International Journal of Mathematical Archive-8(4), 2017, 129-136
Available online through www.ijma.info ISSN 2229 – 5046

ON COC-LIGHT FUNCTIONS

HAIDER JEBUR ALI*[1], HUDA FADEL ABASS[2]

Al-Mustansiriyah University,
College of Science, Department of Mathematics, Baghdad, Iraqi.

(Received On: 27-03-17; Revised & Accepted On: 25-04-17)

ABSTRACT

In this paper we study coc-open sets which used to introduce some concepts namely coc-disconnected, coc-totally disconnected spaces, coc*-continuous, Inversely coc (coc*, coc**) totally disconnected and coc-light functions. Some facts, Examples and propositions have been given to support our work.

***Key words**: coc-open sets, coc-compact and coc-T_2 spaces.coc-continuous, coc-homeomorphism functions.*

1-INTRODACTION

In [5] S.AlGHour and S.Samarah introduced the concept coc-open sets in topological spaces and in [3] a space X is said to be totally disconnected if for every pair of distinct points a, b ЄX has a disconnection A∪B to X such that a ϵ A and b ϵ B and the author in[6] introduced a concept namely light mapping (= A surjective mapping f:X→Y is called light mapping if for every y ∈ Y, $f^{-1}(y)$ is totally disconnected set) we used the concept coc-open set to define some types of spaces and functions like coc-disconnected, coc-totally disconnected spaces and coc-light, coc*-continuous, Inversely coc (coc*, coc**) totally disconnected and coc-homeomorphism functions. Throw our work X is mean a topological space.

Definition (1), [5]: Asub set A of a space (X,Ʈ) is called co -compact open set (briefly coc-open) if for every x∈A, there exists an open set U⊆X and a compact subset K such that x∈U-K⊆A. The complement of coc-open subset is called coc-closed.

Remarks 1:

1. The set of all coc-open sets forms a topology on X denoted by $Ʈ^{K}$**[5]**
2. Every open set is coc-open set but the converse may be not true in general as in the following:

Example 1: (R, $Ʈ_{ind}$), then Q and Q^{C} are coc-open sets which they not open, where $Ʈ_{ind}$ is the indiscrete topology.

Definition 2, [2]: Let f be a function from a space (X, Ʈ) into an space (Y,Ʈ), then f is said to be coc-continuous function if the inverse image of every open set U in Y is coc-open in X.

Clearly, every continuous function is coc-continuous function but the converse may be not true.

Example 2: Let f: (R, $Ʈ_{ind}$) → (R, $Ʈ_{D}$) such that f(x) = x for each $x \in R$

Since any subset of (R, $Ʈ_{D}$) is open.

If {1} ∈ $Ʈ_{D}$, f^{-1}({1})={1} is not open in (R, $Ʈ_{ind}$), since the only open sets in (R, $Ʈ_{ind}$) are ∅ and R.

Now if we take A = {1}

For each x ∈ A ∃ just 1∈A such that 1∈R-(R, {1}) = {1}⊆A={1} clearly R-{1} is compact in (R, $Ʈ_{ind}$). So f^{-1}(A) = A is coc-open set.

Therefore f is coc-continuous but not continuous.

Corresponding Author: Haider Jebur Ali[1]*
Al-Mustansiriyah University, College of Science, Department of Mathematics, Baghdad, Iraqi.

Definition 3 [1]: A space X is said to be coc-compact if every coc-open cover of X has finite sub cover.

Remark: Every coc-compact space is compact.

Example 3: (R, τ_{ind}) is compact but not coc-compact

Since the space is compact

To prove its not coc-compact.

Let $\{\{x\}\}_{\in R}$ be coc –open cover to R, Where {x} is coc-open set for each x ∈ X, there exists th only open set R which contain x and R-{x} is compact subset of R such that x∈{x}=R-(R-{x})⊆{x}.

But we cannot reduce this cover to a finite sub cover. Since if we remove one coc-open set $\{x_0\}$ so the remind coc-open cover is cover R-$\{x_0\}$.which is not finite sub cover to R.

So (R, τ_{ind}) is not coc-compact space.

Definition 4[2]:A space X is said to be coc-hausdorff space if for each x≠y in X there exist two distinct points an coc-open sets U, V such that x∈U, y∈V.

Remark 2, [2]: It is clear that every hausdorff space is coc-hausdorff space. But the converse is not true in general as in the following.

