, \tau_j) \text{SInt}_{\mathfrak{C}}(\lambda)) = (\tau_i, \tau_j) \text{Scl}_{\mathfrak{C}}(\mathfrak{C}\lambda).$
- (ii)  $\mathfrak{C}((\tau_i, \tau_j) \operatorname{Scl}_\mathfrak{C}(\lambda)) = (\tau_i, \tau_j) \operatorname{SInt}_\mathfrak{C}(\mathfrak{C}\lambda).$

#### **Proof:**

- (i) By Definition 4.1, (τ<sub>i</sub>, τ<sub>j</sub>) SInt ε(λ) = ν {μ : μ ≤ λ, μ is fuzzy 𝔅 (τ<sub>i</sub>, τ<sub>j</sub>) semi open}. Taking complement on both sides, we get 𝔅 ((τ<sub>i</sub>, τ<sub>j</sub>) SInt ε(λ)(x)) = 𝔅 {Sup {μ(x): μ(x) ≤ λ(x), μ is fuzzy 𝔅 (τ<sub>i</sub>, τ<sub>j</sub>) semi open}}. Since 𝔅 satisfies the monotonic and involutive properties, By Lemma 2.5, 𝔅 ((τ<sub>i</sub>, τ<sub>j</sub>) SInt ε(λ)(x)) = Inf {𝔅 μ(x) : μ(x) ≤ λ(x), μ is fuzzy 𝔅 (τ<sub>i</sub>, τ<sub>j</sub>) semi open} = Inf {𝔅 μ(x) : μ(x) ≤ 𝔅 λ(x), μ is fuzzy 𝔅 (τ<sub>i</sub>, τ<sub>j</sub>) semi open} = Inf {𝔅 μ(x) : 𝔅 μ(x) ≥ 𝔅 λ(x), μ is fuzzy 𝔅 (τ<sub>i</sub>, τ<sub>j</sub>) semi open} = Inf {𝑘(x) : η(x) ≥ 𝔅 λ(x), η is fuzzy 𝔅 (τ<sub>i</sub>, τ<sub>j</sub>) semi closed}. By Definition 4.2, 𝔅 ((τ<sub>i</sub>, τ<sub>i</sub>) SInt ε(λ)(x)) = (τ<sub>i</sub>, τ<sub>i</sub>) S cl ε(𝔅 λ)(x). Therefore 𝔅 ((τ<sub>i</sub>, τ<sub>i</sub>) SInt ε(λ)) = (τ<sub>i</sub>, τ<sub>i</sub>) S cl ε(𝔅 λ).
- (ii) By Definition 4.2,  $(\tau_i, \tau_j) S \operatorname{cl}_{\mathbb{G}}(\lambda) = \wedge \{\mu : \mu \ge \lambda, \mu \text{ is fuzzy } \mathfrak{C} (\tau_i, \tau_j) \operatorname{semi closed}\}$ . Taking complement on both sides, we get  $\mathfrak{C}((\tau_i, \tau_j) S \operatorname{cl}_{\mathbb{G}}(\lambda)(x)) = \mathfrak{C} \{ \operatorname{Inf} \{\mu(x) : \mu(x) \ge \lambda(x), \mu \text{ is fuzzy } \mathfrak{C} (\tau_i, \tau_j) \operatorname{semi closed} \} \}$ . Since  $\mathfrak{C}$  satisfies the monotonic and involutive properties, By using Lemma 2.6,  $\mathfrak{C}((\tau_i, \tau_j) - S \operatorname{cl}_{\mathbb{G}}(\lambda)(x)) = \operatorname{Sup} \{ \mathfrak{C} \mu(x) : \mu(x) \ge \lambda(x), \mu \text{ is fuzzy } \mathfrak{C} - (\tau_i, \tau_j) - \operatorname{semi closed} \} \}$ .  $\{ \mathfrak{C} \mu(x) : \mu(x) \ge \lambda(x), \mu \text{ is fuzzy } \mathfrak{C} - (\tau_i, \tau_j) - \operatorname{semi closed} \}$ . By Definition  $\mathfrak{C}((\tau_i, \tau_j) - S \operatorname{cl}_{\mathbb{G}}(\lambda)(x)) = \operatorname{Sup} \{ \mathfrak{C} \mu(x) : \mathfrak{C} \mu(x) \le \mathfrak{C} \lambda(x), \mu \text{ is fuzzy } \mathfrak{C} - (\tau_i, \tau_j) - \operatorname{semi closed} \} = \operatorname{Sup} \{ \eta(x) : \eta(x) \le \mathfrak{C} \lambda(x), \eta \text{ is fuzzy } \mathfrak{C} - (\tau_i, \tau_j) - \operatorname{semi closed} \} = \operatorname{Sup} \{ \eta(x) : \eta(x) \le \mathfrak{C} \lambda(x), \eta \text{ is fuzzy } \mathfrak{C} - (\tau_i, \tau_j) - \operatorname{semi closed} \} = \operatorname{Sup} \{ \eta(x) : \eta(x) \le \mathfrak{C} \lambda(x), \eta \text{ is fuzzy } \mathfrak{C} - (\tau_i, \tau_j) - \operatorname{semi closed} \} = \operatorname{Sup} \{ \eta(x) : \eta(x) \le \mathfrak{C} \lambda(x), \eta \text{ is fuzzy } \mathfrak{C} - (\tau_i, \tau_j) - \operatorname{semi closed} \} = \operatorname{Sup} \{ \eta(x) : \eta(x) \le \mathfrak{C} \lambda(x), \eta \text{ is fuzzy } \mathfrak{C} - (\tau_i, \tau_j) - \operatorname{semi closed} \} = \operatorname{Sup} \{ \eta(x) : \eta(x) \le \mathfrak{C} \lambda(x), \eta \text{ is fuzzy } \mathfrak{C} - (\tau_i, \tau_j) - \operatorname{semi closed} \} = \operatorname{Sup} \{ \eta(x) : \eta(x) \le \mathfrak{C} \lambda(x), \eta \text{ is fuzzy } \mathfrak{C} - (\tau_i, \tau_j) - \operatorname{Semi} \mathfrak{C} (\mathfrak{C} \lambda)(x) \} = (\tau_i, \tau_j) - \operatorname{Semi} \mathfrak{C} (\mathfrak{C} \lambda)(x) = (\tau_i, \tau_j) - \operatorname{Semi} \mathfrak{C} (\mathfrak{C} \lambda)(x) \} = (\tau_i, \tau_j) - \operatorname{Semi} \mathfrak{C} (\mathfrak{C} \lambda)$

