

STATISTICAL ANALYSIS OF NON-PREEMPTIVE PRIORITY IN FUZZY QUEUEING SYSTEM

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ABSTRACT

In General a management does not like an arriving customer to wait for service in a system. Most of the situations it becomes impossible to avoid the difficulty of waiting for service. Thus the management is forced to go for a priority queueing system which is helpful to rectify this difficulty. There are two types of priority schemes namely preemptive and Non preemptive. In the Non preemptive priority discipline, there is no interruption and the highest priority customer just goes to the head of the queue to wait for his turn. In this discipline customers with higher priority and lower priority have their own service times. Practically the queue parameters are not deterministic. So, in this paper we discussed the non-preemptive priority of fuzzy queueing system with equal and unequal service times separately and we derived the steady state balance equation. So, first we construct the range of uncertainty values through inverse membership function and then analyze the interval optimal values using a suitable statistical inference. A Numerical example is illustrated.

Keywords: Fuzzy sets, Membership function, multiple servers, Nonlinear programming..

AMS Classification Number: MSC 60.

INTRODUCTION

At any situation, an arriving customer does not like waiting in any queue. Unavoidably situation warrants a customer to wait. Since the management could not take a good decision in such situations. We try to simulate the above problem with the support of the priority queueing system. Generally, the priority queueing system consists of two cases non-preemptive and preemptive. In the preemptive priority system interruptions are allowed to overtake the lower priority customers even without for the service to complete. So, in this paper we consider a non-preemptive priority technique in which the higher priority customers may not break up the service time of a lower priority customer. Many authors [2, 3, 4, 6, 7] have already discussed the priority queueing system model. But some authors [5, 8, 16, 19, 22, 27] have done in preemptive priority queueing model. Since the model depends only on the service time, which is always lacking the decision maker could not take a correct decision, to recover from this drawback; we prefer the non –preemptive model. [18] Proposed some optimal policies for the multi-server of non-preemptive priority queues. [23] Stated that system capacity of the wireless system in batch arrival M/G/1 non-preemptive priority of queueing model.[1] Consist the multiple item of two group priority classes for each class considered as a own arrival and service time, and estimated that the steady state probabilities for the approximation error. [13] Studied the simultaneous queueing system of single server in M/G/1 and M/D/1 with the help of non-preemptive priority rule. [20] Studied the discrete – time queue in high and low priority arrival of binomial distribution, and analyzed the system performance level used weighted cost function method.

For the unexpected situation, queues parameters (arrival, service, waiting discipline etc.,) are uncertain because of the natural calamities. In this case zadeh extension principle is reduced to the fuzzy queue into classical queue. Many authors [14, 15, 17, 24] are described application on classical queueing model into fuzzy queueing model. [10, 11, 12] Developed the exponential time based queueing theory in non- preemptive model and [9] Constructed the membership function of a fuzzy priority queue and considered the arrival rates are Trapezoidal fuzzy numbers. In previous paper we have solved statistical analysis in K-phase Erlang arrival and service in infinite capacity. But Priority for K-phase Erlang distribution model is very tedious once. So we take K=1, then the K-phase Erlang model comes to M/M/1:∞/FIFO model.

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2. PRELIMINARIES [25, 26]

In this section, Basic definition and Results on fuzzy numbers are summarized which are needed in this sequel.

2.1 Definition [20, 22]: Let X be a nonempty set. A fuzzy set “A” in X is defined as the map $A: X \rightarrow [0, 1]$. A fuzzy set A in X is characterized by its membership function $A(x)$ and is interpreted as the degree of membership of element x in the fuzzy set A , for each $x \in X$.