Example 2-4: Let (R, τ_{ind}) be indiscrete space.
Then (R, τ_{ind}) is not T_2-space, but (R, τ_{ind}) is coc T_2-space, since for each x, y∈ R with x≠y we have {x} and {y} are disjoint coc-open sets containing x and y respectively.

Definitions 5: Let f:X→Y be function of a space X into a space Y then:

i: f is called an coc-closed function if f(A) is an coc-closed set in Y for every closed set A in X **[2]**.
ii f is called an coc*-closed function if f(A) is an closed set in Y for every coc-closed set A in X.
iii f is called an coc**-closed function if f(A) is an coc-closed set in Y for every coc-closed set A in X **[2]**.

Definition 6 [2]: Let X and Y be spaces and let f be a function from X into Y then f is said to be coc-homeomorphism if:

1. f is bijective.
2. f is coc-continuous.
3. f is coc-closed (coc-open).

It is clear that every homeomorphism is an coc-homeomorphism.

Example 5: I_R: (R, τ_{ind}) →(R, τ_D) where I_R be the identity function .Then I_R is bijective, coc-open and coc-continuous.

So it is coc-homeomorphism but not homeomorphism since if it is not continuous.

Definition 7: Let (X,τ) be a space and let A, B be nonempty coc -open sets on X, then A∪B is said to be coc-disconnection to X if and only if A∪B=X and A∩B=ϕ.

Definition 8: A space X is said to be coc-disconnected if there is coc-disconnection A∪B to X.

So a space X is coc-connected if it is not coc-disconnected.

Example 6: (R, τ_D) is coc-disconnected space.

Since R=Q∪Q^C, Q∩Q^C=ϕ where Q and Q^C are coc-open sets.

Clearly the space is also disconnected.

Remark 3: If the set is both coc-open and coc-closed, then we say that its coc-clopen as in discrete space.

Lemma 1: A space X is said to be coc-disconnected if it has nonempty proper subset which is coc-clopen set.

Proof: Let A be a nonempty proper subset of X which is coc-clopen.

To prove X is coc-disconnected. let $B=A^c$, then B is nonempty (since A is proper subset of X) moreover, $A\cup B = X$ and $A\cap B = \phi$.

Since A is coc-clopen that is, A is coc-closed so B is coc-open but A is coc-open

So X is coc-disconnected.

Conversely:

Let X is coc-disconnected. Then there exist nonempty subsets A and B are coc-open sets in X such that $A\cap B=\emptyset$, $A\cup B=X$. Since B is coc-open in X $\Longrightarrow$ B^C is coc-closed in X but $A\cap B=\emptyset$, so $B^C=A$ (A, B are nonempty sets) $\Longrightarrow$A is coc-clopen in X. So B is similarly.

Then A and B are coc-clopen sets.

Definition 9: A space X is said to be coc-totally disconnected if for every pair of distinct points a, b $\in$X has coc-disconnection $A\cup B$ to X such that $a \in A$ and $b \in B$.

Example 7: (R, τ_D) is coc-totally disconnected.

let R be the set of all real number with discrete space, then for every two distinct points s, n we have {s}, R-{s} are two coc-open sets containing s, n respectively.

Remark 4: Every coc-totally disconnected is coc-disconnected but the converse may be not true in general.

Example 8: Let $X = (m, n)\cup(r, s)$ such that $m<n<r<s$, m, n, r, s distint point in R (the set of all real numbers). Let τ be topological space define to X.

But $X = (m, n)\cup(r, s)$, $(m, n)\cap(r, s) = \phi$, so $(m, n)\cup(r, s)$ is coc-disconnection to X.

Then X is coc-disconnected but X may be not coc-totally disconnected.

Proposition 1: Let X be a space and let $Y\subseteq X$ if X is coc-totally disconnected, then Y is also coc-totally disconnected space.

Proof: Let m, n are different points in Y but $Y\subseteq X$, then $m, n \in X$, which is coc-totally disconnected, then there exists coc-disconnection $M\cup N$ to X such that $m \in M$ and $n\in N$, so $m \in M\cap Y$ and $n \in N\cap Y$.

But $(M\cap Y)\cup(N\cap Y) = (M\cup N)\cap Y=X\cap Y=Y$
$(M\cap Y)\cap (N\cap Y) = (M\cap N) \cap Y=\phi \cap Y=\phi$.

