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TWO-LAYERED NEWTONIAN MODEL FOR BLOOD FLOW MODEL IN CATHETERIZED ASYMMETRIC STENOSED ARTERY WITH VELOCITY SLIP AT INTERFACE

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ABSTRACT

The paper deals with a mathematical model for two-layered blood flow inside a catheterized asymmetric stenosed artery and velocity slip at the interface. The model consists of a core region of red blood cell suspension in the middle layer and the cell poor region peripheral plasma layer (PPL) in the outer region. It is assumed that both the core and the peripheral plasma layer are represented by a Newtonian fluid with different viscosities μ_1 and μ_2 respectively. Analytical expressions are obtained for axial velocity, flow rate, and wall shear stresses. The behaviour of three flow variables have been discussed. By employing velocity slip at interface, axial velocity and flow rate can be accelerated on one hand and impedance to flow can be retarded on the other. The present analysis includes some mathematical models as its special cases. Physiological implications of this theoretical modeling to blood flow situations, are also discussed in brief.

Keywords*: Blood, two-layered model, asymmetric stenosis, velocity slip, catheter.*

1. FLOW GEOMETRY

A two layered model for blood flow in a catheterized asymmetric stenosis has been developed. The model basically consists of a core of red blood cell suspension in the middle layer and the peripheral plasma layer in the outer layer (as shown in figure1). It is assume that both the core and the peripheral layer are represented by a Newtonian fluid with different viscosities μ_1 and μ_2 respectively.

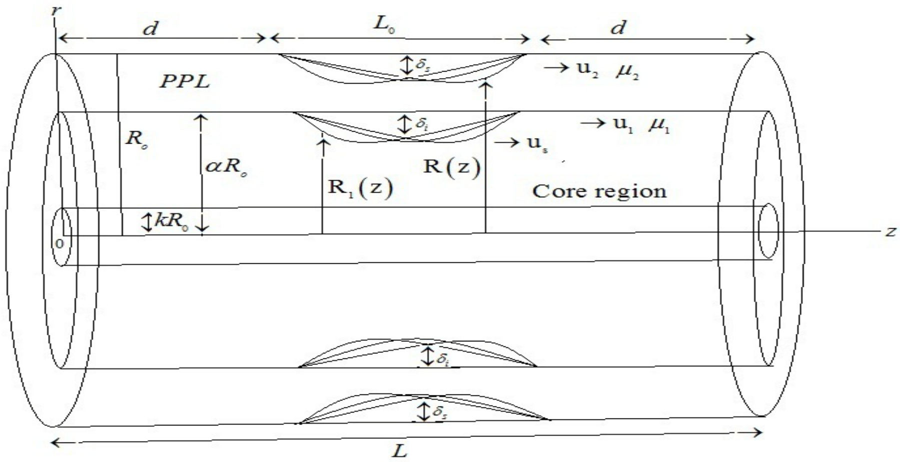

Fig 1:Schematic diagram for Annular Flow of Two-Layered Newtonian Fluid within catheterized Asymmetric Stenosed Artery

The geometry of the stenosis which is developed at the arterial wall in an axially non symmetric but radially symmetric manner - a rigid tube with a circular section and a catheter kR_0 ($k<<1$) coaxial to it as in from (fig.1) is mathematically modeled as given by (Ponalagusamy 1986) for PPL

$$\frac{R(z)}{R_0} = 1 - \mathrm{A}\left[\mathrm{L}_0^{\text{n-1}}(z-d) - (z-d)^n\right], \qquad \mathrm{d} \le \mathrm{z} \le \mathrm{d}\ \ \mathrm{L}_0$$
$$= 1\,, \quad \text{otherwise} \tag{1}$$