2.2 Definition: A triangular fuzzy numbers “A” is represented by the three points as follows:

$A = (a_1, a_2, a_3)$, where $a_i \in \mathbb{R}$. This representation is interpreted as membership function satisfying (Fig. 1) the following conditions:

- (i) a_1 to a_2 is increasing function
- (ii) a_2 to a_3 is decreasing function
- (iii) $a_1 \leq a_2 \leq a_3$,

$$\mu_a(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \frac{x-a_1}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{for } x > a_3 \end{cases}$$

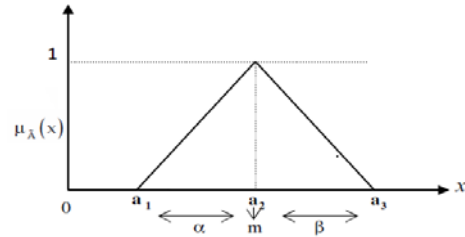


Figure-1: Triangular fuzzy number

Fuzzy Interval Correlation [21]:

Correlation coefficient is nothing but to study the relationship between the variables X and Y , which can be classified into Positive correlation, negative correlation or non - correlation. Mostly we are dealing with Pearson Correlation coefficient for two random variables linearly related in a sample. The population correlation coefficient ρ , is defined for

two variables X and Y by the formula $\rho = \frac{\text{cov}(X, Y)}{\sigma_x \sigma_y}$ where (x_i, y_i) is the i^{th} pair observation value, $i = 1, 2, 3 \dots n$.

\bar{x} and \bar{y} are the sample mean for X and Y respectively.

MODEL DESCRIPTION FOR NON-PRE EMPTIVE SYSTEM

Suppose that Customers arrive as a Poisson Process to a single exponential Channel and that each Customer, upon arrival to the system, is assigned to one of two priority classes. The usual convention is determined to number the priority classes so that smaller numbers correspond to the higher priorities. Suppose that the arrivals of the first or higher priority class have mean arrival rate λ_1 and that those of the second or lower priority class have mean arrival rate λ_2 . The total arrival rate is $\lambda = \lambda_1 + \lambda_2$. Also suppose that the first priority customer is served ahead of the second priority customers, but that there is no pre emption.

From these assumptions, a system of balance equations may be established for the steady- state probabilities, defined as follows (where m and n are not both 0)

Let $P_{mnr} = \Pr \{m, n \text{ are the priority of the Customer in the system 1 and 2 (ie, priority } r = 1 \text{ or } 2)\}$

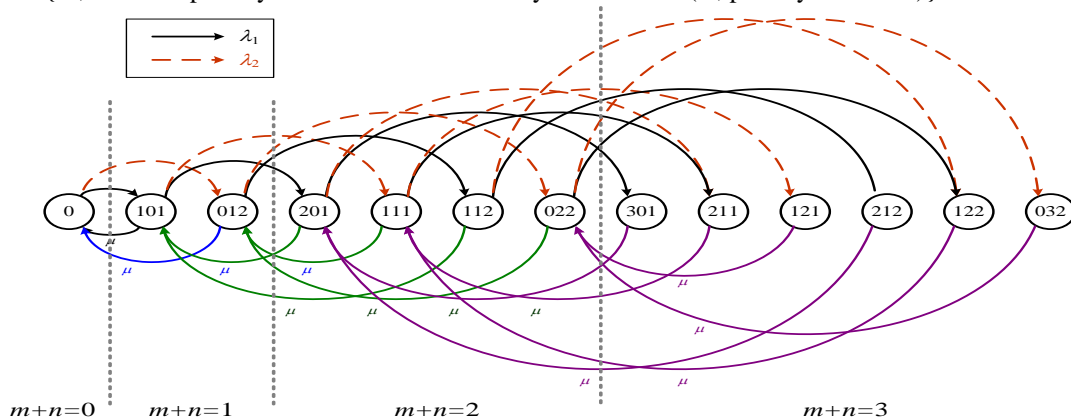


Figure-2

Let P_0 be the probability of the system is idle. Let $L^{(i)}$ be the expected number of customers in the i^{th} system.

