

## TENSOR PRODUCT OF A NEAR-FIELD SPACE AND SUB NEAR-FIELD SPACE OVER A NEAR-FIELD

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### ABSTRACT

A tensor product of near-field space and sub near-field space over a near-field defined. But it turned out to be an abelian group. In order to avoid this special situation, this concept is generalized further by Dr N V Nagendram in this paper. Now there are two tensor products for the same pair of sub near-field spaces of a near-field space over a near-field. This situation fits better in the theory of near-field spaces and their sub near-field spaces over a near-field. It has been seen here that there may be two dual near-field spaces for a pair of sub near-field spaces over a near-field.

**Keywords:** sub near-field space, near-field space, semi simple near-field space, ordered Euclidean near-field spaces, vector lattices.

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### SECTION 1: INTRODUCTION

We write maps on the right and hence use left near-field spaces and the traditional sub near-field spaces are right sub near-field spaces.

**Definition 1.1 (N-sub near-field space):** Let  $(N, +, \cdot)$  be a left near-field space. A sub near-field space  $(M, +)$  is called an N-sub near-field space i.e. traditional one if there is a near-field space homo-morphism  $\theta : N \rightarrow \text{Map}(M)$ . As usual, we write  $gn$  to mean  $g(n\theta)$  for  $g \in M$  and  $n \in N$ . In this case the group elements distribute over the near-field spaces.

**Definition 1.2 (Complementary N-near-field space):**  $M$  is called a complementary N-sub near-field space or  $N - \text{co}$  sub near-field space, for short, if there is a semi sub near-field space elements distribute over the sub near-field space elements and the action of  $N$  is usually written on the left of the elements of  $M$ .

**Definition 1.3 ((N, T) – bi sub near-field space):** Let  $N$  and  $T$  be two left near-field spaces. A sub near-field space  $M$  is called an  $(N, T) - \text{bi}$  sub near-field space if

- (a)  $M$  is an  $N$ -co sub near field space
- (b)  $M$  is an  $T$ -sub near-field space and (c)  $(ng)t = n(gt), \forall g \in M, n \in N, t \in T$ .

**Definition 1.4 (left strong N-sub near-field space):**  $M$  is called left strong N-sub near-field space if the action of  $N$  is defined on the left of  $M$  satisfying the following conditions  $\forall n, n' \in N$  and  $g, g' \in M$

- (a)  $(nn')g = n(n'g)$
- (b)  $n(g + g') = ng + ng'$  and (c)  $(n + n')g = ng + n'g$ .

**Note-1.5:** A right strong N-sub near-field space is defined similarly.  $(N, +)$  is an  $(N - N) - \text{bi}$  sub near-field space for the left as well as right near-field space  $N$  over a near-field. If  $N$  is distributive near-field space then  $(N, +)$  is a left as well as right strong N-sub near-field space. Many more examples of these structures are given in near-field space related topic.

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**Definition 1.6 (N-homomorphism):** Let  $M$  and  $K$  be two  $N$ -sub near-field spaces ( $N$ -co sub near-field space). A sub near-field space homomorphism  $\theta : M \rightarrow K$  is called an  $N$ -homomorphism if for any  $g \in M$  and  $n \in N$ ,  $(gn)\theta = (g\theta)n$ ,  $((rg)\theta = r(g\theta))$ .

**Note-1.7:** An  $(N - T) -$  homomorphism for  $(N - T)$ -bi sub near-field space are defined in a similar way.

## SECTION 2: TENSOR PRODUCT

Let  $N$  be left near-field space,  $A$  an  $N$ -sub near-field space and  $B$  an  $N$ -co sub near-field space. Let  $E$  be the free collection of sub near-field spaces on  $A \times B$ . Let  $P$  and  $Q$  be the normal sub near-field spaces of  $E$  generated by  $\{(a + a', b) - (a', b) - (a, b), (ar, b) - (a, rb) \mid a, a' \in A, b \in B, r \in N\}$  and  $\{(a, b + b') - (a, b') - (a, b), (ar, b) - (a, rb) \mid a \in A, b, b' \in B, r \in N\}$  respectively.

We call  $E/P$  the left tensor product of  $A$  and  $B$  and denote it by  $A_N \otimes B$  and call  $E/Q$  the right tensor product of  $A$  and  $B$  and denote it by  $A \otimes_N B$ . The coset  $(a, b) + P$  is denoted by  $a_l \otimes b$  and  $(a, b) + Q$  is denoted by  $a \otimes_r b$ . The coset  $P$  is denoted by  $0$  in both cases. Since  $E$  is generated by  $A \times B$ ,  $E/P = A_N \otimes B$  and  $E/Q = A \otimes_N B$  are generated by  $\{a_l \otimes b \mid a \in A, b \in B\}$  and  $\{a \otimes_r b \mid a \in A, b \in B\}$  respectively. An element of  $A_N \otimes B$  ( $A \otimes_N B$ ) is a finite sum of the form  $\sum \epsilon_i (a_i l \otimes b_i)$  ( $\sum \epsilon_i (a_i \otimes_r b_i)$ ), where each  $\epsilon_i = \pm 1$ .

