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THE ENLARGEMENT OF THE UNIVERSE DESCRIBED BY COMPRESSED NUMBERS

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ABSTRACT.

Using the hypotheses of Einstein (1905) and Lemaitre (1927) by a certain compression and explosion of real numbers we construct a theoretical mathematical model which can show the permanent inflation of the universe from a „small" to „big" state. Our model is limited to the time-dimension and the three space- dimensions. and a new apparatus of exploded and compressed numbers was used [1]. We offer a justification of compression and explosion of real numbers, model the inflation of the cube universe and perform computations in it.

INTRODUCTION

We imagine the universe in the abstract state, as the familiar three dimensional Euclidean space

$$\mathbb{R}^3 = \left\{ P = (x, y, z) \,\middle|\, \begin{Bmatrix} -\infty < x < \infty \\ -\infty < y < \infty \\ -\infty < z < \infty \end{Bmatrix} \right\},$$

with its well known apparatus, among others

- the ordered field $(\mathbb{R}, <, +, \cdot)$ of real numbers,
- the vector algebra of the multiplication $c \cdot P = (cx. cy, cz), c \in \mathbb{R}$ and addition $P_1 + P_2 = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$ with their consequences, for example the norm
$$\|P\| = \sqrt{x^2 + y^2 + z^2}$$
and distance
$$d(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2},$$
- the concept of Cauchy – convergence and limit of functions, for example the sequence of real numbers $\{x_n\}_{n=1}^{\infty}$ converges to the real number x_0 if for any positive ε there exist a threshold – number ν, such that if $n > \nu$ then $|x_n - x_0| < \varepsilon$, or $\lim_{n\to\infty} x_n = \infty$ means that for any real number K there exist a threshold – number ν, such that if $n > \nu$ then $x_n > K$.

Moreover, a new apparatus of exploded and compressed numbers is used. (See [1]).

Here, we offer a justification of the compression and explosion of real numbers, represented by the number line. First we need to determine the place of 0 (origo) then that of 1, which will determine the unit of distance - measurement. By doing that, we have determined the place of each real number *x*. Let us call the direction towards 1 the positive direction, the other the negative direction. But how does this method work in practise? Let us take the position of 0 from which we look at the three-dimensional space. In this step we focus on the positive direction, only. To find the point *x* we send a point-like satellite from the origo in the positive direction. We are in the origo. How can we obtain the information on *x* from the satellite? If we get it then, at time when the satellite reaches the point $\boldsymbol{x}$, it is late. It is because, by the time the information reaches the origo the satellite will be over *x*. So, the satellite has to give the sign „I am in *x*" earlier, from the point ξ. This is the compressed of *x*, denoted by $\underline{x}$. So,

(0.1) $$\xi = \underline{x}.$$

Conversely, x is called the exploded ξ, denoted by

(0.2) $$x = \check{\xi}.$$

Using these notations mutually, we have the inversion formulas:

(0.3) $$\xi = \underline{(\check{\xi})},$$ where ξ is an arbitrary compressed number,

and

(0.4) $$x = \overset{\smile}{(\underline{x})},$$ where x is an arbitrary real number.

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Clearly,

(0.5) $$\underline{0} = 0 \quad and \quad \check{0} = 0$$

but if $x \neq 0$ then

(0.6) $|\underline{x}| < |x|$, (x may be negative real number, too)

and if $\xi \neq 0$ then

(0.7) $$|\xi| < |\check{\xi}|.$$

If we accept that the universe was created by the„Big Bang" (Lemaitre, 1927) then from the beginning to our days a certain time, denoted by $t_{universe} \approx 13,7 \cdot 10^9\ year$, passed. Assuming that there exists a supreme speed (Einstein, 1905), denoted by $v_{supreme}$, for any real number x we have that its compressed ξ remains under the bound $v_{supreme} \cdot t_{universe}$. Nowadays our universe seems to be enlarging. Nobody knows that the time of enlargement is finite or infinite. If $t_{universe}$ tends to get closer to infinity, then the universe enlarges, eternally. (If the inflation of the universe has a final time $t_{inflation}$, then in the case of $t > t_{inflation}$ the collapse of the universe is coming, perhaps.) Of course, we can compute shorter time $(0 <) t < t_{universe}$, from the beginning, too. In this case we have the compression - parameter

(0.8) $$\sigma(t) = v_{supreme} \cdot t$$

and

(0.9) $$|\xi| < \sigma(t)$$

is obtained. Hence, $\underline{\mathbb{R}}_\sigma$ denotes the set of compresseds of real numbers, represented by the open interval $(-\sigma, \sigma)$ of the abstract number line. We may interpret this open interval as compressed number line. The σ - compressed of the real number x will be denoted by $\underline{x}_\sigma$. Considering a point $P = (x, y, z) \in \mathbb{R}^3$ we define its $\sigma -$ compressed, as