**Example 4.5:** Let X = {a, b, c},  $\tau_1 = \{0, 1, \{c_{.3}\}, \{a_{.8}, b_{.7}\}, \{a_{.8}, b_{.7}, c_{.3}\}, 1\}$  and  $\tau_2 = \{0, 1, \{c_{.9}\}, \{a_{.8}, b_{.9}\}, \{a_{.8}, b_{.9} c_{.9}\}, \{a_{.9}, b_{.9}, c_{.9}\}\}$ . Let  $\mathfrak{C}(x) = \frac{1}{1+3x}, 0 \le x \le 1$ , be a complement function does not satisfies involute property. Then the family of all fuzzy  $\mathfrak{C}$ -  $\tau_i$  - closed sets are  $\mathfrak{C}(\tau_1) = \{1, \{a_{.25}, b_{.25}, c_{.25}\}, \{a_1, b_1, c_{.526}\}, \{a_{.294}, b_{.3225}, c_1\}, \{a_{.294}, b_{.3225}, c_{.526}\}\}$  and  $\mathfrak{C}(\tau_2) = \{1, \{a_{.25}, b_{.25}, c_{.25}\}, \{a_1, b_1, c_{.27}\}, \{a_{.29}, b_{.27}, c_1\}, \{a_{.29}, b_{.27}, c_{.27}\}, \{a_{.29}, b_{.29}, c_{.27}\}, \{a_{.29}, b_{.29}, c_{.27}\}, \{a_{.29}, b_{.27}, c_{.27}\}, a_{.27}, b_{.27}, b_{.27}, c_{.27}\}$ . Let  $\lambda = \{a_{.2}, b_{.2}, c_{.8}\}$ . Then  $(\tau_1, \tau_2) - S \operatorname{cl}_{\mathfrak{c}}(\mathfrak{C}\lambda) = \{a_{.8}, b_{.8}, c_{.5}\}$ . Therefore  $\mathfrak{C}(\tau_1, \tau_2) - S \operatorname{cl}_{\mathfrak{c}}(\lambda)=\{a_{.25}, b_{.25}, c_{.25}\}$  and  $\mathfrak{C}(\tau_1, \tau_2) - S \operatorname{cl}_{\mathfrak{c}}(\mathfrak{C}\lambda) = \{a_{.2}, b_{.2}, c_{.25}\}$ . This implies that  $\mathfrak{C}(\tau_1, \tau_2) - S \operatorname{cl}_{\mathfrak{c}}(\mathfrak{C}\lambda)$  and  $(\tau_1, \tau_2) - S \operatorname{cl}_{\mathfrak{c}}(\mathfrak{C}\lambda)$  are not equal.