The following result is a direct consequence of the definition of tensor product of near-field space and sub near-field space over a near-field.

**Theorem 2.1:**  $\forall a, a' \in A, b \in B, r \in N$  the following are satisfied in  $A_N \otimes B$ :

- (i)  $(a + a')_l \otimes b = a_l \otimes b + a'_l \otimes b$
- (ii)  $ar_l \otimes b = a_l \otimes rb$
- (iii)  $0_{Al} \otimes b = 0$
- (iv)  $(-a)_l \otimes b = -(a_l \otimes b)$ .

**Theorem 2.2:**  $\forall a \in A, b, b' \in B, r \in N$  the following are satisfied in  $A \otimes_N B$ :

- (i)  $a \otimes (b + b') = a \otimes_r b + a \otimes_r b'$
- (ii)  $ar \otimes_r b = a \otimes_r rb$
- (iii)  $a \otimes_r Bb = 0$
- (iv)  $a \otimes_r (-b) = -(a \otimes_r b)$ .

**Remark 2.3:** In general  $(a + a')r \neq ar + a'r$  in  $A$  as  $A$  is an  $N$ -sub near-field space. But we have

$$\begin{aligned} (a + a')r_l \otimes b &= (a + a')_l \otimes rb = a_l \otimes rb + a'_l \otimes rb \\ &= ar_l \otimes b + a'r_l \otimes b \\ &= (ar + a'r)_l \otimes b. \end{aligned}$$

This shows that in  $A_N \otimes B$  with  $b \neq 0$ .

**Remark 2.4:** It is possible that  $a \otimes_r b = 0$  in  $A \otimes_N B$ , with  $b \neq 0$ . Later on we will show these by examples and we generalize the definition of a middle linear map.

**Definition 2.5 (left N-middle linear map(LNMLM)):** Let  $N, A$  and  $B$  be as before and let  $C$  be any sub near-field space with a map  $f : A \times B \rightarrow C$ . then we call  $f$  a left  $N$ -middle linear map if  $(a + a', b)f = (a, b)f + (a', b)f$ ,  $(ar, b)f = (a, rb)f$ ,  $\forall a, a' \in A, b \in B, r \in N$ .

**Definition 2.6 (Right N-middle linear map(RNMLM)):** Let  $N, A$  and  $B$  be as before and let  $C$  be any sub near-field space with a map  $f : A \times B \rightarrow C$ . then we call  $f$  a right  $N$ -middle linear map if  $(a, b + b')f = (a, b)f + (a, b')f$ ,  $(ar, b)f = (a, rb)f$ ,  $\forall a, a' \in A, b, b' \in B, r \in N$ .

It is easy to see that  $\theta_1 = j\pi_P : A \times B \rightarrow A_N \otimes B = E/P$ ,  $(a, b) \mapsto a_l \otimes b$  and  $\theta_1 = j\pi_Q : A \times B \rightarrow A \otimes_N B = E/Q$ ,  $(a, b) \mapsto a \otimes_r b$  are

LNMLM and RNMLM respectively. Here  $j : A \times B \rightarrow E$  is the inclusion map and  $\pi_P : E \rightarrow E/P$  and  $\pi_Q : E \rightarrow E/Q$  are the natural homomorphisms. We call  $\theta_1$  ( $\theta_r$ ) the canonical LNMLM (RNMLM).

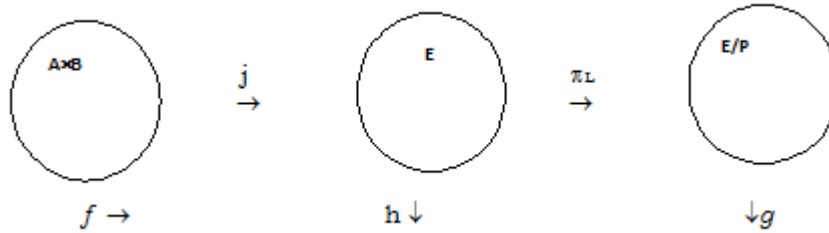
Now we prove the universal property of the tensor product of near-field space and sub near-field space over a near-field  $N$ .

**Theorem 2.7:** Let  $N$  be a left-near field space,  $A$  be an  $N$ -sub near-field space and  $B$  an  $N$ -co sub near-field space. Let  $C$  be a group with a function  $f: A \times B \rightarrow C$ , and  $\theta_l (\theta_r)$  be as above.