(0.10) $$\underline{P}_\sigma = \left(\underline{x}_\sigma, \underline{y}_\sigma, \underline{z}_\sigma\right).$$

If $\mathbb{S}$ is a subset of $\mathbb{R}^3$ then its $\sigma -$ compressed is

(0.11) $$\underline{\mathbb{S}}_\sigma = \{\underline{P}_\sigma | P \in \mathbb{S}\}.$$

If $\mathbb{S}$ is a line, plane, circle, ball, we are speaking about a sub – line, sub – plane, sub – circle, sub – ball and so on. Moreover,

(0.12) $$\underline{\mathbb{R}^3}_\sigma = \{\underline{P}_\sigma | P \in \mathbb{R}^3\}$$

is the compressed three – dimensional space, represented by the open cube

(0.13) $$\underline{\mathbb{R}^3}_\sigma = \left\{ P = (x, y, z) | \begin{cases} -\sigma < x < \sigma \\ -\sigma < y < \sigma \\ -\sigma < z < \sigma \end{cases} \right\}.$$

The open cube $\underline{\mathbb{R}^3}_\sigma$ may be extremely small. For example if $t = 10^{-43}\ sec$ then (0.8) gives that $\sigma(t) \approx 3 \cdot 10^{-27} millimetre$, which is a measure for quantum mechanics. On the other hand computing with $t \approx t_{universe}$ the $\sigma(t) \approx 13,7 \cdot 10^9\ light - year$ is obtained. This is an extremely big open cube in the abstract three dimensional space $\mathbb{R}^3$. Athough the sizes of this open cube are finite, under today's circumstances its border is not perceptible, practically infinite. We are moving in the finite universe $\underline{\mathbb{R}^3}_\sigma$ but thinking in the infinite universe $\mathbb{R}^3$.

1.THE CUBE – UNIVERSE WITH SOME SUB – LINES

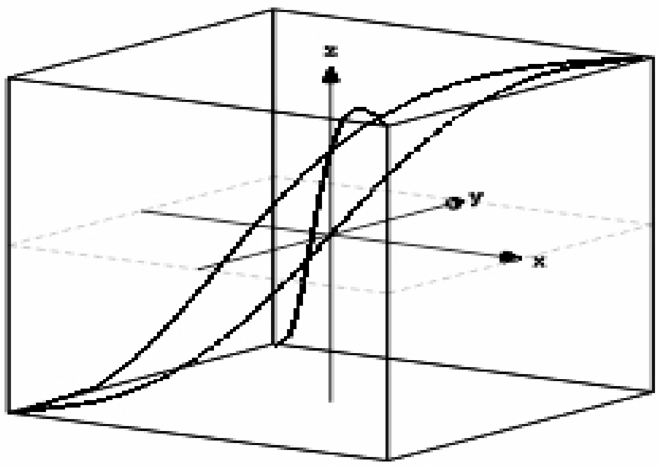

Figure- 1.1

As the real compression is unknown for us, we choose a comfortable compression. So, we give the $\sigma - compressed$ of real number x:

(1.2) $$\underline{x}_\sigma = \sigma \cdot \tanh \frac{x}{\sigma} \left(= \sigma \cdot \frac{e^{\frac{x}{\sigma}} - e^{-\frac{x}{\sigma}}}{e^{\frac{x}{\sigma}} + e^{-\frac{x}{\sigma}}} \right). -\infty < x < \infty, \quad \text{(See (0.1).)}$$

It is easy to check the validity of (0.6) and the first part of (0.5). For the $\sigma - exploded$ of compressed number $\xi \in \underline{\mathbb{R}}_\sigma$ we give

(1.3) $$\breve{\xi}^\sigma = \sigma \cdot \tanh^{-1} \frac{\xi}{\sigma} \left(= \frac{\sigma}{2} \cdot \ln \frac{1+\frac{\xi}{\sigma}}{1-\frac{\xi}{\sigma}} \right), -\sigma < \xi < \sigma. \quad \text{(See (0.2).)}$$

We may check the validity of (0.7) and the second part of (0.5) By the inverse connection given by (1.2) and (1.3) we can check the inversion formulas (0.3) and (0.4).

Introducing the sub-addition

(1.4) $$x \oplus_\sigma y = \underline{\breve{x}^\sigma + \breve{y}^\sigma}_\sigma \left(= \frac{x+y}{1+\frac{x \cdot y}{\sigma^2}} \right) \quad , \quad x, y \in \underline{\mathbb{R}}_\sigma,$$

and sub-multiplication

(1.5) $$x \odot_\sigma y = \underline{\breve{x}^\sigma \cdot \breve{y}^\sigma}_\sigma \left(= \sigma \cdot \tanh \left(\sigma \cdot \left(\tanh^{-1} \frac{x}{\sigma} \right) \cdot \left(\tanh^{-1} \frac{y}{\sigma} \right) \right) \right), x, y \in \underline{\mathbb{R}}_\sigma,$$

we give an algebraic structure for $\underline{\mathbb{R}}_\sigma$. By (1.4) and (1.5) we have an isomorphy between $(\mathbb{R}, <, +, \cdot)$ and $(\underline{\mathbb{R}}_\sigma, <, \oplus_\sigma, \odot_\sigma)$. So, the latter is an ordered field, too.

