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CRYPTOGRAPHY BY APPLYING SUMUDU TRANSFORM TO TRIGONOMETRIC SINE FUNCTION

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ABSTRACT

In this paper we introduce the application of Sumudu transform & inverse Sumudu transform in the process of encryption &decryption (i.e Cryptography) respectively. In the first part of the paper we consider one plain text and convert it to clipher text by applying Sumudu transform to trigonometric sine function and in the second part we convert clipher text to plain text by applying inverse sumudu transform. Finally we generalize some results for encryption and decryption.

Keywords: *Sumudu transform, encryption, decryption, Cryptography, trigonometric sine function.*

1. INTRODUCTION

In Mathematical Sciences transformation is such a device which is used to convert an integrable function $f(x)$to function of new variable like $f(s), f(p)$,etc.There are several integral transforms as Laplace transform, Sumudu transform, Elzaki transform etc.

In 1990 Gamage K. Watugala has introduced new integral transform which is similar to Laplace transform [1] which is defined by

$$G(w) = \int_0^\infty \frac{1}{w} e^{\frac{-x}{w}} f(x)dx$$

He introduced this transform to solve differential equations and control engineering problems [1]. Some fundamental properties of Sumudu transform were established [7].

The word Cryptography comes from the Greek word kryptos which means hidden and graphein means 'to write". After demonetization in 2016 people prefers cashless transactions like A.T.M., Paytm, Mobile banking, internet banking etc.To operate these facilities password is required. Cryptography is the branch in which we study some techniques to secure communication between two persons or to keep confidentiality of their communication. Some integral transforms plays an important role in the process of cryptography. A new cryptographic scheme was developed by applying Laplace transform in [3], [4] and obtained key which is the number of multiples of mod n therefore it is not easy for eyedropper to trace the key by any attack

In this work we present cryptography by applying Sumudu transform and inverse Sumudu transform to trigonometric sine function.

Generally scientists, software engineers works in various projects and password is required for communication between two scientists confidentially. As Engineers and scientists have knowledge of transforms theory so they can use process of cryptography applying integral transforms easily.

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Some definitions and theorems are given below

Theorem 1.1: Let $H_0, H_1\ H_2 H_3, H_4, \ldots\ldots$ be coefficients $of\ t^2 \sinh 2t$ then given plaintext in terms of H_i i=0, 1, 2, 3, 4,…… under Laplace transform of H$t^2 \sinh 2t$ can be converted to cipher text $H_i{}'$ =r_i-26k_i for i =0, 1, 2, 3,……. where

r_i = 2^{2i+1}(2i+3) (2i+2) H_i for i=1, 2, 3, 4… and a key is given by k_i =$\frac{r_i - H_i{}'}{26}$ for i =0, 1, 2, 3, 4…

Theorem 1.2: The given cipher text in terms of $H_i{}'$, i=0,1,2,3,4,……. With a given key $k_i\ for$ i = 0,1,2,3,4, … …. can be converted to plain text H_i under the inverse Laplace transform of

H$\frac{d2}{dp2}\frac{2}{p^2-2^2}$ =$\sum_{i=0}^{\infty}\frac{r_i}{p^{2i+4}}$ where H_i =$\frac{26k_i+H_i{}'}{2^{2i+1}(2i+2)(2i+3)}$ for i=1, 2, 3, 4….. and $r_i = 26k_i + H_i{}'$

The above theorems are proved in [3] by using Laplace transform and inverse Laplace transform of hyperbolic sine and cosine functions. Here we will prove the above theorems by applying Sumudu transform to trigonometric sine function.

Definition 1.1: Cryptology: It is the study of secrecy systems which can be traced back to the early Egyptians.

Definition 1.2: Plain text: The original message which is to be transmitted in such a form having secrecy.

Definition 1.3: Cipher text: when we convert the original message in the form having secrecy then this new form is said to be cipher text.

Definition 1.4: cipher: The method of converting plain text to cipher text is called cipher.

Definition 1.5: Encrypting: The process of converting plain text to cipher text is known as encrypting.

