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ON THE SUM CONNECTIVITY GOURAVA INDEX

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ABSTRACT

We introduce the sum connectivity Gourava index of a molecular graph. In this paper, we determine the sum connectivity Gourava index of some standard classes of graphs. We also compute the sum conenctivity Gourava index of linear [n]-Tetracene, V-Tetracenic nanotube, H-Tetracenic nanotube and Tetracenic nanotori.

Mathematics Subject Classification: *05C05, 05C12, 05C35.*

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1. INTRODUCTION

Let G be a finite, simple and connected graph with a vertex set $V(G)$ and an edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. We refer to [1] for undefined term and notation.

A molecular graph is a simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edges to the bonds between atoms . A topological index is a numerical parameter mathematically derived from the graph structure. These indices are useful for establishing correlation between the structure of a molecular compound and its physico-chemical properties, see [2].

The first and second Zagreb indices [2] of a molecular graph G are defined as

$$M_1(G) = \sum_{uv\in E(G)} \left[d_G(u) + d_G(v)\right], \qquad M_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v).$$

Motivated by the definition of the Zagreb indices and their wide applications, Kulli introduced the first Gourava index of a molecular graph in [3] as follows:

The first Gourava index of a graph G is defined as

$$GO_1(G) = \sum_{uv\in E(G)} \left[\left(d_G(u) + d_G(v)\right) + \left(d_G(u)d_G(v)\right)\right].$$

The second Gourava index [3] of a molecular graph G is defined as

$$GO_2(G) = \sum_{uv\in E(G)} \left(d_G(u) + d_G(v)\right)\left(d_G(u)d_G(v)\right).$$

In [4], Kulli introduced the product connectivity Gourava index of a graph G and it is defined as

$$PGO(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\left(d_G(u) + d_G(v)\right)\left(d_G(u)d_G(v)\right)}}.$$

In [5], Zhou and Trinajstić introduced the sum connectivity index of a graph G and it is defined as

$$S(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u) + d_G(v)}}$$

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Motivated by the definition of the sum connectivity index, we define the sum connectivity Gourava index of a graph *G* and it is defined as

$$SGO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\left(d_G(u) + d_G(v)\right) + \left(d_G(u) d_G(v)\right)}}.$$

Recently, many other topological indices were studied, for example, in [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17].

In this paper, the sum connectivity Gourava index of certain nanostructures are determined. Also the sum connectivity Gourava index of some standard classes of graphs are determined.

2. RESULTS FOR SOME STANDARD CLASSES OF GRAPHS

Proposition 1: If C_n is a cycle with n∈3 vertices, then $SGO(C_n) = \frac{n}{2\sqrt{2}}$.

Proof: Let C_n be a cycle with *n* vertices and *n* edges. Then

$$SGO(C_n) = n \frac{1}{\sqrt{(2+2)+(2\times 2)}} \; \frac{n}{2\sqrt{2}}$$

Proposition 2: If K_n is a complete graph with n∈2 edges, then $SGO(C_n) = \frac{n\sqrt{(n-1)}}{2\sqrt{(n+1)}}$

Proof: Let K_n be a complete graph. Then $E(K_n) = \frac{n(n-1)}{2}$.

$$SGO(K_n) = \frac{n(n-1)}{2} \frac{1}{\sqrt{\left[(n-1)+(n-1)\right]+(n-1)(n-1)}} = \frac{n\sqrt{(n-1)}}{2\sqrt{(n+1)}}.$$

Proposition 3: If $K_{m,n}$ is a complete bipartite graph with $1 \le m \le n$, then $SGO(K_{m,n}) = \frac{mn}{\sqrt{mn+m+n}}$.

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m + n$ vertices and mn edges such that $|V_1| = m$, $|V_2| = n$, $V(K_{m,n}) = V_1 \cup V_2$. Every vertex of V_1 is adjacent with *n* vertices and evey vertex of V_2 is adjacent with *m* vertices.

$$SGO(K_{m,n}) = \frac{mn}{\sqrt{(m+n)+mn}} = \frac{mn}{\sqrt{mn+m+n}}.$$

Corollary 3.1: Let $K_{n,n}$ be a complete bipartite graph. Then $SGO(K_{n,n}) = \frac{n\sqrt{n}}{\sqrt{2+n}}$.