It is remarkable that the sub-addition is similar to the Lorentz – addition of speed. On the other hand there exist an essential difference: in the latter the speed of light c appears instead of the compression – parameter σ which has a distance dimension. The isomorphy between the universe $\mathbb{R}^3$ and the universe $\underline{\mathbb{R}^3}_\sigma$ given by the mappings

$$(x, y, z) \mapsto \left(\underline{x}_\sigma, \underline{y}_\sigma, \underline{z}_\sigma \right) \qquad , x, y, z \in \mathbb{R}$$

and

$$(\xi, \eta, \zeta) \mapsto \left(\breve{\xi}^\sigma, \breve{\eta}^\sigma, \breve{\zeta}^\sigma \right) \qquad , \xi, \eta, \zeta \in \underline{\mathbb{R}}_\sigma,$$

is rather general. For example, the sub-distance of points $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2) \in \underline{\mathbb{R}^3}_\sigma$ is defined by

(1.6) $$d_\sigma(P_1, P_2) = \underline{d\left(\breve{P_1}^\sigma, \breve{P_2}^\sigma \right)}_\sigma,$$

where for $P = (x, y, z) \in \underline{\mathbb{R}^3}_\sigma$ we say that

(1.7) $$\breve{P}^\sigma = (\breve{x}^\sigma, \breve{y}^\sigma, \breve{z}^\sigma) \in \mathbb{R}^3:$$

With respect to universe $\underline{\mathbb{R}^3}_\sigma$ the compression – parameter σ plays the role of ∞. For example the point $P_{border} = (0, 0, \sigma) \in \mathbb{R}^3$ being on the border of $\underline{\mathbb{R}^3}_\sigma$ is outside universe $\underline{\mathbb{R}^3}_\sigma$. So, $d_\sigma(O, P_{border})$ does not exist. But, considering the point $P = (x, 0, 0)$, $0 < x < \sigma$ we have that $P \in \underline{\mathbb{R}^3}_\sigma$. So, by (1.6), (1.7), (0.5), (1.3) and (0.3) we can write

$$d_\sigma(O, P) = \underline{d\left(O, \breve{P}^\sigma \right)}_\sigma = \underline{|\breve{x}^\sigma|}_\sigma = \underline{\left| \sigma \cdot \tanh^{-1} \frac{x}{\sigma} \right|}_\sigma = \underline{(\breve{x}^\sigma)}_\sigma = x.$$

Hence, $\lim_{\substack{x \to \sigma \\ x < \sigma}} d_\sigma(O, P) = \sigma \notin \underline{\mathbb{R}}_\sigma$. (The point $P_{border} = (0, 0, \sigma)$ is invisible in $\underline{\mathbb{R}^3}_\sigma$.)

Choosing the $\sigma = 1$,we give some examples for the geometry of universe $\underline{\mathbb{R}^3}_1$, demonstrated by Fig. 1.1. (A complete discussion is in [2].) Compressing the linea

$$\mathbb{L} = \left\{ P = (x^*, y^*, z^*) \in \mathbb{R}^3 \middle| \begin{cases} x^* = \frac{1}{\sqrt{6}} t \\ y^* = \frac{1}{\sqrt{6}} t \qquad , -\infty < t < \infty \\ z^* = \breve{\left(\frac{1}{2} \right)}^1 + \frac{2}{\sqrt{6}} t \end{cases} \right\}$$

and writing that $\underline{(x^*)_1} = x$, $\underline{(y^*)_1} = y$ and $\underline{(z^*)_1} = z$ by (0.10) and (1.2) we have the sub line

$$\underline{\mathbb{L}}_1 = \left\{ P = (x,y,z) \in \mathbb{R}^3 \middle| \begin{cases} x = \tanh\dfrac{t}{\sqrt{6}} \\ y = \tanh\dfrac{t}{\sqrt{6}} \\ z = \dfrac{1 + 2\tanh\dfrac{2t}{\sqrt{6}}}{2 + \tanh\dfrac{2t}{\sqrt{6}}} \end{cases}, -\infty < t < \infty \right\}$$

which for $t = 0$ has the point $(0,0,\frac{1}{2})$, moreover its border-points are $(-1,-1,-1)$ *and* $(1,1,1)$. (See Fig. 1.1)