Case- (i): Equal Service Rate

In this case, the arriving customers has to be considered as a different manner at the same time we should consider the service rate as equal for n^{th} stage ($n=2$)

$$\text{We take } \rho_1 = \frac{\lambda_1}{\mu}, \rho_2 = \frac{\lambda_2}{\mu}, \rho = \rho_1 + \rho_2 = \frac{\lambda}{\mu}, \lambda = \lambda_1 + \lambda_2$$

From the Figure 2 we get, the balance equation,

$$\begin{aligned} (\lambda + \mu) p_{mn2} &= \lambda_1 p_{m-1,n,2} + \lambda_2 p_{m,n-1,2} \quad (m \geq 1, n \geq 2), \\ (\lambda + \mu) p_{mn1} &= \lambda_1 p_{m-1,n,1} + \lambda_2 p_{m,n-1,1} + \mu(p_{m+1,n,1} + p_{m,n+1,2}) \quad (m \geq 2, n \geq 1) \\ (\lambda + \mu) p_{m12} &= \lambda_1 p_{m-1,1,2} \quad (m \geq 1) \\ (\lambda + \mu) p_{1n1} &= \lambda_2 p_{1,n-1,1} + \mu(p_{2n1} + p_{1,n+1,2}) \quad (n \geq 1) \\ (\lambda + \mu) p_{0n2} &= \lambda_2 p_{0,n-1,2} + \mu(p_{1n1} + p_{0,n+1,2}) \quad (n \geq 2) \\ (\lambda + \mu) p_{m01} &= \lambda_1 p_{m-1,0,1} + \mu(p_{m+1,0,1} + p_{m,1,2}) \quad (m \geq 2) \\ (\lambda + \mu) p_{012} &= \lambda_2 p_0 + \mu(p_{111} + p_{022}) \\ (\lambda + \mu) p_{101} &= \lambda_1 p_0 + \mu(p_{201} + p_{122}) \\ \lambda p_0 &= \mu(p_{101} + p_{012}) \end{aligned}$$

The service rates are considered as an exponential distribution with the same rate μ , the total number of customers in service has the same steady state distribution as an M/M/1 queue.

Therefore, $p_n = (1 - \rho)\rho^n$, ($n > 0$), where $\rho = \rho_1 + \rho_2$ is the fraction of time the server is busy.

$$\text{Now we define } P_{m1}(z) = \sum_{n=0}^{\infty} z^n p_{mn1} \quad (m \geq 1) \quad P_{m2}(z) = \sum_{n=0}^{\infty} z^n p_{mn2} \quad (m \geq 0)$$

Then the joint generating function of two classes is

$$H(y, z) = H_1(y, z) + H_2(y, z) + P_0.$$

$$\text{Where, } H_1(y, z) = \sum_{m=1}^{\infty} y^m P_{m1}(z) \quad \text{with } H_1(1,1) = \rho_1, \quad H_2(y, z) = \sum_{m=0}^{\infty} y^m P_{m2}(z) \quad \text{with } H_2(1,1) = \rho_2$$

$$\text{After simplifying the equations, we get, } L_q = L_q^{(1)} + L_q^{(2)} = \frac{\rho^2}{1 - \rho},$$

$$\text{where, } L_q^{(1)} = \frac{\lambda_1 \rho}{\mu - \lambda_1}, \quad L_q^{(2)} = \frac{\lambda_2 \rho}{(\mu - \lambda_1)(1 - \rho)}$$

UNEQUAL SERVICE RATE

Similar for the case (i), but we assume that the service rates of the two classes are not necessarily equal. Specifically, Priority -1 customers are served at a rate and priority-2 customers are served at a rate for this queue, define

$$\rho_1 = \frac{\lambda_1}{\mu_1}, \rho_2 = \frac{\lambda_2}{\mu_2}, \rho = \rho_1 + \rho_2$$

$$\text{Using the above balance equation, we get, } L_q = L_q^{(1)} + L_q^{(2)},$$

$$\text{where } L_q^{(1)} = \frac{\lambda_1 (\rho_1 / \mu_1 + \rho_2 / \mu_2)}{1 - \rho_1}, \quad L_q^{(2)} = \frac{\lambda_2 (\rho_1 / \mu_1 + \rho_2 / \mu_2)}{(1 - \rho_1)(1 - \rho)}$$