- (a). If  $f$  is a LNMLM, then there exists a unique sub near-field space homomorphism  $g: A_N \otimes B \rightarrow C$ , such that  $\theta_l g = f$   
 (b). If  $f$  is a RNMLM, then there exists a unique sub near-field space homomorphism  $g: A \otimes_N B \rightarrow C$ , such that  $\theta_r g = f$ .

**Proof:** Given  $N$  be a left-near field space,  $A$  be an  $N$ -sub near-field space and  $B$  an  $N$ -co sub near-field space. Let  $C$  be a group with a function  $f: A \times B \rightarrow C$ .

**To prove (a):** Let us consider the following diagrammatic expression of near-field spaces over a near-field  $N$ .



where  $h$  is the unique homomorphism extending  $f$ , as  $E$  is the free sub near-field space on  $A \times B$ . Since  $f$  is an LNMLM,  $P \subseteq \text{Ker } h$ . This gives us a unique sub near-field space homomorphism  $g: E/P \rightarrow C$ , such that  $\pi_P g = h$ . It follows then  $\theta_l g = j\pi_L g = jh = f$ . Now for the uniqueness let  $g'$  be another homomorphism from  $E/P$  to  $C$  with  $\theta_l g' = f$ . then

$$(a_l \otimes b)g' = (a, b)\theta_l g' = (a, b)f = (a, b)jh = (a, b)j\pi_L g = (a, b)\theta_l g = (a_l \otimes b)g.$$

Therefore,  $g$  and  $g'$  agree on the generators of  $A_N \otimes B$  and hence are equal.

Hence proved (b). In similar manner (b) can be proved. This completes the proof of the theorem.

**Corollary 2.8:** Let  $A, A'$  be  $N$ -sub near-field spaces and  $B, B'$  be  $N$ -co sub near-field spaces over a near-field  $N$ ,  $f: A \rightarrow A'$  and  $g: B \rightarrow B'$  be  $N$ -homomorphisms of  $N$ -sub near-field spaces and  $N$ -co sub near-field spaces respectively. Then there are unique sub near-field space homomorphisms  $\phi: A_N \otimes B \rightarrow A'_N \otimes B'$  and  $\Psi: A \otimes_N B \rightarrow A' \otimes_N B'$  such that  $(a_l \otimes b)\phi = af_l \otimes bg$  and  $(a \otimes_r b)\Psi = af \otimes_r bg$ .

**Note-2.9:** The homomorphism  $\phi$  and  $\Psi$  in the above corollary are denoted by  $f_l \otimes g$  and  $f \otimes_r g$  respectively.

If  $A, A', A''$  are  $N$ -sub near-field spaces of a near-field space over a near-field  $N$  and  $B, B', B''$  are  $N$ -sub near-field spaces of a near-field space over a near-field  $N$  with  $N$ -homomorphisms  $f: A \rightarrow A', f': A \rightarrow A'', g: B \rightarrow B'$  and  $g': B \rightarrow B''$ , then  $(f_l \otimes g)(f'_l \otimes g') = ff'_l \otimes gg'$  and  $(f \otimes_r g)(f' \otimes_r g') = ff' \otimes_r gg'$ . Moreover, if  $f$  and  $g$  are isomorphisms then  $f_l \otimes g$  and  $f \otimes_r g$  are isomorphisms.

**Theorem 2.10:** Let  $N$  and  $T$  be left near-field spaces over a near-field,  $A$  an  $(N - T)$ -bi sub near-field space and  $B$  an  $T$ -co sub near-field space over a near-field. The  $A_T \otimes B$  and  $A \otimes_T B$  are  $N$ -co sub near-field spaces over a near-field  $N$ .

**Proof:**  $\forall n \in N$ , define  $\alpha_n: A \times B \rightarrow A_T \otimes B$  by  $(a, b)\alpha_n = na_l \otimes b$ , for all  $(a, b) \in A \times B$ . We claim that  $\alpha_n$  is a LTMPM. For all  $a, a' \in A, b \in B$  and  $t \in T$  then we have the following relation as

$$(a + a', b)\alpha_n = n(a + a')_l \otimes b = (na + na')_l \otimes b = na_l \otimes b + na'_l \otimes b = (a, b)\alpha_n + (a', b)\alpha_n.$$

$$(at, b)\alpha_n = n(at)_l \otimes b = (na)_l \otimes b = na_l \otimes sb = (a, sb)\alpha_n.$$