Similarly, the sub – line

$$\underline{L^+}_1 = \left\{ P = (x,y,z) \in \mathbb{R}^3 \middle| \begin{cases} x = \tanh\dfrac{t}{\sqrt{6}} \\ y = -\tanh\dfrac{t}{\sqrt{6}} \\ z = \dfrac{1 + 2\tanh\dfrac{2t}{\sqrt{6}}}{2 + \tanh\dfrac{2t}{\sqrt{6}}} \end{cases}, -\infty < t < \infty \right\}$$

is obtained. It has the point $(0,0,\frac{1}{2})$, moreover its border-points are $(-1,1,-1)$ *and* $(1,-1,1)$. (See Fig. 1.1.) The identity

$$\frac{\left(\tanh\dfrac{1}{\sqrt{6}}\right)^2 + 4\tanh\dfrac{1}{\sqrt{6}} + 1}{2\left(\tanh\dfrac{1}{\sqrt{6}}\right)^2 + 2\tanh\dfrac{1}{\sqrt{6}} + 2} = \frac{1 + 2\tanh\dfrac{2t}{\sqrt{6}}}{2 + \tanh\dfrac{2t}{\sqrt{6}}}, -\infty < t < \infty,$$

proves, that $\underline{\mathbb{L}}_1 \cup \underline{L^+}_1$ is a subset of the sub – plane

$$\underline{\mathbb{S}}_1 = \left\{ P = (x,y,z) \in \mathbb{R}^3 \middle| \begin{cases} -1 < x < 1 \\ -1 < y < 1 \\ z = \frac{x^2+4x+1}{2x^2+2x+2} \end{cases} \right\}$$

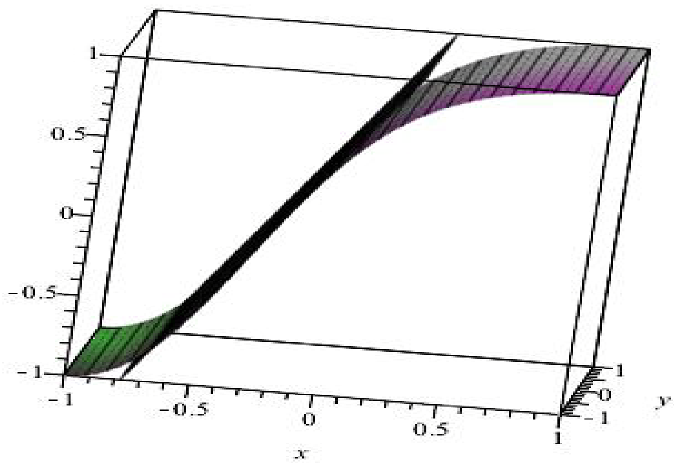

Figure-1.8

where $\mathbb{S} = \left\{ P = (x,y,z) \in \mathbb{R}^3 \middle| \begin{cases} -\infty < x < \infty \\ -\infty < y < \infty \\ z = 2x + \overset{\smile}{\left(\frac{1}{2}\right)}^1 \end{cases} \right\}.$

2.THE INFLATION OF THE CUBE UNIVERSE

We start from a primitive cube universe with extremely small parameter σ_0. For the sake of comvenience we assume that $t_{inflation}$ is not a final time. So, by (0.8) we can write

(2.1) $$\lim_{\substack{t\to 0\\ t>0}} \sigma(t) = 0 \quad and \quad \lim_{t\to\infty} \sigma(t) = \infty.$$

Using (1.2) and consdering the function

(2.2) $$\underline{x}_{\sigma(t)} = v_{supreme} \cdot t \cdot \tanh \frac{x}{v_{supreme} \cdot t} \quad , 0 < t < \infty, \; x \in \mathbb{R}$$

we can see that for any arbitray but fixed real number *x*

(2.3) $$\lim_{\substack{t\to 0\\ t>0}} \underline{x}_{\sigma(t)} = 0.$$

Moreover, the fuction $\underline{x}_{\sigma(t)}$ for positive *x* has the upper bound *x*, and for negative *x* has the lower bound *x* ,such that

(2.4) $$\lim_{t\to\infty} \underline{x}_{\sigma(t)} = x$$

holds.

Discussing the function $\underline{x}_{\sigma(t)}$ we compute the firt and second derivatives

(2.5) $$\frac{d\underline{x}_{\sigma(t)}}{dt} = v_{supreme} \left(\tanh \frac{x}{v_{supreme} \cdot t} - \frac{\frac{x}{v_{supreme} \cdot t}}{\left(\cosh \frac{x}{v_{supreme} \cdot t}\right)^2} \right), \quad 0 < t < \infty,$$

and

(2.6) $$\frac{d^2 \underline{x}_{\sigma(t)}}{dt^2} = -2 \cdot \frac{v_{supreme}}{t} \cdot \left(\frac{\frac{x}{v_{supreme} \cdot t}}{\cosh \frac{x}{v_{supreme} \cdot t}} \right)^2 \cdot \tanh \frac{x}{v_{supreme} \cdot t}, \; 0 < t < \infty.$$

As for any negative *x* we have that

(2.7) $$\underline{x}_{\sigma(t)} = -\underline{|x|}_{\sigma(t)}, \quad 0 < t < \infty,$$

in the following we may assume that $x > 0$.