Solution Procedure:

A queueing system consists of a single phase in general discipline. The queueing parameters are considered as a fuzzy numbers are denoted by $\bar{\lambda}$ and $\bar{\mu}$ and defined as $\bar{\lambda} = \{(x, \mu_{\bar{\lambda}}(x)) / x \in X\}$, $\bar{\mu} = \{(y, \mu_{\bar{\mu}}(y)) / y \in Y\}$ where X & Y are the Universal Sets. Using α -cuts, we construct the fuzzy membership function of $\bar{\lambda}$ and $\bar{\mu}$ is

$$\hat{\lambda}_{\alpha} = [x_{\alpha}^l, x_{\alpha}^u] = \left[\min_{x \in X} \{x / \mu_{\bar{\lambda}}(x) \geq \alpha\}, \max_{x \in X} \{x / \mu_{\bar{\lambda}}(x) \geq \alpha\} \right]$$

$$\hat{\mu}_{\alpha} = [y_{\alpha}^l, y_{\alpha}^u] = \left[\min_{y \in Y} \{y / \mu_{\bar{\mu}}(y) \geq \alpha\}, \max_{y \in Y} \{y / \mu_{\bar{\mu}}(y) \geq \alpha\} \right]$$

Here, $x_{\alpha}^l, x_{\alpha}^u, y_{\alpha}^l$ and y_{α}^u are lower and upper bound of the arrival and service rates. In this paper, first we construct the inverse membership function through α -cuts and the following conditions are satisfied $\mu_{\bar{\lambda}}(x) = \alpha$ and $\mu_{\bar{\mu}}(y) \geq \alpha$ or $\mu_{\bar{\lambda}}(x) \geq \alpha$ and $\mu_{\bar{\mu}}(y) = \alpha$. In Sec. 2 we derived the average number of customer in the queue for equal and unequal service behavior. Using membership function of \bar{L}_q for unequal and equal service rate is $\mu_{\bar{L}_q}(z) = \sup \{ \mu_{\bar{\lambda}}(x), \mu_{\bar{\mu}}(y) / z = L_q^1 + L_q^2 \}$, where $L_q^1 = \frac{\lambda_1((\rho_1/\mu_1) + (\rho_2/\mu_2))}{1 - \rho_1}$ and $L_q^2 = \frac{\lambda_2((\rho_1/\mu_1) + (\rho_2/\mu_2))}{(1 - \rho_1)(1 - \rho)}$ and $\mu_{\bar{L}_q}(z) = \sup \{ \mu_{\bar{\lambda}}(x), \mu_{\bar{\mu}}(y) / z = L_q^1 + L_q^2 \}$ where $L_q^{(1)} = \frac{\lambda_1 \rho}{\mu - \lambda_1}$ and $L_q^{(2)} = \frac{\lambda_2 \rho}{(\mu - \lambda_1)(1 - \rho)}$. so, we establish a mathematical programming technique, we constructed the lower and upper bound of L_q for un equal and equal service rate in nonlinear programming problem.

NUMERICAL EXAMPLE

Consider the system in Non-Preemptive technique and the rate of service parameters are different possibility level [equal and unequal]. Consider the queue parameters in triangular fuzzy number and its represented by $\lambda_1 = [1 \ 2 \ 3]$, $\lambda_2 = [2 \ 5 \ 6]$, $\mu_1 = [10, 11, 12]$ and $\mu_2 = [12 \ 14 \ 15]$. The system manager wants to evaluate the performance measures of the system such as the expected number of customers in the queue and to analyze the optimality level of the system.