By theorem 2.9 there is a unique endomorphism  $\beta_n$  of  $A_T \otimes B$  such that  $\theta_l \beta_n = \alpha_n$ , where  $\theta_l$  is the canonical LTMPM:  $A \times B \rightarrow A_T \otimes B$ . The action of  $N$  on  $A_T \otimes B$  is now defined by  $nu = u\beta_n$  for  $n \in N$  and  $u \in A_T \otimes B$ . We claim that this action defines  $A_T \otimes B$  as an  $N$ -co sub near-field space over a near-field  $N$ . For all  $u, u' \in A_T \otimes B$  and  $n, n' \in N$  we have  $n(u + u') = (u + u')\beta_n = u\beta_n + u'\beta_n = nu + nu'$ . In order to prove that  $(nn')u = n(n'u)$ , it is enough to prove that  $\beta_{nn'} = \beta_n \beta_{n'}$  for all  $n, n' \in N$ . we look at their action on generators of  $A_T \otimes B$ .

$$(a_l \otimes b)\beta_{nn'} = (a, b)\theta_l \beta_{nn'} = (a, b)\alpha_{nn'} = (nn')a_l \otimes b = n(n'a)_l \otimes b = (n'a, b)\alpha_n = (n'a_l \otimes b)\beta_n = (a, b)\alpha_{n'}\beta_n = (a_l \otimes b)\beta_n \beta_{n'}.$$

The fact that  $A_T \otimes B$  is  $N$ -co sub near-field space over a near-field and similar manner one can prove that  $A \otimes_T B$  is an  $N$ -co sub near-field space over a near-field. This completes the proof of the theorem.

**Note-2.11:**  $N_N \otimes B$  and  $N \otimes_N B$  are co sub near-field spaces for any N-co sub near-field space B over a near-field.

**Note-2.12:** Let N be a left near-field space with 1, and B be any unital N-co sub near-field space. Then for  $n \in N$  and  $b \in B$ , we have  $n_l \otimes b = 1_l \otimes nb$  and  $n \otimes_n b = 1 \otimes_n nb$ . Therefore we have the following results:

- $N_N \otimes B$  is generated by  $\{1_l \otimes b \mid b \in B\}$
- $N \otimes_N B$  is generated by  $\{0 \otimes_n b, 1 \otimes_n b \mid b \in B \setminus \{0\}\}$ .
- $\theta_b : N \rightarrow N_N \otimes B$  defined by  $n\theta_b = n_l \otimes b$  is a sub near-field space homomorphism.
- $\theta_b : N \rightarrow N \otimes_N B$  defined by  $n\theta_b = n \otimes_n b$  is not a sub near-field space homomorphism in general.

**Theorem 2.13:** let N and T be left near-field spaces, A an N-sub near-field space and B an (N – T)-bi sub near-field space. Then  $A_N \otimes B$  is an T-sub near-field space with T acting on the right. If in addition B is a right strong T-sub near-field space then  $A \otimes_N B$  is also a T-co sub near-field space with T acting on the right.

**Proof:** For  $t \in T$  define  $\alpha_t : A \times B \rightarrow A_N \otimes B$  by  $(a, b) \alpha_t = a_l \otimes bt$ . It is easy to see that  $\alpha_t$  is a LNMLM.

This gives us a unique endomorphism  $\beta_t$  of  $A_N \otimes B$  such that  $\theta_l \beta_t = \alpha_t$ , where  $\theta_l : A \times B \rightarrow A_N \otimes B$  is the canonical LNMLM. For all  $(a_l \otimes b) \in A_N \otimes B$ , we have  $(a_l \otimes b) \beta_t = (a, b) \theta_l \beta_t = (a, b) \alpha_t = a_l \otimes (bt)$ .

Now we define an action of T on  $A_N \otimes B$  by  $ut = u\beta_t \forall u \in A_N \otimes B$ . Clearly,  $(u + u')t = ut + u't, \forall u, u' \in A_N \otimes B$ .

For the other condition we need to show that  $\beta_{tt'} = \beta_t \beta_{t'} \forall t, t' \in T$ . It is enough to look at their behaviour on the generators.

$(a_l \otimes b) \beta_{tt'} = a_l \otimes b(tt') = a_l \otimes (bt)t' = (a_l \otimes (bt)) \beta_{t'} = (a_l \otimes b) \beta_t \beta_{t'}$ . The second part can be proved in similar manner. This completes the proof of the theorem.

**Corollary 2.14:**  $A_N \otimes B$  is an N-co sub near-field space with N acting on the right. If N is distributive near-field space then  $A_N \otimes N$  and  $A \otimes_N N$  is an N-co sub near-field space with N acting on the right.

**Note-2.15:** Let N be a left near-field space with 1 and A be a unital N-sub near-field space. Then we have  $A_N \otimes N$  is generated by notation of a set  $\{a_l \otimes 1, a_l \otimes 0 \mid a \in A \setminus \{0\}\}$  and  $A \otimes_N N$  is generated by  $\{a \otimes_n 1 \mid a \in A\}$ .