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The function $R_1(z)$ which represent the shape of the central layer has been assumed to be of the from

$$\frac{R_1(z)}{R_0} = \alpha - A_1\left[L_0^{n-1}(z-d)-(z-d)^n\right], \qquad d \le z \le d+L_0$$
$$= \alpha, \quad \text{otherwise} \tag{2}$$

where $R(z)$ is the radius of the tube with stenosis, R_0 is the constant radius of the false, L_0 is the length of the stenosis, L the length of the tube, d the stenosis location, δ_s and δ_i are the maximum height of the stenosis in the PPL and that in the Core region respectively at $z = d + \frac{L_0}{2}$ is a parameter determining the shape of the stenosis,

$$A = \frac{\delta s}{R_0 L_0^n}\cdot\frac{n^{\frac{n}{(n-1)}}}{(n-1)},$$

$$A_1 = \frac{\delta i}{R_0 L_0^n}\cdot\frac{n^{\frac{n}{(n-1)}}}{(n-1)},$$

It is of interest to note that an increase in the value of n leads to the change of stenosis shape. When n = 2, the geometry of stenosis becomes symmetrical at $z = d + \frac{L_0}{2}$ and $\alpha = \frac{\delta i}{\delta s}$.

3. MATHEMATICAL ANALYSIS

Let us consider a steady, Laminar flow of blood through an axially non symmetric but radially symmetric stenosed artery - a circular tube with a catheter of radius kR_0 (k<<1) co-axial to it and one-dimensional flow obeying the constitutive equation for a Newtonian fluid. Fluid velocity vector has the form $\vec{V} = (0,0,u(r))$ in cylindrical polar system (r,θ,z) representing the radial, circumferential and axial coordinate respectively. The equations of motion governing the fluid flow in (r,θ,z) coordinate system (schlichting, 1968) are written as follows

$$\frac{\partial p}{\partial r} = 0, \tag{3}$$

$$\frac{\partial p}{\partial \theta} = 0, \tag{4}$$

$$\frac{\mu}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \frac{\partial p}{\partial z} \tag{5}$$

Where $\mathrm{u} = \mathrm{u}(\mathrm{r})$ denotes the axial velocity, μ is the viscosity of blood and p the pressure.

As a result of equation (3-5), the governing equation of fluid flow is given by

$$C + \frac{\mu}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial \mu}{\partial r}\right) = 0\ , \qquad kR_0 \le r \le R(z) \tag{6}$$

Where $C = -\frac{dp}{dz}$, is the pressure gradient.

4. BOUNDARY CONDITIONS

The boundary conditions for the present problem are given by

(i) $u_2 = 0$ at $r = R(z)$, (zero-slip at stenotic wall)

(ii) $u_1 - u_2 = u_s$ at $r = R_1(z)$,, (slip at interface)

(iii) $u_1 = 0$ at $r = kR_0$, (zero-slip at catheter wall) (7)

(iv) $\mu_1\frac{\partial u_1}{\partial r} = \mu_2\frac{\partial u_2}{\partial r}$ at $r = kR_0$, (stresses equal at interface)

Where μ_1 and μ_2 are viscosities of blood in core and peripheral plasma layer respectively. The equation (6) with boundary conditions (7), is a boundary value problem.

5. SOLUTIONS OF THE PROBLEM

Integrating the equation (6) twice and by employing the boundary conditions (7), the analytic expression for velocity function for the core region-

$$\mathrm{u}_1(r)=\frac{c}{4\mu_1}+\left((kR_0)^2-r^2\right)+\frac{\mu_2 H}{4H_1}\ln\left(\frac{r}{kR_0}\right), kR_0 \le r \le R_1(z) \tag{8}$$

$$\mathrm{u}_2(r)=\frac{c}{4\mu_2}+\left(R^2(z)-r^2\right)+\frac{\mu_1 H}{4H_1}\ln\left(\frac{r}{R(z)}\right), R_1 \le r \le R(z) \tag{9}$$

where $$\mathrm{H}=u_s+\frac{c}{4}\left\{R_1^2(z)-r^2\left(\frac{1}{\mu_1}-\frac{1}{\mu_2}\right)-\left(\frac{(kR_0)^2}{\mu_1}-\frac{R^2(z)}{\mu_2}\right)\right\} \tag{10}$$

$$\mathrm{H}_1=\mu_2\ln\frac{R_1(z)}{kR_0}-\mu_1\ln\left(\frac{R_1(z)}{R(z)}\right) \tag{11}$$