**Theorem 4.6:** Let  $(X, \tau_i, \tau_j)$  be a fuzzy bitopological space and  $\mathfrak{C}$  be a complement function that satisfies the monotonic and involutive properties. Then for a fuzzy subset  $\lambda$  and  $\mu$  of a fuzzy bitopological space X, we have

- (i)  $\lambda \leq (\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda)$ .
- (ii)  $\lambda$  is fuzzy  $\mathfrak{C} (\tau_i, \tau_j)$  semi closed  $\Leftrightarrow (\tau_i, \tau_j)$  S cl<sub> $\mathfrak{C}$ </sub>  $(\lambda) = \lambda$ .
- (iii)  $(\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}((\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda)) = (\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda).$
- (iv) If  $\lambda \leq \mu$  then  $(\tau_i, \tau_j)$  S cl<sub>c</sub> $(\lambda) \leq (\tau_i, \tau_j)$  S cl<sub>c</sub> $(\mu)$ .

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#### **Proof:**

- (i) follows from Definition 4.2.
- (ii) Let λ be a fuzzy 𝔅 (τ<sub>i</sub>, τ<sub>j</sub>) semi closed. Since 𝔅 satisfies the monotonic and involutive conditions, By using Theorem 3.2, 𝔅 λ is fuzzy 𝔅 (τ<sub>i</sub>, τ<sub>j</sub>) semi open. Therefore by Theorem 4.3, we get (τ<sub>i</sub>, τ<sub>j</sub>) SInt 𝔅 (𝔅 λ) = 𝔅 λ. By Proposition 4.4, 𝔅 ((τ<sub>i</sub>, τ<sub>j</sub>) S cl<sub>𝔅</sub>(λ)) = 𝔅 λ. Taking complement on both sides, we get 𝔅 (𝔅 ((τ<sub>i</sub>, τ<sub>j</sub>) S cl<sub>𝔅</sub>(λ))) = 𝔅 (𝔅 λ). Therefore (τ<sub>i</sub>, τ<sub>j</sub>) S cl<sub>𝔅</sub>(λ) = 𝔅 λ. Conversely, we assume that (τ<sub>i</sub>, τ<sub>j</sub>) S cl<sub>𝔅</sub>(λ) = 𝔅 λ. Taking complement on both sides, we get 𝔅 (𝔅 ((τ<sub>i</sub>, τ<sub>j</sub>) S cl<sub>𝔅</sub>(λ))) = 𝔅 (𝔅 λ). Therefore (τ<sub>i</sub>, τ<sub>j</sub>) S cl<sub>𝔅</sub>(λ) = 𝔅 λ. Conversely, we assume that (τ<sub>i</sub>, τ<sub>j</sub>) S cl<sub>𝔅</sub>(λ) = 𝔅 λ. By Theorem 4.3, 𝔅 λ is fuzzy 𝔅 (τ<sub>i</sub>, τ<sub>j</sub>) semi open. Again By using Theorem 3.2, λ is fuzzy 𝔅 (τ<sub>i</sub>, τ<sub>j</sub>) semi closed.
- (iii) By Theorem 4.4,  $\mathfrak{C}((\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda)) = (\tau_i, \tau_j) \operatorname{SInt}_{\mathfrak{C}}(\mathfrak{C} \lambda)$ . This implies that  $\mathfrak{C}((\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda))$  is fuzzy  $\mathfrak{C} (\tau_i, \tau_j) semi$  open. By using Theorem 3.2,  $(\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda)$  is fuzzy  $\mathfrak{C} (\tau_i, \tau_j) semi$  closed. Therefore By using (ii), we get  $(\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda) = (\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda)$ .
- (iv) Suppose  $\lambda \le \mu$  then  $\mathfrak{C} \ \lambda \ge \mathfrak{C} \ \mu$ . This implies that By Theorem 4.3,  $(\tau_i, \tau_j) S$  Int  $\mathfrak{c} (\mathfrak{C} \ \lambda) \ge (\tau_i, \tau_j) S$  Int  $\mathfrak{c} (\mathfrak{C} \ \mu)$ . Since  $\mathfrak{C}$  satisfies the monotonic and involutive and taking complement on both sides, we get  $(\tau_i, \tau_j) - S \operatorname{cl}_{\mathfrak{C}} (\lambda) \le (\tau_i, \tau_j) - S \operatorname{cl}_{\mathfrak{C}} (\lambda)$ .