**Note-2.16:** If A is any sub near-field space and B is an abelian sub near-field space then  $A_Z \otimes B$  and  $A \otimes_Z B$  are right Z-co sub near-field spaces and hence are abelian sub near-field spaces over a near-field N.

**Note-2.17:** If N is a near-field and A is an N-sub near-field space in the near-field spaces sense, that is  $(A, +)$  is not necessarily abelian, then  $A_N \otimes B$  and  $A \otimes_N B$  can be constructed, which may be different.

The structure of tensor products of near-field spaces can be explored further and these proofs here to show the importance of this concept.

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## REFERENCES

1. A I Lihtman, Rings that are radical over a commutative subring, math, Sbormik (N.S.) 83, (1970) 513-523.
2. Clay, J R, Near-rings-Genses and applications, Oxford University Press, Oxford, 1992.
3. Graninger G, Left modules for left near—rings, Doctoral theis, Univ. Of Arizona, 1988.
4. Hungerford T W, Algebra, springer-verlag, New Yark, 1974.
5. Mahmood S J and Mansouri M F, Tensor products of near—ring Modules, Kulwer academic Publishers, Netherland, 1997.
6. Mathna N M Q, Near-rings and their modules, Master's Thesis, King Saud Univ. KSA, 1990.
7. I N Herstein, two remarks on the commutativity of rings, contd. Math 7, 1955, 411-412.
8. I N Herstein Topiccs in ring theory Chicago lectures in Mathematics, University of Chicago press 11 1969.
9. I kaplansky A theorem on division rings contd., J Math, N.S. 83, 1970, 513 – 523.
10. N V Nagendram, T V Pradeep Kumar and Y V Reddy On “Semi Noetherian Regular Matrix  $\delta$ -Near Rings and their extensions”, International Journal of Advances in Algebra (IJAA), Jordan, ISSN 0973 - 6964, Vol.4, No.1, (2011), pp.51-55.