Decrypting: The reverse process by the beneficiary who knows key is known as decrypting and is accomplished by a decrypt.

The encrypt and decrypt or may be algorithms executed by people or computers.

2. CONVERSION OF PLAINTEXT TO CLIPHERTEXT BY APPLYING SUMUDU TRANSFORM TO TRIGONOMETRIC SINE FUNCTION

Consider the trigonometric sine series given by

$\sin nx == nx - \frac{n^3x^3}{3!}+\frac{n^5x^5}{5!} - \frac{n^7x^7}{7!} + \frac{n^9x^9}{9!} + \cdots$ then we have

$$x^m \sin nx == nx^{m+1} - \frac{n^3x^{m+3}}{3!}+\frac{n^5x^{m+5}}{5!} - \frac{n^7x^{m+7}}{7!}+\ldots\ldots\ldots\ldots \quad (2.1)$$

First we consider the case when $m = 1 \& n = 1\ then\ we\ have$

$$x \sin x = x^2 - \frac{x^4}{3!}+\frac{x^6}{5!} - \frac{8}{7!}+\frac{x^{10}}{9!} - \frac{x^{12}}{11!} + \cdots \quad (2.2)$$

In the process to convert given plaintext to cipher text we will

allocate $0 to A, 1\ tO\ B, 2to\ C, 3to\ D$ … … … … … … … … .25 $to\ Z$

Example 4. 1: Let the given original message (plain text) be 'B A H I R J I' and suppose that this plain text be equivalent to

1 0 7 8 17 9 8

Let us assume that H_0=1, H_1=0, H_2=7,H_3=8, H_4=17,H_5=9, H_6=8

writing $H_0,\ H_1,\ H_2,\ H_3,\ H_4,\ H_5,\ H_6$ as the coefficients of $x \sin x$ equation (5.3.2)becomes

$$Hx \sin x = Hx \sin x = H_0x^2 - H_1\frac{x^4}{3!}+H_2\frac{x^6}{5!} - H_3\frac{8}{7!}+H_4\frac{x^{10}}{9!} - H_5\frac{x^{12}}{11!}+, H_6\frac{x^{14}}{13!} = \sum_{i=0}^{6}(-1)^i\frac{x^{2i+2}}{2i+1}H_i$$

Therefore we have

$$Hx \sin x = 1x^2 - 0\frac{x^4}{3!}+7\frac{x^6}{5!} - 8\frac{x^8}{7!}+17\frac{x^{10}}{9!} - 9\frac{x^{12}}{11!}+, 8\frac{x^{14}}{13!} \quad (2.3)$$

Applying Sumudu transform to equation (2.3) we have

$$S[Hx \sin x] = S[x^2] - 0S[x^4] + \frac{7}{5!}S[x^6] - \frac{8}{7!}S[x^8] + \frac{17}{9!}S[x^{10}] - \frac{9}{11!}S[x^{12}] + \frac{8}{13!}S[x^{14}]$$

$$S[Hx \sin x] = 2!\,w^2 - 0w^4 + 42w^6 - 64w^8 + 170w^{10} - 108w^{12} + 112w^{14} \quad (2.4)$$

Adjusting the resulting values 2, 0, 42,-64, 170,-108,112 modulo 26 we have
$2 \equiv 2(mod\ 26)$, $0 \equiv 0(mod\ 26)$, $42 \equiv -10(mod\ 26)$, -64$\equiv -12(mod\ 26)$, $170 \equiv 14(mod\ 26)$, -108$\equiv -4(mod\ 26)$, 112$\equiv 8(mod\ 26)$ we have

$H_0{}' = 2,\quad H_1{}' = 0,\quad H_2{}' = -10,\ H_3{}' = -12,\ H_4{}' = 14$
$H_5{}' = 4,\quad H_6{}' = 8\quad H_\alpha{}' = 0$ for $\alpha \geq 7$

Thus the given plaintext is converted in to 2 0 10 12 14 4 8

i.e. The cliphertext is C A K M O E I

Assuming r_0== 2, $r_1 = 0$, $r_2 = 42$, $r_3 = -64$, $r_4 = 170$, $r_5 = -108$, $r_6 = 112$ and by using the formula $k_i = \frac{r_i - H_i{}'}{26}$ for i=0, 1, 2……. We obtained the key
0 0 2 -2 6 -4 4