By (2.5) we can see that

(2.8) $$\lim_{\substack{t\to 0\\ t>0}} \frac{d\underline{x}_{\sigma(t)}}{dt} = v_{supreme} \qquad and \qquad \lim_{t\to\infty} \frac{d\underline{x}_{\sigma(t)}}{dt} = 0.$$

Moreover by (2.6) we can easily prove

(2.9) $$\lim_{\substack{t\to 0\\ t>0}} \frac{d^2 \underline{x}_{\sigma(t)}}{dt^2} = 0 \qquad and \qquad \lim_{t\to\infty} \frac{d^2 \underline{x}_{\sigma(t)}}{dt^2} = 0.$$

By the left hand side of (2.1) we mention

Consequence 1: There is no smallest cube universe. The origin of cube universes is only a point of the abstract universe $\mathbb{R}^3$.

Intoducing the proposition

(2.10) $$u = \frac{x}{v_{supreme} \cdot t} \; with \; x > 0 \; and \; 0 < t < \infty,$$

and having that

$$\tanh u - \frac{u}{(\cosh u)^2} > 0 \Leftrightarrow \sinh 2u > 2u,$$

by (2.5) the inequality $\frac{d\underline{x}_{\sigma(t)}}{dt} > 0$ is obtained. So, we have that the fuction $\underline{x}_{\sigma(t)}$ with positive *x* (see (2.2)) is strictly monotonic increasing on the open interval $(0, \infty)$. As the fuction $\underline{x}_{\sigma(t)}$ is continuous, we get

Conseqence 2: If the time passes, the enlargement of cube universe continuously increases.
By the right hand side of (2.1) we mention

Consequence 3: There is no biggest cube universe. If the time $t_{universe}$ becomes increasingly long the cube universes tend to get closer to the abstract universe $\mathbb{R}^3$.

Clearly, $\frac{d^2 \underline{x}_{\sigma(t)}}{dt^2} < 0$ (see (2.6) with $x > 0 \; and \; 0 < t < \infty$). So, we have that the fuction $\underline{x}_{\sigma(t)}$ with positive *x* (see (2.2)) is concave on the open interval $(0, \infty)$. Moreover, with respect to (2.3), (2.4) and (2.7) we have the following graph

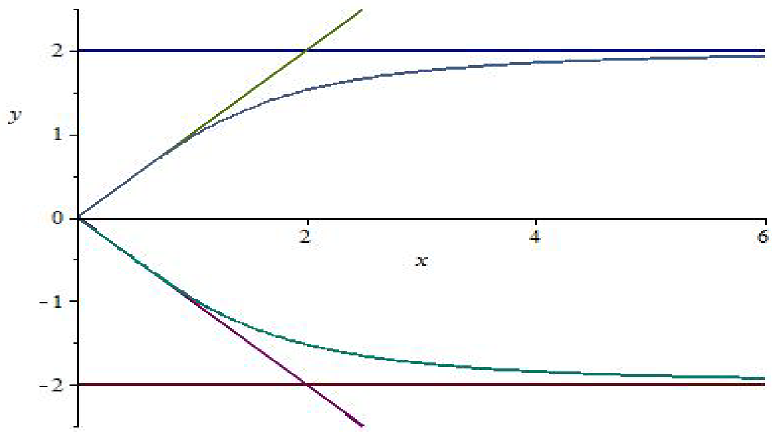

Figure-2.11

(In Figure 2.11 the $v_{supreme} = 1\frac{lighyear}{year}$ and $|x| = 2\ lightyear$)

Considering (2.8) and (2.9) we mention

Consequence 4: If $t_{universe}$ is near to 0 then the universe enlarges approximatey linearly and the speed of the enlargement is approximately constant, namely it is almost $v_{supreme}$(which is a little bit greater than the speed of light).

If $t_{universe}$ is close to $t_{inflation}$.(which in our case is going forward for ever) the enlargement has the bound *x* and the speed of enlargement almost 0, but it is always greater than 0.

This means that for constant *x* the inflation of cube universe is slowly increasing.

Remark 1: If the $t_{universe}$ is close to 0, Fig. 2.11 shows some similarity to the enlargement of Einstein – de Sitter universe. (See [3], p.95. Fig. 3.16.)