Corollary 3.2: Let $K_{1,n}$ be a Star. Then $SGO(K_{1,n}) = \frac{n}{\sqrt{1+2n}}$.

Proposition 4: If *G* is an *r*-regular graph with *n* vertices, then $SGO(G) = \frac{n\sqrt{r}}{2\sqrt{2+r}}$.

Proof: Let *G* be an r-regular graph with *n* vertices and $\frac{nr}{2}$ edges. Then the degree of each vertex of *G* is *r*.

$$SGO(G) = \frac{nr}{2} \frac{1}{\sqrt{(r+r)+r^2}} = \frac{n\sqrt{r}}{2\sqrt{2+r}}.$$

Proposition 5: Let P_n be a path with $n \geq 3$ vertices. Then $SGO(P_n) = \frac{n}{2\sqrt{2}} + \frac{2}{\sqrt{5}} - \frac{3}{2\sqrt{2}}$.

Proof: Let $G=P_n$ be a path with $n \geq 3$ vertices. We obtain two partitions of the edge set of P_n as follows:

$E_3 = \{uv \in E(G) \mid d_G(u)=1, d_G(v)=2\}, \mid E_3\mid = 2.$

$E_4 = \{uv \in E(G) \mid d_G(u) = d_G(v)=2\}, \mid E_4\mid = n - 3.$

To compute $SGO(P_n)$, we see that

$$SGO(P_n) = \frac{1}{\sqrt{(1+2)+(1\times 2)}}2 + \frac{1}{\sqrt{(2+2)+(2\times 2)}}(n-3) = \frac{n}{2\sqrt{2}} + \frac{2}{\sqrt{5}} - \frac{3}{2\sqrt{2}}.$$

3. SUM CONNECTIVITY GOURAVA INDEX OF SOME NANOSTRUCTURES

3.1 Linear [*n*] – Tetracene.

The molecular graph of a linear [n]-Tetracene is shown in Figure -1.

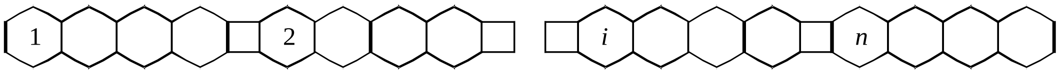

Figure-1: The molecular graph of a linear [n]-Tetracene

We compute the sum connectivity Gourava index of a linear [n]-Tetracene.

Theorem 1: Let T be a linear [n]-Tetracene. Then

$$SGO(T) = \left(\frac{16}{\sqrt{11}} + \frac{7}{\sqrt{15}}\right)n + \left(\frac{3}{\sqrt{2}} - \frac{4}{\sqrt{11}} - \frac{4}{\sqrt{15}}\right).$$

Proof: From Figure 1, by algebraic method, we obtain $|V(T)| = 18n$ and $|E(T)| = 23n - 2$. Also we obtain three partitions of the edge set of T as follows:

$E_{22} = \{uv \in E(T) \mid d_T(u) = d_T(v)=2\}, \mid E_{22}\mid = 6.$

$E_{23} = \{uv \in E(T) \mid d_T(u) = 2, d_T(v)=3\}, \mid E_{23}\mid = 16n - 4.$

$E_{33} = \{uv \in E(T) \mid d_T(u) = d_T(v)=3\}, \mid E_{33}\mid = 7n - 4.$

To compute $SGO(T)$, we see that

$$SGO(T) = \sum_{uv\in E(T)} \frac{1}{\sqrt{\left(d_T(u)+d_T(v)\right)+\left(d_T(u)d_T(v)\right)}}.$$

$$= 6\frac{1}{\sqrt{(2+2)+(2\times 2)}} + (16n-4)\frac{1}{\sqrt{(2+3)+(2\times 3)}} + (7n-4)\frac{1}{\sqrt{(3+3)+(3\times 3)}}$$

$$= \left(\frac{16}{\sqrt{11}} + \frac{7}{\sqrt{15}}\right)n + \left(\frac{3}{\sqrt{2}} - \frac{4}{\sqrt{11}} - \frac{4}{\sqrt{15}}\right).$$

3.2. Nanostructure *F* = *F*[*p,q*]

The molecular graph of 2-D lattice of $F = F[p,q]$ with $p = 2$ and $q = 4$ is shown in Figure 2.