Table-2.1

i	H_i	$k_i = \frac{r_i - H_i{}'}{26}$	$r_i = (-1)^i(2i+2)H_i$	$H_i{}' = r_i - 26k_i$
0	1	0	2	2
1	0	0	0	0
2	7	2	42	-10
3	8	-2	-64	-12
4	17	6	170	14
5	9	-4	-108	4
6	8	4	112	8

From the above table (2.1) we have

Theorem 2.1: The given plain text in terms of $H_i, i = 0,1,2,3,4, \ldots\ldots$. Under the Sumudu transform of $Hx\sin x$ assuming $H_0, H_1, H_2, \ldots\ldots$ as non negative coefficients of $x\sin x$ can be converted to clipher text $H_i{}' = r_i - 26k_i$, $i = 0,1,2,3,4, \ldots\ldots$. Where $r_i = (-1)^i(2i+2)H_i$ and a key is given by $k_i = \frac{r_i - H_i{}'}{26}$ for i=0, 1, 2, 3……..

3. CONVERSION OF CLIPHERTEXT TO PLAINTEXT BY APPLYING INVERSE SUMUDU RANSFORMS

Applying inverse Sumudu transform to equation (2.4) we get again equation (2.3) i.e.

$$Hx\sin x = 1x^2 - 0\frac{x^4}{3!} + 7\frac{x^6}{5!} - 8\frac{x^8}{7!} + 17\frac{x^{10}}{9!} - 9\frac{x^{12}}{11!} +, 8\frac{x^{14}}{13!}$$

Assuming coefficients of the above equation we get the plaintext B A H I R J I

Table-3.1

i	H_i	$(-1)\left[\frac{26k_i + H_i{}'}{2i+2}\right]^i$
0	1	1
1	0	0
2	7	7
3	8	8
4	17	17
5	9	9
6	8	8

From the above table (3.1) we have

Theorem 3.1: The given clipher text $H_i{}'$ for $i = 0,1,2,3,4, \ldots\ldots$. with a given key k_i can be converted to plain text H_i, $i = 0,1,2,3,4, \ldots\ldots$ .under the inverse Sumudu transform of S$[Hx\sin x]$ =$\sum_{i=0}^{\infty}(-1)^i r_i w^{2i+2}$
where

$$H_i = (-1)^i\left[\frac{26k_i + H_i{}'}{2i+2}\right]$$

4. In this case we consider m = 1 & n = 2 then equation (2.1) becomes

$$Hx\sin 2x = 2x^2 - 0x^4 + \frac{224}{5!}x^6 - \frac{1024}{7!}x^8 + \frac{8704}{9!}x^{10} - \frac{18432}{11!}x^{12} + \frac{65536}{13!}x^{14} \quad (4.1)$$

Applying Sumudu transform to equation (4.1) we have

$$G(w) = 4w^2 - 0w^4 + 1344w^6 - 8192w^8 + 87040w^{10} - 221184w^{12} + 917504w^{14} \quad (4.2)$$

Adjusting the resulting values 4, 0, 1344,-8192, 87040,-221184, 917504 modulo 26 we have
$4 \equiv 4(mod\ 26)$, $0 \equiv 0(mod\ 26)$, $1344 \equiv -8(mod\ 26)$, $-8192 \equiv -2(mod\ 26)$, $87040 \equiv -8(mod\ 26)$,
$-221184 \equiv -2(mod\ 26)$, $917504 \equiv -10(mod\ 26)$ we have

$H_0{}' = 4$, $H_1{}' = 0$, $H_2{}' = -8$, $H_3{}' = -2$, $H_4{}' = -8$
$H_5{}' = -2$, $H_6{}' = -10$ $H_\alpha{}' = 0$ for $\alpha \geq 7$

Thus the plaintext is converted to cliphertext 4 0 8 2 8 2 10 i.e., E A I C I C K