3. COMPUTATION IN THE CUBE UNIVERSE

We consider the cube universe with compression parameter $\sigma = \sigma(t)$ given by (0.8) and use the operations given by (1.4) and (1.5). Moreover, we use the sub-subtraction

(3.1) $$x \ominus_\sigma y = \frac{x-y}{1-\frac{x\cdot y}{\sigma^2}}, x, y \in \underline{\mathbb{R}}_\sigma$$

and sub-division

(3.2) $$x \oslash_\sigma y = \sigma \cdot \tanh\frac{\tanh^{-1}\frac{x}{\sigma}}{\sigma\cdot\tanh^{-1}\frac{y}{\sigma}} \quad , x, y(\neq 0) \in \underline{\mathbb{R}}_\sigma.$$

Let f be a given familiar double variable function with its domain $\mathbb{D}_f \subseteq \mathbb{R}^2$ and $\mathbb{R}_f = \{z \in \mathbb{R} | z = f(x,y)\ , (x,y) \in \mathbb{D}_f\}$. Having a compression – parameter σ we say that a point $(x,y) \in \underline{\mathbb{R}^2}_\sigma$ belongs to the domain $\mathbb{D}_{\underline{f_\sigma}} \left(\subseteq \underline{\mathbb{R}^2}_\sigma\right)$ of the double variable sub – function $\underline{f_\sigma}$ if $(\breve{x}^\sigma, \breve{y}^\sigma) \in \mathbb{D}_f$. Moreover, $\mathbb{R}_{\underline{f_\sigma}} = \left\{z \in \mathbb{R} | z = \underline{f(\breve{x}^\sigma, \breve{y}^\sigma)}_\sigma,\ (x,y) \in \mathbb{D}_{\underline{f_\sigma}}\right\}$. Shortly,

(3.3) $$\underline{f_\sigma}(x,y) = \underline{f(\breve{x}^\sigma, \breve{y}^\sigma)}_\sigma,\ (x,y) \in \mathbb{D}_{\underline{f_\sigma}}.$$

Theorem 1: If the point $(x,y) \in \left(\mathbb{D}_f \cap \mathbb{D}_{\underline{f_\sigma}}\right)$ and the double variable function f is continuous at the point (x,y) then

(3.5) $$\lim_{\sigma\to\infty} \underline{f_\sigma}(x,y) = f(x,y).$$

Proof: It is already known that

(3.6) $$\lim_{\sigma\to\infty} \breve{x}^\sigma = x,\quad x \in \mathbb{R}$$

is valid. (See, e.g. [4], Theorem 20.). Using (1.2), and (1.3) by (3.3) we have

$$\underline{f_\sigma}(x,y) = \sigma \tanh\frac{f(\breve{x}^\sigma, \breve{y}^\sigma)}{\sigma} (= \sigma \tanh\frac{f\left(\frac{\sigma}{2}\cdot\ln\frac{1+\frac{x}{\sigma}}{1-\frac{x}{\sigma}}, \frac{\sigma}{2}\cdot\ln\frac{1+\frac{y}{\sigma}}{1-\frac{y}{\sigma}}\right)}{\sigma}), -\sigma < x, y < \sigma.$$

By (3.6) we see, that $\lim_{\sigma\to\infty}(\breve{x}^\sigma,\breve{y}^\sigma)=(x,y)$. Moreover by the continuity of f, $\lim_{\sigma\to\infty} f(\breve{x}^\sigma,\breve{y}^\sigma)=f(x,y)\in\mathbb{R}$ is obtained. Hence $\lim_{\sigma\mapsto\infty}\frac{f(\breve{x}^\sigma,\breve{y}^\sigma)}{\sigma}=0$. Finally applying that $\lim_{u\to 0}\frac{\tanh u}{u}=1$, by

$$\sigma\tanh\frac{f(\breve{x}^\sigma,\breve{y}^\sigma)}{\sigma}=f(\breve{x}^\sigma,\breve{y}^\sigma)\cdot\frac{\tanh\frac{f(\breve{x}^\sigma,\breve{y}^\sigma)}{\sigma}}{\frac{f(\breve{x}^\sigma,\breve{y}^\sigma)}{\sigma}}$$

gives the statement (3.5). (If $f(\breve{x}^\sigma,\breve{y}^\sigma)=0$ then the statement is obtained by the first step.)

Corollary 1: Considering the double variable functions
$f_{addition}(x,y)=x+y$, $f_{multiplication}(x,y)=xy$, $f_{subraction}(x,y)=x-y$ *and* $f_{division}(x,y)=\frac{x}{y},(y\neq 0)$ and assuming that $(x.y)\in\underline{\mathbb{R}^2}_\sigma$ the statement (3.5) yields

(3.7) $\lim_{\sigma\to\infty}(x\oplus_\sigma y)=x+y$, see (1.4),

(3.8) $\lim_{\sigma\to\infty}(x\odot_\sigma y)=x\cdot y$, see (1.5),

(3.9) $\lim_{\sigma\to\infty}(x\ominus_\sigma y)=\lim_{\sigma\to\infty}\frac{x-y}{1-\frac{x\cdot y}{\sigma^2}}=x-y$

and

(3.10) $\lim_{\sigma\to\infty}(x\oslash_\sigma y)=\lim_{\sigma\to\infty}\sigma\cdot\tanh\frac{\tanh^{-1}\frac{x}{\sigma}}{\sigma\cdot\tanh^{-1}\frac{y}{\sigma}}=\frac{x}{y},y\neq 0$

hold, respectively.