11. N V Nagendram,T V Pradeep Kumar and Y V Reddy “A Note on Bounded Matrices over a Noetherian Regular Delta Near Rings”, (BMNR-delta-NR) published in International Journal of Contemporary Mathematics, Vol.2, No.1, June 2011, Copyright@MindReaderPublications, ISSNNo:0973-6298, pp.13-19.
12. N V Nagendram,T V Pradeep Kumar and Y V Reddy “A Note on Boolean Regular Near-Rings and Boolean Regular  $\delta$ -Near Rings”, (BR-delta-NR) published in International Journal of Contemporary Mathematics, IJCM Int. J. of Contemporary Mathematics, Vol. 2, No. 1, June 2011,Copyright@Mind Reader Publications, ISSN No: 0973-6298, pp. 29 - 34.
13. N V Nagendram,T V Pradeep Kumar and Y V Reddy “on p-Regular  $\delta$ -Near-Rings and their extensions”, (PR-delta-NR) accepted and to be published in int. J. Contemporary Mathematics (IJCM), 0973-6298,vol.1, no.2, pp.81-85, June 2011.
14. N V Nagendram, T V Pradeep Kumar and Y V Reddy “On Strongly Semi –Prime over Noetherian Regular  $\delta$ -Near Rings and their extensions”, (SSPNR-delta-NR) published in International Journal of Contemporary Mathematics, Vol.2, No.1, June 2011, pp.83-90.
15. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Structure Theory and Planar of Noetherian Regular  $\delta$ -Near-Rings (STPLNR-delta-NR)”, International Journal of Contemporary Mathematics, IJCM, published by IJSMA, pp.79-83, Dec, 2011.
16. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Matrix’s Maps over Planar of Noetherian Regular  $\delta$ -Near-Rings (MMPLNR-delta-NR)”, International Journal of Contemporary Mathematics, IJCM, published by IJSMA, pp.203-211, Dec, 2011.
17. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On IFP Ideals on Noetherian Regular- $\delta$ - Near Rings (IFPINR-delta-NR)”, Int. J. of Contemporary Mathematics, Copyright @ Mind Reader Publications, ISSN No: 0973-6298, Vol. 2, No. 1, pp.53-58, June 2011.
18. N V Nagendram, B Ramesh paper "A Note on Asymptotic value of the Maximal size of a Graph with rainbow connection number  $2*(AVM-GR-CN2*)$ " published in an International Journal of Advances in Algebra(IJAA) Jordan @ Research India Publications, Rohini , New Delhi, ISSN 0973-6964 Volume 5, Number 2 (2012), pp. 103-112.
19. N V Nagendram research paper on "Near Left Almost Near-Fields (N-LA-NF)" communicated to for 2nd international conference by International Journal of Mathematical Sciences and Applications, IJMSA@ mindreader publications, New Delhi on 23-04-2012 also for publication.
20. N V Nagendram, T Radha Rani, Dr T V Pradeep Kumar and Dr Y V Reddy “A Generalized Near Fields and (m, n) Bi-Ideals over Noetherian regular Delta-near rings (GNF-(m, n) BI-NR-delta-NR)", published in an International Journal of Theoretical Mathematics and Applications (TMA), Greece, Athens, dated 08-04-2012.
21. N V Nagendram,Smt.T.Radha Rani, Dr T V Pradeep Kumar and Dr Y V Reddy "Applications of Linear Programming on optimization of cool freezers(ALP-on-OCF)" Published in International Journal of Pure and Applied Mathematics, IJPAM-2012-17-670 ISSN-1314-0744 Vol-75 No-3(2011).
22. N V Nagendram "A Note on Algebra to spatial objects and Data Models(ASO-DM)" Published in international Journal American Journal of Mathematics and Mathematical Sciences, AJMMS,USA, Copyright @ Mind Reader Publications, Rohini , New Delhi, ISSN. 2250-3102, Vol.1, No.2 (Dec. 2012), pp. 233 – 247.
23. N V Nagendram, Ch Padma, Dr T V Pradeep Kumar and Dr Y V Reddy "A Note on Pi-Regularity and Pi-S-Unity over Noetherian Regular Delta Near Rings (PI-R-PI-S-U-NR-Delta-NR)" Published in International Electronic Journal of Pure and Applied Mathematics, IeJPAM-2012-17-669 ISSN-1314-0744 Vol-75 No-4 (2011).
24. N V Nagendram, Ch Padma, Dr T V Pradeep Kumar and Dr Y V Reddy "Ideal Comparability over Noetherian Regular Delta Near Rings(IC-NR-Delta-NR)" Published in International Journal of Advances in Algebra (IJAA, Jordan), ISSN 0973-6964 Vol:5,NO:1(2012), pp.43-53@Research India publications, Rohini, New Delhi.
25. N. V. Nagendram , S. Venu Madava Sarma and T. V. Pradeep Kumar, “A Note On Sufficient Condition Of Hamiltonian Path To Complete Graphs (SC-HPCG)”, IJMA-2(11), 2011, pp.1-6.
26. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Noetherian Regular Delta Near Rings and their Extensions(NR-delta-NR)”, IJCMS, Bulgaria, IJCMS-5-8-2011,Vol.6,2011,No.6,255-262.
27. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Semi Noetherian Regular Matrix Delta Near Rings and their Extensions (SNRM-delta-NR)”,Jordan,@ResearchIndiaPublications,AdvancesinAlgebraISSN 0973-6964 Volume 4, Number 1 (2011), pp.51-55© Research India Publicationspp.51-55.
28. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Boolean Noetherian Regular Delta Near Ring (BNR-delta-NR)s”, International Journal of Contemporary Mathematics, IJCM Int. J. of Contemporary Mathematics, Vol. 2, No. 1-2, Jan-Dec 2011, Mind Reader Publications, ISSN No: 0973-6298, pp. 23-27.
29. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Bounded Matrix over a Noetherian Regular Delta Near Rings (BMNR-delta-NR)”, Int. J. of Contemporary Mathematics, Vol. 2, No. 1-2, Jan-Dec 2011 ,Copyright @ Mind Reader Publications, ISSN No: 0973-6298, pp.11-16.
30. N V Nagendram,Dr T V Pradeep Kumar and Dr Y V Reddy “On Strongly Semi Prime over Noetherian Regular Delta Near Rings and their Extensions(SSPNR-delta-NR)”, Int. J. of Contemporary Mathematics,Vol. 2, No. 1, Jan-Dec 2011 , Copyright @ Mind Reader Publications ,ISSN No: 0973-6298, pp.69-74.

31. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On IFP Ideals on Noetherian Regular Delta Near Rings(IFPINR-delta-NR)", Int. J. of Contemporary Mathematics, Vol. 2, No. 1-2, Jan-Dec 2011, Copyright @ Mind Reader Publications, ISSN No: 0973-6298, pp.43-46.
32. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On Structure Thoery and Planar of Noetherian Regular delta-Near-Rings (STPLNR-delta-NR)",International Journal of Contemporary Mathematics, IJCM ,accepted for Ist international conference conducted by IJSMA, New Delhi December 18,2011,pp:79-83,Copyright @ Mind Reader Publications and to be published in the month of Jan 2011.
33. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On Matrix's Maps over Planar of Noetherian Regular delta-Near-Rings (MMPLNR-delta-NR)",International Journal of Contemporary Mathematics, IJCM, accepted for Ist international conference conducted by IJSMA, New Delhi December 18,2011, pp:203-211, Copyright @ Mind Reader Publications and to be published in the month of Jan 2011.
34. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "Some Fundamental Results on P- Regular delta-Near-Rings and their extensions (PNR-delta-NR)", International Journal of Contemporary Mathematics ,IJCM,Jan-December'2011,Copyright@MindReader Publications,ISSN:0973-6298, vol.2, No.1-2, Pp.81-85.
35. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "A Generalized ideal based-zero divisor graphs of Noetherian regular Delta-near rings (GIBDNR- d-NR)" , International Journal of Theoretical Mathematics and Applications (TMA)accepted and published by TMA, Greece, Athens,ISSN:1792- 9687 (print),vol.1, no.1, 2011, 59-71, 1792-9709 (online), International Scientific Press, 2011.
36. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "Inversive Localization of Noetherian regular Delta-near rings (ILNR- Delta-NR)" , International Journal of Pure And Applied Mathematics published by IJPAM-2012-17-668, ISSN.1314-0744 vol-75 No-3,SOFIA, Bulgaria.
37. N V Nagendram1, N Chandra Sekhara Rao2 "Optical Near field Mapping of Plasmonic Nano Prisms over Noetherian Regular Delta Near Fields (ONFMPN-NR-Delta-NR)" accepted for 2nd international Conference by International Journal of Mathematical Sciences and Applications, IJMSA @ mind reader publications, New Delhi going to conduct on 15 – 16 th December 2012 also for publication.
38. N V Nagendram, K V S K Murthy (Yoga), "A Note on Present Trends on Yoga Apart From Medicine Usage and Its Applications (PTYAFMUIA)" Published by the International Association of Journal of Yoga Therapy, IAYT 18 th August, 2012.
39. N V Nagendram, B Ramesh, Ch Padma, T Radha Rani and S V M Sarma research article "A Note on Finite Pseudo Artinian Regular Delta Near Fields (FP AR-Delta-NF)" communicated to International Journal of Advances in Algebra, IJAA, Jordan on 22 nd August 2012.
40. N V Nagendram "Amenability for dual concrete complete near-field spaces over a regular delta near-rings (ADC-NFS-R- $\delta$ -NR)" accepted for 3rd international Conference by International Journal of Mathematical Sciences and Applications, IJMSA@mind reader publications, New Delhi going to conduct on 15 – 16 th December 2014 also for publication.
41. N V Nagendram "Characterization of near-field spaces over Baer-ideals" accepted for 4th international Conference by International Journal Conference of Mathematical Sciences and Applications, IJCMSA @ mind reader publications, New Delhi going to conduct on 19 – 20 th December 2015 at Asian Institute of Technology AIT, Klaung Lange 12120, Bangkok, Thailand.
42. N V Nagendram, S V M Sarma Dr T V Pradeep Kumar "A note on sufficient condition of Hamiltonian path to Complete Graphs" published in International Journal of Mathematical archive IJMA, ISSN 2229-5046, Vol.2, No.2, Pg. 2113 – 2118, 2011.
43. N V Nagendram, S V M Sarma, Dr T V Pradeep Kumar "A note on Relations between Barnette and Sparse Graphs" publishd in an International Journal of Mathematical Archive (IJMA), An International Peer Review Journal for Mathematical, Science & Computing Professionals, 2(12), 2011, pg no.2538-2542, ISSN 2229 – 5046.
44. N V Nagendram "On Semi Modules over Artinian Regular Delta Near Rings(S Modules-AR-Delta-NR) Accepted and published in an International Journal of Mathematical Archive (IJMA)", An International Peer Review Journal for Mathematical, Science & Computing Professionals ISSN 2229-5046, IJMA-3-474, 2012.
45. N V Nagendram "A note on Generating Near-field efficiently Theorem from Algebraic K - Theory" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.3, No.10, Pg. 1 – 8, 2012.
46. N V Nagendram and B Ramesh on "Polynomials over Euclidean Domain in Noetherian Regular Delta Near Ring Some Problems related to Near Fields of Mappings (PED-NR-Delta-NR & SPR-NF)" Accepted and published in an International Journal of Mathematical Archive (IJMA), An International Peer Review Journal for Mathematical, Science & Computing Professionals ISSN 2229-5046, vol.3, no.8, pp no. 2998-3002, 2012.
47. N V Nagendram "Semi Simple near-fields Generating efficiently Theorem from Algebraic K - Theory" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.3, No.12, Pg. 1 – 7, 2012.
48. N V Nagendram "-----" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.3, No.10, Pg. 3612 – 3619, 2012.
49. N V Nagendram, E Sudeeshna Susila, "Applications of linear infinite dimensional system in a Hilbert space and its characterizations in engg. Maths(AL FD S HS & EM)", IJMA, ISSN.2229-5046, Vol.4, No.7, Pg. 1– 11 (19 – 29), 2013.