The volumetric flow rate for the core region $kR_0 \le r \le R_1(z)$ is given by

$$Q_1=2\pi\int_{r=kR_0}^{R_1(z)} r u_1(r)\,dr$$

$$=\frac{\pi\mu_2}{2H_1}\left(u_s+\frac{c}{4}H_2\right)+\left\{\left(2R_1^2(z)\ln\frac{R_1(z)}{kR_0}\right)-\left(R_1^2(z)\right)-(kR_0)^2\right\}-\frac{\pi c}{8\mu_1}\left\{\left(R_1^2(z)\right)-(kR_0)^2\right\}^2 \tag{12}$$

And for PPL $R_1(z)\le r\le R(z)$

$$Q_2=2\pi\int_{r=R_1(z)}^{R(z)} r u_2\,dr$$

$$=\frac{\pi c}{8\mu_2}\left(R^2(z)-R_1^2(z)\right)^2-\frac{\pi\mu_1}{2H_1}\left(u_s+\frac{c}{4}H_2\right)\left\{2R_1^2(z)\ln\left(\frac{R_1(z)}{kR_0}\right)-\left(R^2(z)-R_1^2(z)\right)\right\} \tag{13}$$

where $$\mathrm{H}_2=R_1^2(z)\left(\frac{1}{\mu_1}-\frac{1}{\mu_2}\right)-\left(\frac{(kR_0)^2}{\mu_1}-\frac{R^2(z)}{\mu_2}\right)$$

The total flow rate $\overline{Q}$ is expressed as

$$=\frac{\pi c}{8}\left[\frac{\left(R^2(z)-R_1^2(z)\right)^2}{\mu_2}-\frac{R_1^2(z)-(kR_0)^2}{\mu_1}+\frac{H_2H_3}{H_1}\right]+\frac{\pi u_s}{2H_1}.H_4 \tag{14}$$

where $$\mathrm{H}_2=R_1^2(z)\left(\frac{1}{\mu_1}-\frac{1}{\mu_2}\right)^2-\frac{(kR_0)^2}{\mu_1}-\frac{R^2(z)}{\mu_1}$$

$$\mathrm{H}_3=2(\mu_2-\mu_1)^2.R_1^2(z)\left(\ln\frac{R_1(z)}{kR_0}-\ln\frac{R_1(z)}{R(z)}\right)$$

$$\mathrm{H}_4=\mu_1\left(R^2(z)-R_1^2(z)\right)^2-\mu_2\left(R_1^2(z)-(kR_0)^2\right)$$

Shear stress at the stenotic wall and at the interface respectively given by

$$\tau_{R_{(z)}} = -\mu_2 \frac{\partial u_2}{\partial r}\bigg]_{r=R(z)}$$

$$= \frac{cR(z)}{2} - \frac{\mu_1\mu_2\left(u_s + \frac{c}{4}H_2\right)}{R_{(z)}H_1} \tag{15}$$

and $\tau_{R_1(z)} = -\mu_1 \frac{\partial u_1}{\partial r}\bigg]_{r=R_1(z)}$

$$= \frac{cR_1(z)}{2} - \frac{\mu_1\mu_2\left(u_s + \frac{c}{4}H_2\right)}{R_1(z)H_1} \tag{16}$$

Apparent viscosity is given by

$\mu_a = \frac{\pi c R^4{}_{(z)}}{8Q}$, where Q is given in eq. (14)

$$\mu_a = R^4(z)\left[\left[\left\{\frac{1}{\mu_2}\left(R^2(z)-R_1^2(z)\right)^2 - \frac{1}{\mu_1}\left(R_1^2(z)-\left(kR_0\right)^2\right)^2\right\} + \frac{\pi}{2H_1}\left(u_s + \frac{c}{4}H_2\right)\right.\right.$$
$$\left.\left\{2\left(\mu_2-\mu_1\right).R_1^2(z)\left(\ln\frac{R_1(z)}{kR_0} - \ln\frac{R_1(z)}{R(z)}\right)\right\} + \left\{\mu_1\left(R^2(z)-R_1^2(z)\right)^2 - \mu_2\left(R_1^2(z)-\left(kR_0\right)^2\right)\right\}\right]^{-1} \tag{17}$$