Then there exists coc-disconnection $M\cup N$ to Y, such that $M\cap Y \neq \phi$ and $N\cap Y \neq \phi$ are coc-open sets in Y.

Then Y is also coc-totally disconnected sub space.

Now we introduce the following definition:

Definition 10:A function f: X→Y is said to be coc*-continuous if the inverse image of coc-open set in Y is open set in X.

Proposition 2: Let f: X→Y be bijective coc*-continuous function, a space Y is coc-compact if X is compact space.

Proof: Let $\{U\alpha\}_{\alpha\in\Omega}$ be a coc-open cover to Y then $f^{-1}\{U\alpha\}_{\alpha\in\Omega}$ be an open cover to X (since f is coc*-continuous function).

But $Y\subseteq\bigcup_{\alpha\in\Omega} U\alpha \Longrightarrow X=f^{-1}(Y) \subseteq f^{-1}(\bigcup_{\alpha\in\Omega} U\alpha) = f^{-1}\{U\alpha\}$

Proposition 3: Every coc-compact sub set of coc-T_2 space is coc-closed.

Proposition 4: Let f: X→Y be coc*- continuous function where X and Y are spaces.

If X is coc-compact and Y is coc- T_2 space then f is coc-closed.

Proof: Let F be Closed Set in X which coc-compact Then X is Compact (by remark 2)

So F compact, but f is coc^*-continuous. Then f (F) is coc-compact in Y, Which coc-T_2 then by (proposition (3))

f(F) is coc-closed set in Y.

Then f is coc –closed function.

Proposition 5: Let f: X→Y be continuous function where X and Y are spaces.

If X is coc-compact and Y is T_2 space then f is coc*-closed.

Proof: Since F is coc-closed set in X which is X is coc-compact so by proposition (every coc-closed subset of coc-compact is coc-compact)

Then F is coc-compact in X (by remark 2) ⟹ F is compact in X

But f is continuous then f(F) is compact in Y by proposition(the continuous image of compact set is also compact)

But Y is T_2-space then by proposition: (every compact subset of T_2 space is closed).

Then f(F) is closed in Y.

Therefore f is coc*-closed function.

Proposition 6: Let f: X→Y be coc**- continuous function where X and Y are spaces.

If X is coc-compact and Y is coc- T_2 space then f is coc**-closed.

Proof: Since F is coc-closed set in X which is X is coc-compact so by proposition (every coc-closed subset of coc-compact is coc-compact)

Then F is coc-compact in X

But f is coc**-continuous then f(F) is compact in Y by proposition(the coc**-continuous image of compact set is coc-compact)

But Y is coc-T_2 space then by proposition: (every coc-compact subset of coc-T_2 space is coc-closed)

Then f(F) is coc-closed in Y

Therefore f is coc*-closed function.

Definition 11: A surjective function f: X→Y is said to be coc-totally disconnected function if and only if for every coc-totally disconnected G⊆X, f(G) is coc-totally disconnected in Y.

Remark 5: The coc-continuous image of coc-totally disconnected set not necessary coc-totally disconnected set for example:

Example 9: Let X = {a, b, c} where τ_D is discrete topology to X.

And Y = {p, q} where τ_{ind} is indiscrete topology to Y.

A coc-continuous f: X→Y define the following:

f(a)=f(b) =p, f(c)=q

Note that f is coc-continuous and X is coc-totally disconnected but y is indiscrete space which is not coc-totally disconnected.

Remark 6: Every totally disconnected is coc-totally disconnected but the converse may be not true in general:

Example 10: Let (R, τ_{ind}) is coc-totally but not totally disconnected.

To prove it is coc-totally disconnected.

If x, y ∈ Q ⊆ R with $x \neq y$, there exist q ∈ Q^C such that x<q<y then U={r ∈ R: r < q} and V={r ∈ R: r ≥ q} so U∪V =R, U∩V=∅, we claim U is coc-open set in R, since there exist only R open in R with x∈ R and U_1={r ∈ R: r < x} so U_1∪V is compact in R.
⟹ x∈R-(U_1∪V) ⊆ U!!!!

Proposition 7: Let X and Y be spaces and let f: X→Y be bijective and coc-open function.

If X be totally disconnected, then Y is coc-totally disconnected.