Assuming $r_0 == 4$, $r_1 = 0$, $r_2 = 1344$, $r_3 = -8192$, $r_4 = 87040$,
$r_5 = -221184$, $r_6 = 917504$ and by using the formula $k_i = \frac{r_i - H_i{}'}{26}$ for i=0, 1, 2……. We obtained the key
0 0 52 -31 5 334 -8507 35289

Table-4.1

i	H_i	$k_i = \frac{r_i - H_i{}'}{26}$	$r_i = (-1)^i n^{2i+1}(2i+2)H_i$	$H_i{}' = r_i - 26k_i$
0	1	0	4	4
1	0	0	0	0
2	7	52	1344	-8
3	8	-315	-8192	-2
4	17	3348	87040	-8
5	9	-8507	-221184	-2
6	8	35289	917504	-10

From the above table (4.1) we have

Theorem 4.1: The given plain text in terms of $H_i, i = 0,1,2,3,4, \ldots\ldots$. under the Sumudu transform of $Hx\sin 2x$ assuming $H_0, H_1, H_2, \ldots\ldots$ as non negative coefficients of $x^2 \sin x$ can be converted to clipher text $H_i{}' = r_i - 26k_i$ where $r_i = (-1)^i n^{2i+1}(2i+2)H_i$ .And a key is given by $k_i = \frac{r_i - H_i{}'}{26}$ for, $i = 0,1,2,3,4, \ldots\ldots$.

Table-4.2

i	H_i	$(-1)^i \left[\frac{26k_i + H_i{}'}{2^{2i+1}(2i+2)}\right]^i$
0	1	1
1	0	0
2	7	7
3	8	8
4	17	17
5	9	9
6	8	8

From the above table (4.2) we have

Theorem 4.2: The given clipher text $H_i{}'$ for $i = 0,1,2,3,4, \ldots\ldots$. With a given key k_i can be converted to plain text$H_i, i = 0,1,2,3,4, \ldots\ldots$ .under the inverse Sumudu transform of S$[Hx\sin 2x]$ =$\sum_{i=0}^{\infty}(-1)^i r_i(2i+2)$ where

$H_i = (-1)^i \left[\frac{26k_i + H_i{}'}{2^{2i+1}(2i+2)}\right]$ for, $i = 0,1,2,3,4, \ldots\ldots$.

From theorems (2.1) & (4.1) we can state more generally

Theorem 4.3: The given plain text in terms of H_i , $i = 0,1,2,3,4, \ldots\ldots$ .under the Sumudu transform of $Hx^m \sin nx$ assuming $H_0, H_1, H_2, \ldots\ldots$ as non negative coefficients of $x^m \sin nx$ can be converted to clipher text $H_i{}' = r_i - 26k_i$ where

$r_i = (-1)^i n^{(2i+1)}(2i+2)(2i+3)\ldots\ldots(2i+m+1)H_i$ .And a key is given by $k_i = \frac{r_i - H_i{}'}{26}$ for, $i = 0,1,2,3,4, \ldots\ldots$.

This is the more generalization for encryption process in cryptography.

From theorem (2.2) & (4.2) we can state more generally

Theorem 4.4: The given clipher text $H_i^{'}$ for $i = 0,1,2,3,4, \ldots \ldots$. With a given key k_i can be converted to plain textH_i, $i = 0,1,2,3,4, \ldots \ldots$ .under the inverse Sumudu transform of S$[Hx^m \sin nx\ ] = \sum_{i=0}^{\infty}(-1)^i r_i w^{2i+m+1}$ where $H_i = (-1)^i \left[\frac{26k_i + H_i^{'}}{n^{2i+1}(2i+2)(2i+3)\ldots\ldots(2i+m+1))}\right]$ for, $i = 0,1,2,3,4, \ldots \ldots$.

5. CONCLUSION

In this paper we have applied Sumudu transform to trigonometric sine function and converted plaintext to clipher text successfully. In the second part we have obtained plaintext from cliphertext by applying inverse Sumudu transform successfully. We have also derived some generalizations which will be applicable in the process of encryption and decryption .Finally we may conclude that Sumudu transform has an important role in cryptography.

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