The non dimensional form of the flow variables and flow geometry can be expressed by using the following non-dimensional variables:

$$\overline{R} = \frac{R(z)}{R_0},\quad \overline{R}_1 = \frac{R_1}{R_0},\ \mathrm{u}_0 = \frac{cR_0{}^2}{4\mu_2},\quad \mathrm{Q}_0 = \frac{\pi c R_0{}^4}{8\mu_2}$$

$$\left(\frac{dp}{dz}\right)_0 = \left(-\frac{8\mu_2 \mathrm{Q}_0}{\pi R_0{}^4}\right),\quad \overline{\mu}_{\mathrm{a}} = \frac{\mu_a}{\mu_2},\quad \mu_2' = \frac{\mu_2}{\mu_1}, \overline{z} = \frac{z}{R_0}$$

$$\lambda_0 = \frac{8\mu L}{\pi R_0{}^4},\quad \overline{\mathrm{u}}_{\mathrm{s}} = \frac{u_s}{u_0},\quad \overline{u}_1 = \frac{u_1}{u_0},\quad \overline{u}_2 = \frac{u_2}{u_0}$$

Flow geometry

$$\overline{R}(z) = 1 - \overline{A}\left[\overline{L}_0{}^{n-1}(\overline{z}-\overline{d}) - \left(\overline{z}-\overline{d}\right)^n\right],\ \ \overline{\mathrm{d}} \le \mathrm{z} \le \overline{\mathrm{d}}+\overline{\mathrm{L}}_0$$
$$= 1,\quad \text{otherwise} \tag{18}$$

$$\overline{\mathrm{R}}_1(z) = \alpha - \overline{A}\left[\overline{L}_0{}^{n-1}(\overline{z}-\overline{d}) - \left(\overline{z}-\overline{d}\right)^n\right],\ \ \overline{\mathrm{d}} \le \mathrm{z} \le \overline{\mathrm{d}}+\overline{\mathrm{L}}_0$$
$$= \alpha,\quad \text{otherwise} \tag{19}$$

Second representations for Velocity function (core region)

$$\overline{u_1}(\overline{r}) = \mu_2'\left\{\left(\overline{KR_0}\right)^2 - \overline{r}^2\right\} + \frac{\mu_2'}{A}\left(\overline{u_s} + B\right)\ln\left(\frac{\overline{r}}{\overline{\overline{KR_0}}}\right) \tag{20}$$

and for ppl layer is

$$\overline{u_2}(\overline{r}) = \left(\overline{R}^2 - \overline{r}^2\right) + \frac{1}{A}\left(\overline{u_s} + B\right)\ln\left(\frac{\overline{r}}{\overline{\overline{R}}}\right) \tag{21}$$

where $A = \mu_2' \ln\left(\dfrac{\overline{R_1}}{\overline{KR_0}}\right) - \ln\left(\dfrac{\overline{R_1}}{\overline{R}}\right)$

and $B = \mu_2'\left\{\overline{R_1}^2 - \left(\overline{KR_0}\right)^2\right\} + \left(\overline{R}^2 - \overline{R_1}^2\right)$

A second representation of flow rate is defined by

$$\overline{Q} = W + \frac{1}{A}\left(\overline{u_s} + B\right)\left(2\overline{R_1}^2 A - B\right) \tag{22}$$

where $W = \left(\overline{R}^2 - \overline{R_1}^2\right)^2 - \mu_2'\left\{\overline{R_1}^2 - \left(\overline{kR_0}\right)^2\right\}^2$