In the following we investigate the safety of computation in the cube universe $\underline{\mathbb{R}^3}_\sigma$.

Theorem 2: If σ is a given compression parameter and *x* a given real number such that

$$-\frac{\pi}{2}\sigma<x<\sigma$$

then the approximation

(3.11) $\left|x-\underline{x}_\sigma\right|<\frac{|x|^3}{3\sigma^2}$

holds.

Proof: Using (1.2) it is easy to see that

(3.12) $\left|x-\underline{x}_\sigma\right|=\sigma\left(\frac{|x|}{\sigma}-\tanh\frac{|x|}{\sigma}\right),\quad x\in\mathbb{R}.$

We use the series

$$\tanh u=\sum_{n=1}^{\infty}\frac{2^{2n}(2^{2n}-1)B_{2n}}{(2n)!}u^{2n-1},\qquad |u|<\frac{\pi}{2},$$

where B_m is the *m*-th Bernoulli – number ($B_0=1,B_1=-\frac{1}{2},B_2=\frac{1}{6},B_4=-\frac{1}{30}$ ...). Denoting

(3.13) $u=\frac{|x|}{\sigma}$

by (3.12) we can write

(3.14)
$$\begin{aligned}\left|x-\underline{x}_\sigma\right|&=\sigma\left(\frac{|x|}{\sigma}-\sum_{n=1}^{\infty}\frac{2^{2n}(2^{2n}-1)B_{2n}}{\sigma^{2n-1}(2n)!}|x|^{2n-1}\right)=\\&=-\sigma\sum_{n=2}^{\infty}\frac{2^{2n}(2^{2n}-1)B_{2n}}{\sigma^{2n-1}(2n)!}|x|^{2n-1}=\\&=\frac{|x|^3}{3\sigma^2}-\sigma\sum_{n=3}^{\infty}\frac{2^{2n}(2^{2n}-1)B_{2n}}{\sigma^{2n-1}(2n)!}|x|^{2n-1}=\\&=\frac{|x|^3}{3\sigma^2}-\frac{|x|^5}{\sigma^4}\sum_{k=0}^{\infty}\frac{2^{2k+6}(2^{2k+6}-1)B_{2k+6}}{\sigma^{2k}(2k+6)!}x^{2k}.\end{aligned}$$

We may express the Bernoulli- numbers by the Riemann „ζ” function:

(3.15) $\zeta(s)=\sum_{k=1}^{\infty}\frac{1}{k^s},\quad 1<s<\infty,$ (see [5]). It is known that

(3.16) $B_{2n}=2(-1)^{n+1}\frac{\zeta(2n)\cdot(2n)!}{(2\pi)^{2n}},n=1,2,3,$ (see [6])

is valid. Moreover, $B_{2n+1}=0$, $n=1,2,3$ ....

We investigate the sign of

$$S=\sum_{k=0}^{\infty}\frac{2^{2k+6}(2^{2k+6}-1)B_{2k+6}}{\sigma^{2k}(2k+6)!}x^{2k}.$$

Applying (3.15) and (3.16) we can write

$$S=\sum_{p=0}^{\infty}\left(\left(\frac{(2^{4p+6}-1)\cdot 2\left(\sum_{q=1}^{\infty}\frac{1}{q^{4p+6}}\right)}{\pi^{4p+6}}\right)\left(\frac{x}{\sigma}\right)^{4p}-\left(\frac{(2^{4p+8}-1)\cdot 2\left(\sum_{q=1}^{\infty}\frac{1}{q^{4p+8}}\right)}{\pi^{4p+8}}\right)\left(\frac{x}{\sigma}\right)^{4p+2}\right)=$$

$$=\sum_{p=0}^{\infty}\left(\frac{2}{\pi^{4p+6}}\right)\left(\frac{x}{\sigma}\right)^{4p}\left((2^{4p+6}-1)\left(\sum_{q=1}^{\infty}\frac{1}{q^{4p+6}}\right)-\frac{1}{\pi^2}\left(\frac{x}{\sigma}\right)^2(2^{4p+8}-1)\left(\sum_{q=1}^{\infty}\frac{1}{q^{4p+8}}\right)\right)>$$

$$>\sum_{p=0}^{\infty}\left(\frac{2}{\pi^{4p+6}}\right)\left(\frac{x}{\sigma}\right)^{4p}\left((2^{4p+6}-1)\left(\sum_{q=1}^{\infty}\frac{1}{q^{4p+6}}\right)-(2^{4p+6}-1)\left(\sum_{q=1}^{\infty}\frac{1}{q^{4p+8}}\right)\right)>0$$

Hence,

$$\frac{|x|^5}{\sigma^4}\sum_{k=0}^{\infty}\frac{2^{2k+6}(2^{2k+6}-1)B_{2k+6}}{\sigma^{2k}(2k+6)!}x^{2k}>0,$$

so, by (3.14) the estimation (3.11) holds.