**Proposition 4.7:** Let  $(X, \tau_1, \tau_2)$  be a fuzzy bitopological space and  $\mathfrak{C}$  be any complement function. Then for a fuzzy subsets  $\lambda$  and  $\mu$  of a fuzzy bitopological space X, we have

- (i)  $(\tau_i, \tau_j) SInt_{\mathfrak{C}}(\lambda \lor \mu) \ge (\tau_i, \tau_j) SInt_{\mathfrak{C}}(\lambda) \lor (\tau_i, \tau_j) SInt_{\mathfrak{C}}(\mu)$  and
- (ii)  $(\tau_i, \tau_j) \operatorname{SInt}_{\mathfrak{C}}(\lambda \wedge \mu) \leq (\tau_i, \tau_j) \operatorname{SInt}_{\mathfrak{C}}(\lambda) \wedge (\tau_i, \tau_j) \operatorname{SInt}_{\mathfrak{C}}(\mu).$

#### **Proof:**

- (i) Since  $\lambda \leq \lambda \vee \mu$  and  $\mu \leq \lambda \vee \mu$ . By Theorem 4.3, we have  $(\tau_i, \tau_j) \text{SInt}_{\mathfrak{C}}(\lambda) \leq (\tau_i, \tau_j) \text{SInt}_{\mathfrak{C}}(\lambda \vee \mu)$  and  $(\tau_i, \tau_j) \text{SInt}_{\mathfrak{C}}(\mu) \leq (\tau_i, \tau_j) \text{SInt}_{\mathfrak{C}}(\lambda \vee \mu)$ . This implies that  $(\tau_i, \tau_j) \text{SInt}_{\mathfrak{C}}(\lambda) \vee (\tau_i, \tau_j) \text{SInt}_{\mathfrak{C}}(\mu) \leq (\tau_i, \tau_j) \text{SInt}_{\mathfrak{C}}(\lambda \vee \mu)$ .
- (ii) Since  $\lambda \ge \lambda \land \mu$  and  $\mu \ge \lambda \land \mu$ . By Theorem 4.3, we have  $(\tau_i, \tau_j)$  SInt<sub> $\mathfrak{C}$ </sub>  $(\lambda \land \mu) \le (\tau_i, \tau_j)$  S Int<sub> $\mathfrak{C}$ </sub>  $(\lambda)$  and  $(\tau_i, \tau_j)$  SInt<sub> $\mathfrak{C}$ </sub>  $(\lambda \land \mu) \le (\tau_i, \tau_j)$  SInt<sub> $\mathfrak{C}$ </sub>  $(\lambda) \land (\tau_i, \tau_j)$  SInt<sub> $\mathfrak{C}$ </sub>  $(\mu)$ .

**Proposition 4.8:** Let  $(X, \tau_1, \tau_2)$  be a fuzzy bitopological space and  $\mathfrak{C}$  be a complement function that satisfies these monotonic and involutive properties. Then for a fuzzy subsets  $\lambda$  and  $\mu$  of a fuzzy bitopological space X, we have

- (i)  $(\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda \lor \mu) = (\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda) \lor (\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\mu)$  and
- (ii)  $(\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda \wedge \mu) \leq (\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda) \wedge (\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\mu).$