A second representation of shear stress in the peripheral region is given by

$$\overline{\tau_{R(z)}} = \frac{\tau_{R(z)}}{(\tau_R)_0} = \overline{R} - \frac{1}{2\overline{R}A}\left(\overline{u_s} + B\right) \tag{23}$$

And for core region is given by

$$\overline{\tau_{R_1(z)}} = \frac{\tau_{R_1(z)}}{(\tau_R)_0} = \overline{R_1} - \frac{1}{2\overline{R_1}A}\left(\overline{u_s} + B\right) \tag{24}$$

where $(\tau_R)_0 = \dfrac{C}{2} R_0$

And the second representation of apparent viscosity as

$$\overline{\mu_a} = \overline{R}^4\left[W + \frac{1}{A}\left(\overline{u_s} + B\right)\left(2\overline{R_1}A - B\right)\right]^{-1} \tag{25}$$

5. RESULTS AND DISCUSSIONS

In carrying out the present work for blood flow through an annular region in (Fig 1) $kR_o \le r \le R(z)$ between a stenotic wall R(z) and a catheterized artery (k << 1, catheter radius $kR_0 << R_0$, artery radius), the following estimates for the constricted region, such as stenosis length L_0 = 8, and d its location in the region $d \le z \le d+L_0$ (Biswas, 2000; Chaturani and Biswas, 1983), stenosis development in asymmetric manner and maximum heights of stenosis $\overline{\delta_s}$ in dimensionless form) equals to $1-\sqrt{3}/2,\ 1-1/\sqrt{2},$ and corresponding to an abnormal growth of 25, 50 and 75 percents in three respective and gradual cases of mild, moderate and severe formations at the lumen of an artery, have been used in developing the current mathematical analysis. It is already reported that knowledge of rheological and fluid dynamic properties of blood and its flow, like velocity, pressure gradient, shear stress at wall, flow rate etc., might play an important role in the fundamental understanding, diagnosis and treatment of many cardiovascular (cvs), renal and arterial diseases (Dintenfass,1981; Punder and Punder, 2006; Cokelet,1972). In view of this, analytical expressions for axial velocity, flow rate, pressure gradient, resistance to flow, wall shear stress, apparent viscosity etc. have been obtained in this study and their graphical representations are shown in Figs. (2-6). It may be noticed that velocity is a function of shear viscosities (μ_1, μ_2), pressure gradient $-\dfrac{dp}{dz}$, tube radii R_0, R(z), $R_1(z)$ and a catheter kR0 ≤ 1, stenosis length L_0 , its location d, δ_s, δ_i , axial coordinate z, radial coordinate r and slip velocity u_s. This is in contrast to Poiseuille flow of blood (behaving as a Newtonian fluid) wherein velocity depends only on R_o, μ_1 and R. Also, the non- uniform radii R(z) and $R_1(z)$ in eqs. (18--19) depends on axial distance z, R_0, δ_s and δ_i in an unobstructed tube.

The present model includes the following cases:

When $kR_0 = 0$ and $R(z) = R_1(z)$, it results in Newtonian flow model of an arterial stenosis with no-slip ($u_s = 0$) and slip $u_s \neq 0$ respectively. In case $R(z) = R_1(z) = R_0$ and $u_s \geq 0$, it reduces to annular flow between co-axial cylindrical tube models of Newtonian fluid with slip and no-slip conditions.

When $R(z) = R_1(z) = R_0$, $kR_0 = 0$ and $u_s = 0$, it expresses a Poiseuille flow of blood inside a uniform tube with slip or zero-slip at the boundary. If $R(z) \neq R_1(z) \neq R_0$, $kR_0 \neq 0$ and $u_s \geq 0$, then it represents a two-layered annular flow of blood through a uniform artery with slip or no-slip at interface.