Proof: Let y_1 and y_2 Є Y with $y_1 \neq y_2$, but f is surjective function, then there exist only two points x_1, x_2Є X such that $f(x_1)=y_1, f(x_2)=y_2$, also X is totally disconnected

Then there exists disconnection G∪H to X such that $x_1 \epsilon G$ and $x_2 \epsilon H$, f is coc-homeomorphism. Where G and H are two open sets.

Then f(G) and f(H)are coc-open sets in Y and f(G)∪f(H)=f(G∪H)=f(X)=Y, but f is bijective function,
f(G)∩f(H) = f(G∩H) = f(ϕ)=ϕ such that $y_1 \epsilon f(G)$, $y_2 \epsilon f(H)$

Then f(G)∪f(H) is coc-disconnection to Y.

That mean Y is coc-totally disconnected.

Corollary 1: Let X and Y are spaces and let f: X→Y be coc-homeomorphism.

If X be totally disconnected, then Y is coc-totally disconnected.

Proposition 8: Let X and Y are spaces and let f: X→Y be coc-homeomorphism.

If X be coc-totally disconnected, then Y is also coc-totally disconnected.

Proof: Let y_1 and y_2 Є Y with $y_1 \neq y_2$, but f is surjective function, then there exist only two points x_1, x_2Є X such that $f(x_1) = y_1$, $f(x_2) = y_2$, also X is coc-totally disconnected

Then there exists coc-disconnection G∪H to X such that $x_1 \epsilon G$ and $x_2 \epsilon H$, f is coc-homeomorphism.so G and H are coc-open sets in X.

Then f(G) and f(H) are coc-open sets in Y and f(G)∪f(H) = f(G∪H) = f(X)=Y, but f is bijective function,
f(G)∩f(H) = f(G∩H) = f(ϕ) = ϕ such that $y_1 \epsilon f(G)$, $y_2 \epsilon f(H)$

Then f(G)∪f(H) is coc-disconnection to Y.

That mean Y is coc-totally disconnected

Definition 12: f: X→Y is said to be coc*-open if f(A) is open in Y whenever A is coc-open in X.

Proposition 9: Let f: X→Y be coc*-open function and bijective, if X is coc-totally disconnected then Y is totally disconnected.

Proof: Let y_1, y_2 ∈Y with $y_1 \neq y_2$ but f is bijective function, then there exist only two point x_1, x_2∈X such that $f(x_1)=y_1$, $f(x_2)=y_2$.

Also X is coc-totally disconnected, then there exist coc-disconnected G∪H to X such that x_1∈G and x_2∈H.f is coc*-open function, then f(G) and f(H)are open in Y. f(G)∪ f(H)=f(G∪H)=f(X)=Y

But f is bijective function, f(G)∩ f(H) = f(G∩H) = f(∅) = ∅

Such that y_1∈f(G)and y_2∈f(H) then f(G)∪f(H) is disconnected to Y.

That mean Y is totally disconnected.

Now we introduce the following definitions:

Definition 13: A surjective function f: (X,Ʈ)→(Y, σ) is said to be inversely coc -totally disconnected mapping if and only if for every totally disconnected set G in Y, $f^{-1}(G)$ is coc-totally disconnected set in X.

Definition 14: A surjective function f: (X,Ʈ)→(Y, σ) is said to be inversely coc* -totally disconnected mapping if and only if for every coc- totally disconnected set G in Y, $f^{-1}(G)$ is totally disconnected set in X.

Definition 15: Asurjective function f: (X,Ʈ)→(Y, σ) is said to be inversely coc** -totally disconnected mapping if and only if for every coc-totally disconnected set G in Y, $f^{-1}(G)$ is coc-totally disconnected set in X.

Propositions 10:

1. Every coc*-totally disconnected is coc-totally disconnected.
2. Every coc**-totally disconnected is coc-totally disconnected.
3. Every coc*-totally disconnected is coc**-totally disconnected.

Proof:

1. Let G is totally disconnected in Y⟹ G is coc-totally disconnected (remark 6).
   $f^{-1}(G)$ is totally disconnected by (definition 14) ⟹ $f^{-1}(G)$ is coc-totally disconnected (by remark 6).
   So f is coc-totally disconnected.
2. Let G is totally disconnected in Y⟹ G is coc-totally disconnected (remark 6).
   $f^{-1}(G)$ is coc-totally disconnected by (definition 15).
   So f is coc-totally disconnected
3. Let G is coc-totally disconnected in Y.
   $f^{-1}(G)$ is totally disconnected by (definition 14) ⟹ $f^{-1}(G)$ is coc-totally disconnected (by remark 6).
   So f is coc**-totally disconnected.