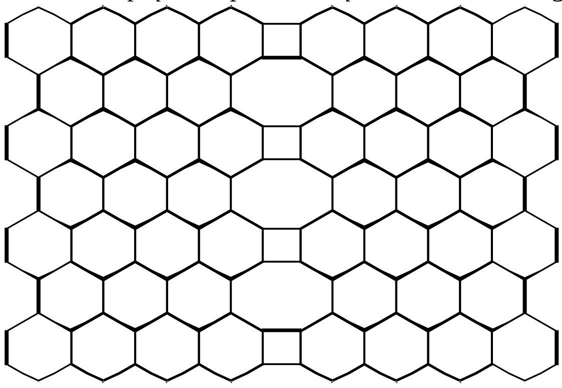

Figure-2: The graph of 2-D lattice of $F = F[p, q]$ with $p = 2$ and $q = 4$

In the following theorem, we compute the sum connectivity Gourava index of a nanostructure $F = F[p, q]$.

Theorem 2: Let $F = F[p, q]$ be a nanostructure. Then

$$SGO\left(F\left[p,q\right]\right)=\frac{27}{\sqrt{15}}pq+\left(\frac{16}{\sqrt{11}}-\frac{20}{\sqrt{15}}\right)p+\left(\frac{1}{\sqrt{2}}+\frac{4}{\sqrt{11}}-\frac{8}{\sqrt{15}}\right)q+\left(\frac{2}{\sqrt{2}}-\frac{8}{\sqrt{11}}-\frac{4}{\sqrt{15}}\right)$$

Proof: Let $F = F[p, q]$ be a nanostructure. By algebraic method, we obtain three partitions of the edge set of F as follows:

$E_{22} = \{uv \in E(F) \mid d_F(u)=d_F(v)=2\}$, $|E_{22}| = 2q+4$.
$E_{23} = \{uv \in E(F) \mid d_F(u) = 2, d_F(v)=3\}$, $|E_{23}| = 16p + 4q - 8$.
$E_{33} = \{uv \in E(F) \mid d_F(u) = d_F(v)=3\}$, $|E_{33}| = 27pq - 20p - 8q + 4$.

To compute $SGO(F[p,q])$, we see that

$$SGO\left(F\left[p,q\right]\right)=\sum_{uv\in E(F)}\frac{1}{\sqrt{\left(d_F(u)+d_F(v)\right)+\left(d_F(u)d_F(v)\right)}}.$$

$$=\left(2q+4\right)\frac{1}{\sqrt{\left(2+2\right)+\left(2\times 2\right)}}+\left(16p+4q-8\right)\frac{1}{\sqrt{\left(2+3\right)+\left(2\times 3\right)}}$$

$$+\left(27pq-20p-8q+4\right)\frac{1}{\sqrt{\left(3+3\right)+\left(3\times 3\right)}}$$

$$=\frac{27}{\sqrt{15}}pq+\left(\frac{16}{\sqrt{11}}-\frac{20}{\sqrt{15}}\right)p+\left(\frac{1}{\sqrt{2}}+\frac{4}{\sqrt{11}}-\frac{8}{\sqrt{15}}\right)q+\left(\frac{2}{\sqrt{2}}-\frac{8}{\sqrt{11}}+\frac{4}{\sqrt{15}}\right).$$

3.3. Nanostructure $G = G[p, q]$.

The molecular graph of 2-*D* lattice of $G=G[p, q]$ with $p = 2$ and $q = 4$ is shown in Figure 3.