#### **Proof:**

- (i) Since  $\mathfrak{C}$  satisfies the involutive condition,  $(\tau_i, \tau_j) S \operatorname{cl}_\mathfrak{C}(\lambda \lor \mu) = (\tau_i, \tau_j) S \operatorname{cl}_\mathfrak{C}(\mathfrak{C}(\mathfrak{C}(\lambda \lor \mu)))$ . Since  $\mathfrak{C}$  satisfies the monotonic and involute properties, by Theorem 4.4,  $(\tau_i, \tau_j) S \operatorname{cl}_\mathfrak{C}(\lambda \lor \mu) = \mathfrak{C}((\tau_i, \tau_j) S \operatorname{In} \mathfrak{t}_\mathfrak{C}(\mathfrak{C}(\lambda \lor \mu)))$ . By Lemma 2.6, we have  $(\tau_i, \tau_j) S \operatorname{cl}_\mathfrak{C}(\lambda \lor \mu) = \mathfrak{C}((\tau_i, \tau_j) S \operatorname{In} \mathfrak{t}_\mathfrak{C}(\mathfrak{C}(\lambda \lor \mu)))$ . Again by Theorem 2.9,  $(\tau_i, \tau_j) S \operatorname{cl}_\mathfrak{C}(\lambda \lor \mu) \leq \mathfrak{C}((\tau_i, \tau_j) \operatorname{SInt}_\mathfrak{C}(\mathfrak{C}(\lambda)))$ .  $(\tau_i, \tau_j) S \operatorname{cl}_\mathfrak{C}(\lambda \lor \mu) \leq \mathfrak{C}((\tau_i, \tau_j) \operatorname{SInt}_\mathfrak{C}(\mathfrak{C}(\lambda)))$ .  $(\mathfrak{C}(\tau_i, \tau_j) S \operatorname{Int}_\mathfrak{C}(\mathfrak{C}(\lambda)))$ . By Theorem 4.4,  $(\tau_i, \tau_j) S \operatorname{cl}_\mathfrak{C}(\lambda \lor \mu) \leq \mathfrak{C}((\tau_i, \tau_j) \operatorname{SInt}_\mathfrak{C}(\mathfrak{C}(\lambda)))$ .  $(\mathfrak{C}(\tau_i, \tau_j) S \operatorname{Int}_\mathfrak{C}(\mathfrak{C}(\lambda)))$ . By Theorem 4.4,  $(\tau_i, \tau_j) S \operatorname{cl}_\mathfrak{C}(\lambda \lor \mu) \leq \mathfrak{C}((\tau_i, \tau_j) \operatorname{SInt}_\mathfrak{C}(\mathfrak{C}(\lambda)))$ . Again by  $(\tau_i, \tau_j) S \operatorname{cl}_\mathfrak{C}(\lambda \lor \mu) \leq \mathfrak{C}((\tau_i, \tau_j) \operatorname{SInt}_\mathfrak{C}(\mathfrak{C}(\lambda)))$ . Also  $(\tau_i, \tau_j) S \operatorname{cl}_\mathfrak{C}(\lambda \lor \mu) \leq \mathfrak{C}(\tau_i, \tau_j) S \operatorname{cl}_\mathfrak{C}(\lambda) \lor (\tau_i, \tau_j) S \operatorname{cl}_\mathfrak{C}(\lambda) \lor (\tau_i, \tau_j) S \operatorname{cl}_\mathfrak{C}(\lambda \lor \mu)$ . This implies that  $(\tau_i, \tau_j) S \operatorname{cl}_\mathfrak{C}(\lambda) \lor (\tau_i, \tau_j) S \operatorname{cl}_\mathfrak{C}$
- (ii) Since  $(\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda \wedge \mu) \leq (\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda)$  and  $(\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda \wedge \mu) \leq (\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\mu)$  which implies that  $(\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda \wedge \mu) \leq (\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda) \wedge (\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\mu)$ .

**Proposition 4.9:** Let  $\mathfrak{C}$  be a complement function that satisfies the monotonic and involutive properties. Then for any family  $\{\lambda_{\alpha}\}$  of fuzzy subsets of a fuzzy bitopological space, we have

- (i)  $\vee((\tau_i \ \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda_{\alpha})) \leq (\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\nu \lambda_{\alpha})$
- (ii)  $(\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\wedge \lambda_{\alpha}) \leq \wedge ((\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda_{\alpha}))$

#### **Proof:**

- (i) For every  $\beta$ ,  $\lambda_{\beta} \leq \nu \lambda_{\alpha} \leq (\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\nu \lambda_{\alpha})$ . By Theorem 4.6,  $(\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda_{\beta}) \leq (\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\nu \lambda_{\alpha})$  for every  $\beta$ . Therefore  $\nu((\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda_{\beta})) \leq (\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\nu \lambda_{\alpha})$ . This proves (i).
- (ii) For every  $\beta$ ,  $\wedge \lambda_{\alpha} \leq \lambda_{\beta}$ . Again by using Theorem 4.6,  $(\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\wedge \lambda_{\alpha}) \leq (\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda_{\beta})$ . This implies that  $(\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\wedge \lambda_{\alpha}) \leq \wedge ((\tau_i, \tau_j) S \operatorname{cl}_{\mathfrak{C}}(\lambda_{\alpha}))$ . This proves (ii).

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