Solution: We know that λ and μ satisfy the condition $\lambda/\mu < 1$

Case - (i): For Equal service Rate:

Using α -cuts we find the upper and lower bounds of the arrival and service rates

$$[x_{\alpha}^L, x_{\alpha}^U] = [\alpha + 1, 3 - \alpha] \text{ and } [y_{\alpha}^L, y_{\alpha}^U] = [\alpha + 10, 12 - \alpha]$$

Applying MINLP technique we get the membership functions of

$$[L_q^{\alpha}, L_q^{\alpha}] \text{ as } {}^l L_q(\alpha) = \frac{16\alpha^2 + 24\alpha + 9}{5\alpha^2 - 69\alpha + 108} \text{ and } {}^u L_q(\alpha) = \frac{4\alpha^2 - 36\alpha + 81}{3\alpha^2 + 31\alpha + 10}$$

The corresponding inverse membership function is

$$\mu_{L_q}(z) = \begin{cases} \frac{(69z + 24) \pm (2601z^2 + 10404z)^{\frac{1}{2}}}{(25z + 6)}, & \text{for } 0.0833 \leq z \leq 1.1136 \\ 1, & \text{for } z = 1.1136 \\ \frac{(31z + 36) \pm (841z^2 + 3364z)^{\frac{1}{2}}}{(6z - 8)}, & \text{for } 1.1136 \leq z \leq 8.1 \end{cases}$$

The 11 values for the α cuts of the performance measures are as follows:

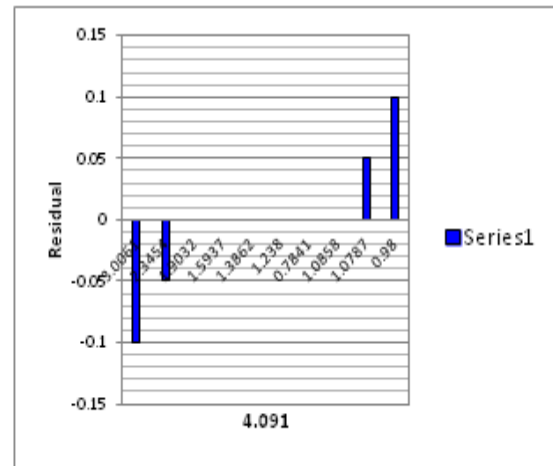
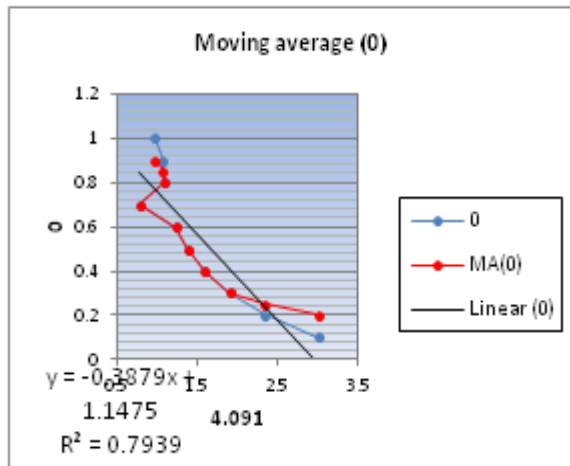
α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
L_q^l	0.0833	0.1143	0.15297	0.20103	0.2606	0.3344	0.4263	0.5413	0.6864	0.8721	1.1136
L_q^u	8.1	5.8979	4.5379	3.6055	2.9388	2.4381	2.04986	1.74133	1.49129	1.2854	1.136

Goodness of fit statistics

Statistic	Observations	DF	SSE	MSE	RMSE	MAPE	MPE	MAE	R ²
Value	10	10	0.025	0.003	0.158	14.056	-10.944	0.030	0.970

Series Before and After Smoothing

4.091	3.0061	2.3454	1.9032	1.5937	1.3862	1.238	0.7841	1.0858	1.0787	0.98
0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Moving Average	0.2	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9
Residuals	-0.1	-0.050	0.000	0.000	0.000	0.000	0.000	0.000	0.050	0.100



Case-(ii): For unequal service rate:

Using α -cuts we find the upper and lower bounds of the arrival and service rates

$$\left[x_{\alpha}^l, x_{\alpha}^u \right] = [\alpha+1, 3-\alpha] \text{ and } [3\alpha+2, 6-\alpha], \left[y_{\alpha}^l, y_{\alpha}^u \right] = [\alpha+10, 12-\alpha] \text{ and } [2\alpha+12] \text{ and } [15-\alpha]$$