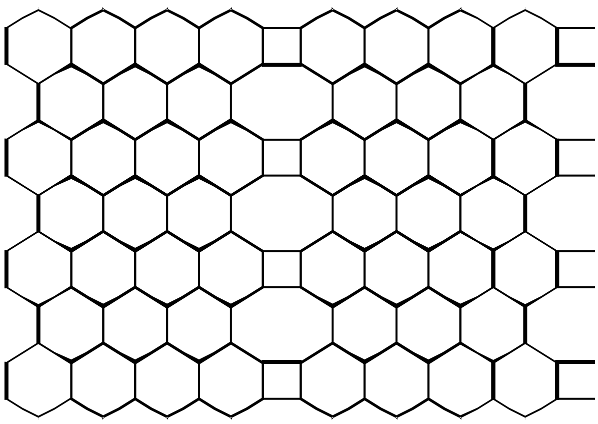

Figure-3: The graph of 2-*D* lattice of $G = G[p,q]$ with $p = 2$ and $q = 4$

In the following theorem, we compute the sum connectivity Gourava index of a nanostructure $G = G[p,q]$.

Theorem 3: Let $G = G[p, q]$ be a nanostructure. Then

$$SGO\left(G\left[p,q\right]\right)=\frac{27}{\sqrt{15}}pq+\left(\frac{16}{\sqrt{11}}-\frac{20}{\sqrt{15}}\right)p.$$

Proof: Let $G = G[p, q]$ be a nanostructure. By algebraic method, we obtain $|V(G)| = 18pq$ and $|E(G)| = 27pq - 4p$. Further, we obtain two partitions of the edge set of G as follows:

$E_{23} = \{uv \in E(G) \mid d_G(u)=2, d_G(v)=3\}$, $|E_{23}| = 16p$.
$E_{33} = \{uv \in E(G) \mid d_G(u) = d_G(v)=3\}$, $|E_{33}| = 27pq - 20p$.

To compute $SGO(G[p,q])$, we see that

$$SGO\left(G\left[p,q\right]\right)=\sum\frac{1}{\sqrt{\left(d_G(u)+d_G(v)\right)+\left(d_G(u)d_G(v)\right)}}.$$

$$= 16p\,\frac{1}{\sqrt{(2+3)+(2\times 3)}} + \left(27pq - 20p\right)\frac{1}{\sqrt{(3+3)+(3\times 3)}}$$
$$= \frac{27}{\sqrt{15}}\,pq + \left(\frac{16}{\sqrt{11}} - \frac{20}{\sqrt{15}}\right)p.$$

3.4. Nanostructure *K* = *K* [*p, q*]

The molecular graph of 2-*D* lattice of $K = K\,[p, q]$ with $p = 2$ and $q = 3$ is shown in Figure 4.

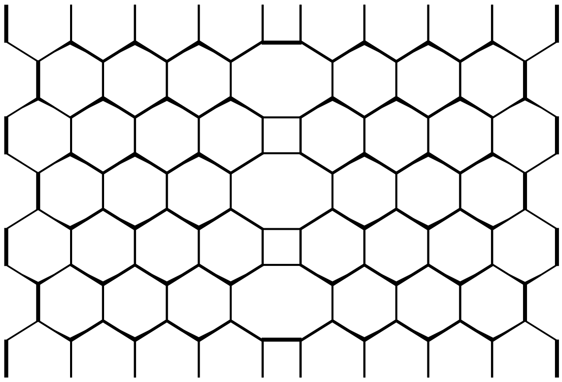

Figure-4: The graph of 2-D lattice of $K = K[p, q]$ with $p = 2$ and $q = 3$

In the following theorem, we compute the the sum connectivity Gourava index of a nanostructure $K = K[p, q]$.