In the analysis, the combined influence of several parameters have been developed, in cases of uniform region. To analyze the quantitative effect of uniform artery, maximum height of stenosis δ_i, δ_s, slip velocities ($u_s \geq 0$) at the interface, Newtonian behaviour of blood, two-layered flow etc., computer codes have been developed for the numerical evaluations of the analytic results obtained for velocity, flow rate, wall shear stress and pressure gradient for parameter values δ_s =0.15, δ_i =0.12, u_s =0.00,.05,.1, Q=0.5, 1.0, 1.5 (Verma and Parihar 2009, 2010) and viscosities μ_1 =1.2 cp, μ_2 = 2 cp, $\mu_2' = .62$,α = 0.82 (< 1) for a full scale location from z = d to d+L_0 , and $0 \leq r \leq R_1(z),\ R_1(z) \leq r \leq R(z)$ for PPL and core regions have been used. In the forgoing analysis, an attempt is taken up to address the variations of velocity, flow rate characteristics etc., due to such parameters.

(i). Axial Velocity profiles

A comparison of velocity profiles that have been obtained from eqs. (20--21), for slip and no-slip cases, maximum heights of stenosis and different axial locations for z = d+L_0/2, d+L_0/4, d+3L_0/4 for shape parameter n=2, 6, 9 and for other parameter values, is shown in Figs (2-3). As tube radius r/R ranges from 0 (at tube axis) to 1 (at wall) on either side of axis, velocity decreases from a greater value at axis to a smaller one slip velocity at interface and, then to a minimum magnitude zero-slip velocity at boundary. As expected, velocity increases with slip at interface. Its values are higher for flows with slip (u_s > 0) than those with no-slip (u_s = 0). Also it is observed from Figs. (2-3) that $\overline{u_1}|_{n=9} < \overline{u_1}|_{n=2} < \overline{u_1}|_{n=6}$. Although, it shows a little deviation from parabolic profile, in to the core region, its behaviour is parabolic in the peripheral region. For symmetric and asymmetric stenoses, it behaves differently. As slip velocity increases, velocity increases in all three forms of stenosis formation at an artery wall.

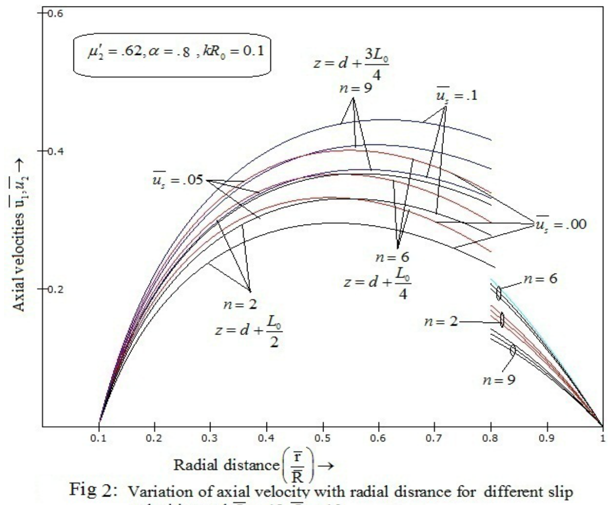

Fig 2: Variation of axial velocity with radial disrance for different slip velocities and $\overline{\delta_s} = .15, \overline{\delta_i} = .12$

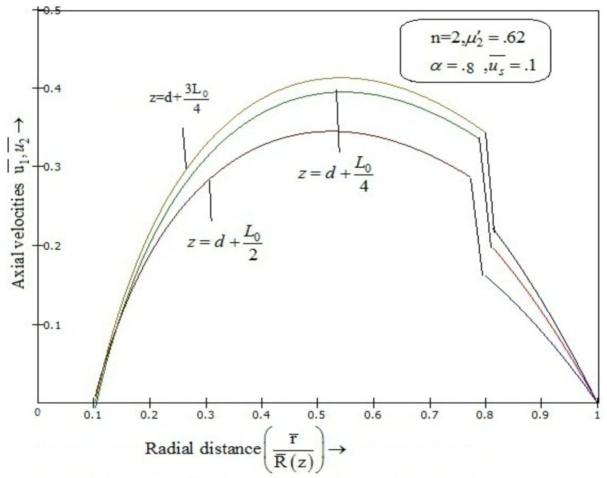

Fig. 3 Variation of axial velocities against radial distance for different heights of stenosis $\overline{\delta_s}, \overline{\delta_i}$ and $\overline{u_s} = .1$, $KR_0 = 0.1$, n=2