Definition 16: Let X and Y be two spaces. A surjictive function f: (X, τ) → (Y, σ) is said to be coc-light function if f(y) is coc-totally disconnected for each y∈Y.

Example 11: Let f:(R,$Ʈ_D$)→(R, $Ʈ_U$) be coc-continuous define the following: f(x)=5 for each x∈R so fis coc-light mapping.

Remarks 7:

1. Every light mapping is coc-light mapping but the converse may be not true in general for example:
2. Every coc-homeomorphism is coc-light mapping but the converse may be not true in general as in example7:

Theorem 1: Let f: X→Y and g:Y→K are surjictive functions then a surjictive function h: X→K such that h=g∘f is light function if f is inversely coc*-totally disconnected and g is coc-light function.

Proof: To prove h is light function, let k∈K but g is coc-light function, then $g^{-1}(k)$ is coc-totally disconnected.

Also f is inversely coc*-totally disconnected mapping, then $f^{-1}(g^{-1}(k))$ is totally disconnected.

But $f^{-1}(g^{-1}(k))=(g\circ f)^{-1}(k)=h^{-1}(k)$.

So h is light mapping.

Theorem 2: Let f: X→Y and g: Y→K are surjictive functions then a surjictive function h:X→K such that h=g∘f is coc-light function if f is inversely coc-totally disconnected and g is light function.

Proof: To prove h is coc-light function, let k ∈ K but g is light function, then $g^{-1}(k)$ is totally disconnected

Also f is inversely coc-totally disconnected function, then $f^{-1}(g^{-1}(k))$ is coc-totally disconnected.

But $f^{-1}(g^{-1}(k))=(g\circ f)^{-1}(k)=h^{-1}(k)$.

So h is coc-light mapping.

Theorem 3: Let f: X→Y and g:Y→K are surjictive functions then a surjictive function h:X→K such that h=g∘f is coc-light function if f is inversely coc**-totally disconnected and g is coc-light function.

Proof: To prove h is coc-light mapping, let k ∈ K but g is coc-light function, then $g^{-1}(k)$ is inversely coc^{**}-totally disconnected.

Also f is inversely coc-totally disconnected function, then $f^{-1}(g^{-1}(k))$ is coc-totally disconnected.

But $f^{-1}(g^{-1}(k)) = (g\circ f)^{-1}(k) = h^{-1}(k)$. So h is coc-light mapping.

Theorem (4): Let $h:X\to Y$ be surjective mapping, where $h=g\circ f$, $f: X\to K$ and $g:K\to Y$ are mappings then:

1. If g bijective mapping and f is coc-light mapping, then h is coc-light mapping.
2. If h coc-light mapping and g is injective mapping, then f is coc-light mapping.
3. If h is coc-light mapping and f is surjective coc**-totally disconnected then g is coc-light mapping again.

Proof:

1. let $y\in Y$

   As g is bijective mapping then there exists one and only one point $k \in K$ such that $g(k) = y$.

   But $h^{-1}(y) = (g\circ f)^{-1}(y) = f^{-1}(g^{-1}(y)) = f^{-1}(g^{-1}(g(k))) = f^{-1}(k)$.

   But f is coc-light mapping, then $f^{-1}(k)$ is coc-totally disconnected in X.

   but $h^{-1}(y) = f^{-1}(k)$, then $h^{-1}(y)$is coc-totally disconnected.

   So we get h is coc-light mapping.

2. let $k\in K$, then $g(k)\in Y$ but h is coc-light mapping, then $h^{-1}(g(k))$ is coc-totally disconnected in X .
   Let $h^{-1}(g(k)) = (g\circ f)^{-1}(g(k)) = f^{-1}(g^{-1}(g(k))) = f^{-1}(k)$ (since g is injective mapping).

   So $f^{-1}(k)$ is coc-totally disconnected in X, then f is coc-light mapping.

3. let $y \in Y$

   But h is coc-light mapping, so $h^{-1}(y)$ is coc-totally disconnected in X.

   But f is coc-totally disconnected, then $f(h^{-1}(y))$ is coc-totally disconnected set in K.

   But $f(h^{-1}(y)) = f((g\circ f)^{-1}(y)) = f(f^{-1}(g^{-1}(y))) = g^{-1}(y)$ (since f is surjictive mapping).