Theorem 4: Let $K = K\,[p, q]$ be a nanostructure. Then
$$SGO\left(K\left[p,q\right]\right) = \frac{27}{\sqrt{15}}\,pq + \left(\frac{1}{\sqrt{2}} + \frac{4}{\sqrt{11}} - \frac{8}{\sqrt{15}}\right)q.$$

Proof: Let $K = K[p, q]$ be a nanostructure. By algebraic method, we obtain $|V(K)| = 18pq$ and $|E(K)| = 27pq - 2q$.

Further, we obtain three partitions of the edge set of K as follows:

$E_{22} = \{uv \in E(K) \mid d_K(u) = d_K(v) = 2\}$, $|E_{22}| = 2q$.
$E_{23} = \{uv \in E(K) \mid d_K(u) = 2, d_K(v) = 3\}$, $|E_{23}| = 4q$.
$E_{33} = \{uv \in E(K) \mid d_K(u) = d_K(v) = 3\}$, $|E_{33}| = 27pq - 8q$.

To compute $SGO(K[p, q])$, we see that
$$SGO\left(K\left[p,q\right]\right) = \sum_{uv\in E(K)} \frac{1}{\sqrt{\left(d_K(u) + d_K(v)\right) + \left(d_K(u)\,d_K(v)\right)}}.$$
$$= 2q\frac{1}{\sqrt{(2+2)+(2\times 2)}} + 4q\frac{1}{\sqrt{(2+3)+(2\times 3)}} + \left(27pq - 8q\right)\frac{1}{\sqrt{(3+3)+(3\times 3)}}$$
$$= \frac{27}{\sqrt{15}}\,pq + \left(\frac{1}{\sqrt{2}} + \frac{4}{\sqrt{11}} - \frac{8}{\sqrt{15}}\right)q.$$

3.5. Nanostructure *L* = *L* [*p, q*]

The molecular graph of 2-D lattice of $L = L[p, q]$ with $p = 2$ and $q = 4$ is shown in Figure 5.

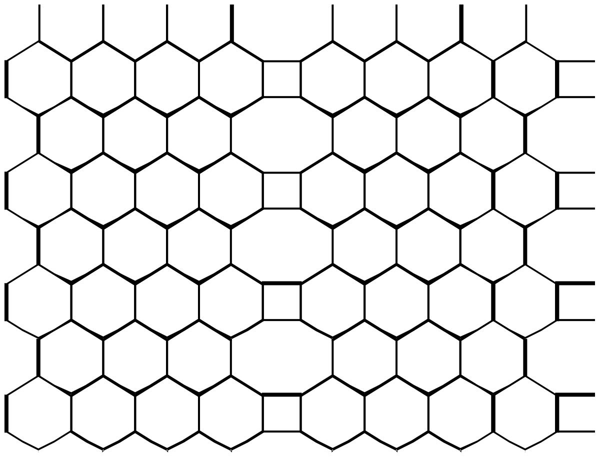

Figure-5: The graph of 2-D lattice of $L = L[p, q]$ with $p = 2$ and $q = 4$.

In the next theorem, we compute the sum connectivity Gourava index of a nanostructure $L = L[p, q]$.

Theorem 5: Let $L = L\,[p, q]$ be a nanostructure. Then

$$SGO\left(L\left[p,q\right]\right)=\frac{27}{\sqrt{15}}pq$$

Proof: Let $L = L[p, q]$ be a nanostructure. By algebraic method, we obtain $|V(L)| = 18pq$ and $|E(L)| = 27pq$. Since the degree of each vertex of L is 3, the edge partition of L is as follows:

$E_{33} = \{uv \in E(L) \mid d_L(u) = d_L(v) = 3\}$, $|E_{33}| = 27pq$.

To compute $SGO(L[p,q])$, we see that

$$SGO\left(L\left[p,q\right]\right)=\sum\frac{1}{\sqrt{\left(d_L\left(u\right)+d_L\left(v\right)\right)+\left(d_L\left(u\right)d_L\left(v\right)\right)}}.$$

$$=27pq\frac{1}{\sqrt{\left(3+3\right)+\left(3\times3\right)}}=\frac{27}{\sqrt{15}}pq.$$

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