(ii). VARIATION OF FLOW RATE

The variations of flow rate with different parameters are shown in Figure 4. It decreases in magnitude from the initiation position to the stenotic throat and there after it increases to the termination position. It is seen that $\overline{Q}$ decreases as shape parameter n decreases. The greatest magnitude is attained at the stenotic throat for n = 2 (symmetric case) at z = d+L_0/2 and away from the throat for n > 2 (asymmetric case) at z = d+3L_0/4. In all cases of stenosis, flow rate increases as slip velocity increases.

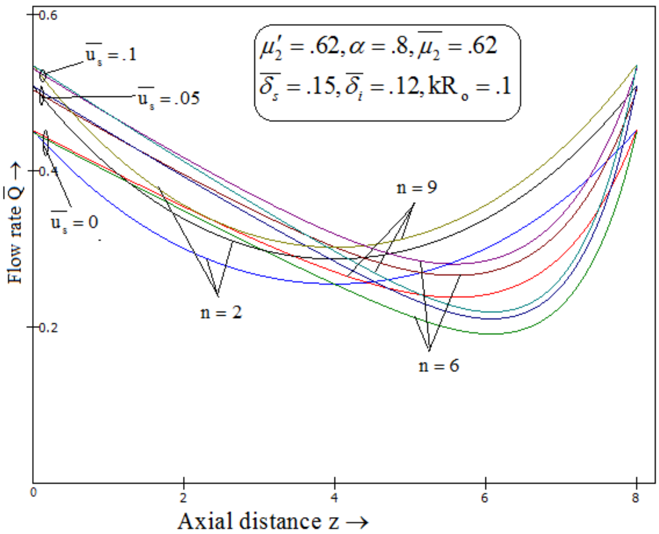

Fig 4:-Variation of flow rate against axial distance for n =2,6,9 ,and $\overline{u_s} = 0, .05, .1$

(iii). VARIATION OF WALL SHEAR STRESS

Figs. (5--6) shows the variation of wall shear stress in the annular region. It decreases with increase in slip velocity. It shows the higher magnitude at the throught of the stanosis and minimum at the both initiation and termination of the stenosis. minimum constriction and therefore, it increases to a higher value at the other end of stenosis.

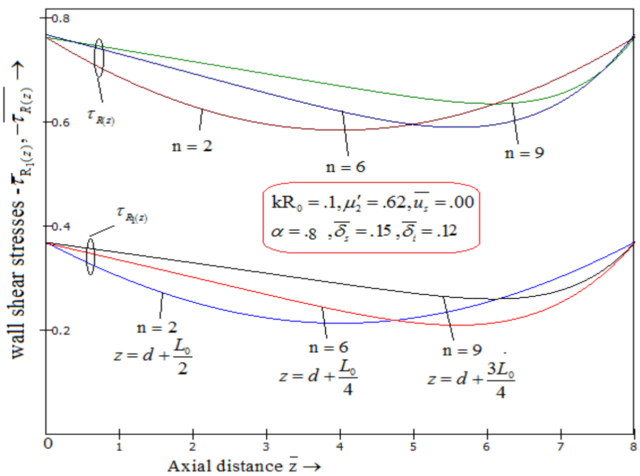

Fig 5 Variation of shear stress against axial distance for different values of n and $\overline{\delta_s} = .15, \overline{\delta_i} = .12$