Applying MINLP technique we get the membership functions of

$${}^u L_q(\alpha) = \frac{5\alpha^4 + 20\alpha^3 - 320\alpha^2 - 912\alpha + 6192}{20\alpha^4 + 358\alpha^3 + 2112\alpha^2 + 4440\alpha + 2016}$$

The 11 values for the α cuts of the performance measures are as follows:

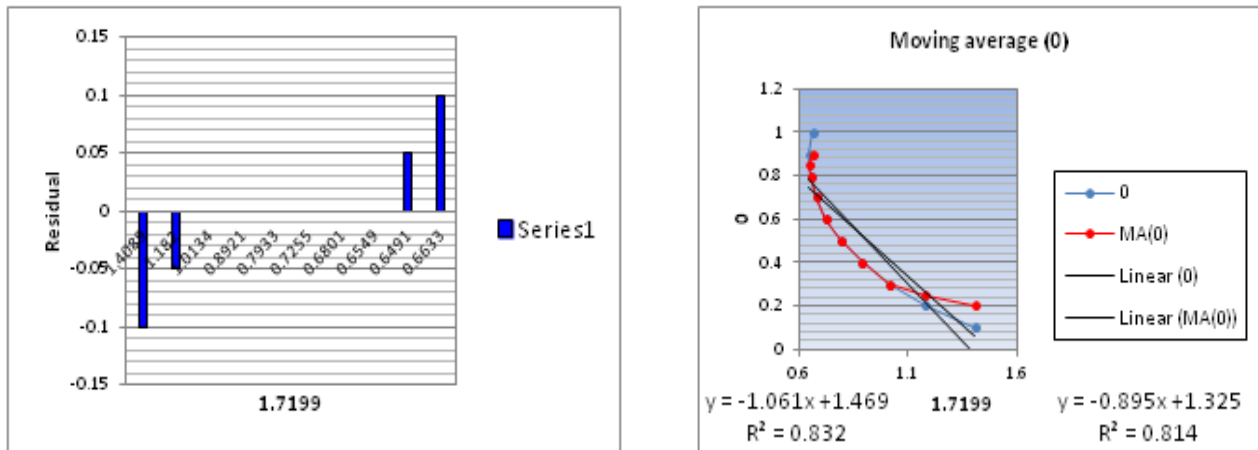
α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
L^l	0.06137	0.08312	0.1098	0.1422	0.1915	0.2289	0.2861	0.3553	0.4391	0.54133	0.6667
L_q^u	3.37857	2.7348	2.2542	1.8846	1.5927	1.3575	1.1649	1.005	0.8707	0.75708	0.6599

Goodness of fit statistics

Statistic	Observations	DF	SSE	MSE	RMSE	MAPE	MPE	MAE	R ²
Value	10	10	0.025	0.003	0.158	14.056	-10.944	0.030	0.970

Series Before and after smoothing

1.7199	1.4089	1.182	1.0134	0.8921	0.7933	0.7255	0.6801	0.6549	0.6491	0.6633
0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Moving average	0.2	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.850	0.9
Residuals	-0.100	-0.050	0.000	0.000	0.000	0.000	0.000	0.000	0.050	0.100



Remark: Suppose, the system consider in an equal service rate, we are easily to obtained the L_q and W_q because the highest degree exists in second order only. But another part of the system did not obtain because the degree exist in fourth order. In this case, we cannot easily to calculate the inverse membership function for unequal service rate.

CONCLUSION

In this paper, we investigate to simulate the arrival pattern and minimize the number of customers in the queueing system. Using alpha cuts method to obtained lower and upper bounds of L_q for equal and unequal service rate. The queueing system performance has been tested through chi-square statistical technique at 5% level of significance. Finally, we analyze the system performance purity level in XLSTAT-2014. This methodology helps to simulate the arrival pattern and provide the best performance in the system.

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