Corollary 2: Let ε be a (small) positive number. If

$$-\sqrt[3]{3\varepsilon\sigma^2}<x<\sqrt[3]{3\varepsilon\sigma^2}$$

then the estimation $\left|x-\underline{x}_\sigma\right|<\varepsilon$ holds.

Theorem 3: If σ is a given compression parameter, x and y are given real numbers such that

$$-\sigma<x+y<\sigma$$

then the approximation

(3.15) $$\left|(x+y)-\left(\underline{x}_\sigma\oplus_\sigma\underline{y}_\sigma\right)\right|<\frac{|x+y|^3}{3\sigma^2}$$

holds.

Proof: Using (1.4) and (1.2) we can write

$$(x+y)-\left(\underline{x}_\sigma\oplus_\sigma\underline{y}_\sigma\right)=(x+y)-\frac{\underline{x}_\sigma+\underline{y}_\sigma}{1+\frac{\underline{x}_\sigma\cdot\underline{y}_\sigma}{\sigma^2}}=$$

$$=(x+y)-\sigma\frac{\tanh\frac{x}{\sigma}+\tanh\frac{y}{\sigma}}{1+\tanh\frac{x}{\sigma}\cdot\tanh\frac{y}{\sigma}}=(x+y)-\sigma\cdot\tanh\frac{x+y}{\sigma}.$$

Hence,

$$\left|(x+y)-\left(\underline{x}_\sigma\oplus_\sigma\underline{y}_\sigma\right)\right|=\sigma\left(\frac{|x+y|}{\sigma}-\tanh\frac{|x+y|}{\sigma}\right).$$

The continuation of the proof is carried out in the same way as in the proof of Theorem 2. More exactly, considering (3.12) , (3.13) , (3.14) and so on, in place of x we write $(x+y)$.

Finally, the approximation (3.15) is obtained.

Theorem 3: If σ is a given compression parameter, x and y are given real numbers such that

$$-\sigma<x\cdot y<\sigma$$

then the approximation

(3.16) $$\left|(x\cdot y)-\left(\underline{x}_\sigma\odot_\sigma\underline{y}_\sigma\right)\right|<\frac{|x\cdot y|^3}{3\sigma^2}$$

holds.

Proof: Using (1.5) with (0.4) and (1.2) we can write

$$(x\cdot y)-\left(\underline{x}_\sigma\odot_\sigma\underline{y}_\sigma\right)=(x\cdot y)-\underline{x\cdot y}_\sigma=$$

$$=(x\cdot y)-\sigma\tanh\frac{x\cdot y}{\sigma}=\sigma\left(\frac{x\cdot y}{\sigma}-\tanh\frac{x\cdot y}{\sigma}\right).$$

Hence,

$$\left|(x\cdot y)-\left(\underline{x}_\sigma\odot_\sigma\underline{y}_\sigma\right)\right|=\sigma\left(\frac{|x\cdot y|}{\sigma}-\tanh\frac{|x\cdot y|}{\sigma}\right).$$

The continuation of the proof is carried out in the same way as in the proof of Theorem 2.

More exactly, considering (3.12) , (3.13) , (3.14) and so on, in place of *x* we write $(x \cdot y)$.

Finally, the approximation (3.16) is obtained. (Istennek Hála! 2016.12.11.(15.33) Szalay István).

We remark that similar results are valid for the $\underline{x}_\sigma \ominus_\sigma \underline{y}_\sigma$ (see 3.1) and $\underline{x}_\sigma \oslash_\sigma \underline{y}_\sigma$ (see (3.2), too.

Consequence 5: The equation (0.8) with Theorems 2, 3 and 4 yields that if the $t_{universe}$ is big enough then being in the cube universe we can compute the familiar addition, multiplication, subtraction and division, too. For example, if the compression parameter $\sigma = 13{,}7 \cdot 10^9$,then the calculator $f_x - 570ES$ says that $\underline{1}_{13,7 \cdot 10^9} = 13{,}7 \cdot 10^9 \cdot \tanh \frac{1}{13{,}7 \cdot 10^9} = 1$. (Of course, we know that $\underline{1}_{13,7 \cdot 10^9} < 1$.)

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