   So $g^{-1}(y)$ is coc-totally disconnected in K.

   Then g is coc-light mapping.

Theorem 5: Let $f: X_1\to Y_1$ and $g: X_2\to Y_2$ be surjective mappings so a mapping $f\times g: X_1\times X_2\to Y_1\times Y_2$ is coc-light mapping if f is coc-homeomorphism and g is coc-light mapping.

Proof: Let $(y_1, y_2) \in Y_1\times Y_2$

So $(f\times g)^{-1}(y_1, y_2) = (f^{-1}\times g^{-1})(y_1, y_2) = f^{-1}(y_1)\times g^{-1}(y_2)$

But f is homeomorphism, so there exist $x_1 \in X_1$ such that $f^{-1}(y_1) = f^{-1}(f(x_1)) = x_1$
I mean $(f\times g)^{-1}(y_1, y_2) = x_1\times g^{-1}(y_2)$

But g is coc-light mapping, so $f_2^{-1}(y_2)$ is coc-totally disconnected.

But $\{x_1\}\times g^{-1}(y_2)$ is homeomorphic to $g^{-1}(y_2)$, then $(f\times g)^{-1}(y_1,y_2)$is coc-totally disconnected. So $f\times g$ is coc-light mapping.

Lemma 2: Let $f: X_1\to Y_1$ and $g: X_2\to Y_2$ are surjictive mappings.

If a mapping $f\times g: X_1\times X_2\to Y_1\times Y_2$ is coc-light mapping then:

1. If f is coc-homeomorphism, then g is coc-light mapping.
2. If g is coc-homeomorphism, then f is coc-light mapping.

Proof:

1. let $(y_1,y_2)\in Y_1\times Y_2$, whereas $(f\times g)^{-1}(y_1,y_2)=(f^{-1}\times g^{-1})(y_1,y_2)=f^{-1}(y_1)\times g^{-1}(y_2)$.

   But f is coc-homeomorphism, then there exists x_1 in X_1 such that $f^{-1}(y_1) = x_1$

$(f\times g)^{-1}(y_1, y_2) = \{x_1\}\times g^{-1}(y_2)$. But $f\times g$ is coc-light mapping, then $(f\times g)^{-1}(y_1, y_2)$ is coc-totally disconnected

So we get $\{x_1\}\times g^{-1}(y_2)$ is coc-totally disconnected.

But $\{x_1\}\times g^{-1}(y_2)$ is coc-homeomorphic to $g^{-1}(y_2)$, then $g^{-1}(y_2)$ is coc-totally disconnected. Hence g is coc- light mapping.

2. let $(y_1, y_2)\in Y_1\times Y_2$.
   $(f\times g)^{-1}(y_1, y_2)=f^{-1}(y_1)\times g^{-1}(y_2)$.

   But g is coc-homeomorphism, then there exists $x_2\in X$ such that $g^{-1}(y_2) = x_2$

   So $(f\times g)^{-1}(y_1, y_2) = f^{-1}(y_1)\times\{x_2\}$
   But $f\times g$ is coc-light mapping, then $(f\times g)^{-1}(y_1, y_2)$ is coc-totally disconnected.

   So we get $f^{-1}\times\{x_{2\}}$ is coc-totally disconnected.

   But $f^{-1}(y_1)\times\{x_2\}$ is coc-homeomorphic to $f^{-1}(y_1)$, then $f^{-1}(y_1)$ is coc-totally disconnected. So f is coc-light mapping.

REFERENCES

1. Dugnndje J, Topology, Allyn and Bacon, Boston (1966).
2. F.H.Jasim" On compactness Via Cocompact open sets" M.Sc. Thesis University of Al-Qadissiya, college of Mathematics and computer science, 2014.
3. G.F.Simmons, "Introduction Topology and Modern Analysis", Mcgraw-Hill Book compapang (1963).
4. N.Bourbaki, Elements of Mathematics "General topology" Chater 1-4, Spring Vorlog, Belin, Heidelberg, New-Yourk, London, Paris, Tokyo 2nd Edition (1989).
5. S.Alghour and s.samarah" cocompact open sets and continuity" Abstract and applied analysis, Volume 2012, Article ID 548912, 9 pages, 2012.
6. Whyburn G.T., Analytic to topology, A.M.S.colloquium Publication, New York 28 (1955).

Source of support: Nil, Conflict of interest: None Declared.