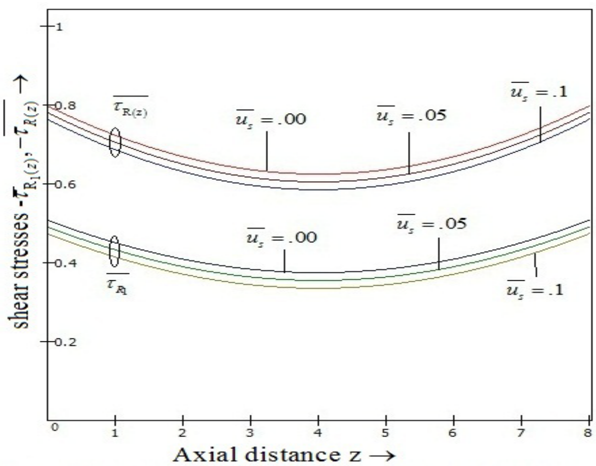

Fig 6 Variation of shear stress against axial distanc for $kR_0 = .1, \overline{u_s} = 0, .05, .1, n = 2$

6. CONCLUSION

In the present paper, steady flow of blood (a Newtonian fluid) through a catheterized stenosed artery subject to the condition of slip at the interface of fluids layers with mild asymmetric stenosis, velocity slip and zero-slip at the catheter and at the tube wall, has been investigated. Analytic expressions for axial velocity, flow rate, wall shear stress have been obtained in this study. It can be noticed that velocity is a function of $\overline{C}$, μ_1 , μ_2, tube radii R_0, $R_1(z)$ and $R(z)$ and a catheter radius $kR_0 << 1$, stenosis length L_0, its location d, heights δ_s, δ_i, axial (z), radial (r) co-ordinates and slip velocity u_s. This is in contrast to Poiseuille flow of blood (behaving as Newtonian fluid) wherein velocity depends only on R_0, μ_1 and R. Here three gradual advances of an abnormal growth (symmetric n = 2, n $\geq$ 2) and lumen of an artery cases of a slip and no-slip at interface and a catheter boundary are dealt with. Important observation of the present analysis may be included in the following:

(a) The current model includes Poiseuille flow of blood (Newtonian Fluid) with slip or zero-slip at vessel wall and non-Newtonian fluid with slip or zero-slip at stenotic wall, annular flow models between co-axial cylindrical tubes and a Newtonian fluid with slip and no-slip and Newtonian fluid models of blood flow in a catheterized stenosed uniform artery with slip or zero-slip conditions.

(b) The velocity increases with an axial slip and it increases to higher magnitudes with the increasing values of slip whereas it decreases with an increase in height of the stenosis, as expected.

(c) The flow rate increases with a slip and attains the greatest magnitudes at either end of the constricted annular region and the least value at the throat of the stenosis.

(d) Wall shear stress decreases with velocity slip and also with an increasing magnitude of slip velocity, but with slip in velocity, it is lowered. It decreases from a higher magnitude at the end of stenosis to the position of minimum constriction and therefore, it increases to a higher value at the other end of stenosis.

It may be worth mentioning that by employing a velocity slip at interface, wall shear stress may be reduced to a considerable extent that in term will help in reducing damage or rupture to the arterial endothelium. Also apparent viscosity $\overline{\mu}_a$ increases as tube radius increases in the annular constricted region decreases, it shows Inverse Fahraeus-Lindqvist Effect (IFLE) for cases of slip or no-slip. Therefore the present model could explain an anomaly in blood flow. Also the reduction in wall shear stress enhancement in velocity and flow rate as a result of introducing at the axial velocity at the slip at interface, may be exploited for better functioning of the diseased arterial system and pressure flow-relationship in a uniform stenosed artery. Hence one may look forward for such device (drugs or tools) which could produce slip and used them for treatment and cure of PPL and arterial diseases as well as for rupture or damage to the arterial endothelium. Further the existing experimental work on blood flow through stenosed vessels, consider only pressure drop. It could be a matter of interest and important to determine the wall shear stress slip at interface in two-layered flow, velocity and flow rate etc. in the stenosed flow as well in the annular flow in a catheterized stenosed artery. Such investigation may be useful in determining the growth, development and progression of an arterial stenosis and investigating the pressure-flow relations and the behaviour of flow variables in the two-layered annular region caused by the invention of a catheter in an arterial stenosis and that in turn may be useful for better understanding of stenotic and arterial diseases like angina, pectoris, myocardial infarction, stroke, thrombosis and Hbss etc.

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