50. N V Nagendram, Dr T V Pradeep Kumar, "Compactness in fuzzy near-field spaces (CN-F-NS)", IJMA, ISSN. 2229 – 5046, Vol.4, No.10, Pg. 1 – 12, 2013.
51. N V Nagendram, Dr T V Pradeep Kumar and Dr Y Venkateswara Reddy, " Fuzzy Bi- $\Gamma$  ideals in  $\Gamma$  semi near – field spaces (F Bi-Gamma I G)" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.4, No.11, Pg. 1 – 11, 2013.
52. N V Nagendram," EIFP Near-fields extension of near-rings and regular delta near-rings (EIFP-NF-E-NR)" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229 - 5046, Vol.4, No.8, Pg. 1 – 11, 2013.
53. N V Nagendram, E Sudeeshna Susila, "Generalization of  $(\in, \in V_q)$  fuzzy sub near-fields and ideals of near-fields (GF-NF-IO-NF)", IJMA, ISSN.2229-5046, Vol.4, No.7, Pg. 1 – 11, 2013.
54. N V Nagendram, Dr T V Pradeep Kumar," A note on Levitzki radical of near-fields (LR-NF)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.4, No.4, Pg.288 – 295, 2013.
55. N V Nagendram, "Amalgamated duplications of some special near-fields (AD-SP-N-F)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.4, No.2, Pg.1 – 7, 2013.
56. N V Nagendram," Infinite sub near-fields of infinite near-fields and near-left almost near-fields (IS-NF-INF-NL-A-NF)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.4, No.2, Pg. 90 – 99, 2013.
57. N V Nagendram "-----" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.4, No.7, Pg. 1 – 11, 2013
58. N V Nagendram, E Sudeeshna Susila, Dr T V Pradeep Kumar "Some problems and applications of ordinary differential equations to Hilbert Spaces in Engg mathematics (SP-O-DE-HS-EM)", IJMA, ISSN.2229-5046, Vol.4, No.4, Pg. 118 – 125, 2013.
59. N V Nagendram, Dr T V Pradeep Kumar and D Venkateswarlu, " Completeness of near-field spaces over near-fields (VNFS-O-NF)" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.5, No.2, Pg. 65 – 74, 2014
60. Dr N V Nagendram "A note on Divided near-field spaces and  $\phi$ -pseudo – valuation near-field spaces over regular  $\delta$ -near-rings (DNF- $\phi$ -PVNFS-O- $\delta$ -NR)" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.6, No.4, Pg. 31 – 38, 2015.
61. Dr. N V Nagendram "A Note on  $B_1$ -Near-fields over R-delta-NR ( $B_1$ -NFS-R- $\delta$ -NR)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.6, No.8, Pg. 144 – 151, 2015.
62. Dr. N V Nagendram " A Note on TL-ideal of Near-fields over R-delta-NR (TL-I-NFS-R- $\delta$ -NR)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.6, No.8, Pg. 51 – 65, 2015.
63. Dr. N V Nagendram "A Note on structure of periodic Near-fields and near-field spaces (ANS-P-NF-NFS)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.7, No.4, Pg. 1 – 7, 2016.
64. Dr. N V Nagendram "Certain Near-field spaces are Near-fields (C-NFS-NF)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.7, No.4, Pg. 1 – 7, 2016.
65. Dr. N V Nagendram "Sum of Annihilators Near-field spaces over Near-rings is Annihilator Near-field space (SA-NFS-O-A-NFS)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.7, No.1, Pg. 125 – 136, 2016.
66. Dr. N V Nagendram " A note on commutativity of periodic near-field spaces", Published by IJMA, ISSN. 2229 - 5046, Vol.7, No. 6, Pg. 27 – 33, 2016.
67. Dr N V Nagendram "Densely Co-Hopfian sub near-field spaces over a near-field, IMA, ISSN No.2229-5046, 2016, Vol.7, No.10, Pg 1-12.
68. Dr N V Nagendram, "Closed (or open) sub near-field spaces of commutative near-field space over a near-field", 2016, Vol.7, No, 9, ISSN NO. 2229 – 5046, Pg No.57 – 72.
69. Dr N V Nagendram, "Homomorphism of near-field spaces over a near-field "IJMA Jan 2017, Vol.8, No, 2, ISSN NO.2229 – 5046, Pg No. 141 – 146.
70. Dr N V Nagendram, "Sigma – toe derivations of near-field spaces over a near-field "IJMA Jan 2017, Vol.8, No, 4, ISSN NO.2229 – 5046, Pg No. 1 – 8.
71. Dr N V Nagendram, "On the hyper center of near-field spaces over a near-field "IJMA Feb 2017, Vol.8, No, 2, ISSN NO.2229 – 5046, Pg No. 113 – 119.

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