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L-MATRIX, LABELED GRAPHS AND FACE BOOK

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ABSTRACT

In this paper we apply L-matrix for labeled graphs. L-Matrices have been defined (reference:4)for labeled vertices. In this paper we extend these to multi labeled vertices and edges consequently getting some new results.

Keywords: *Vertex/Edge labeled Graphs, Colored Graphs, Matrices, (2, d) Sigraph.*

1. INTRODUCTION

In paper I (reference: 5) L-Matrix has been defined for a labeled vertex graph and some theorems have been proved. In our paper we extend the definition of L-Matrix to signed graphs and edge labeled graphs and obtain corresponding results.

2. MATHEMATICAL PRELIMINARY

We recall some standard definitions and introduce some new concepts.

Graph [2]:
A pair consisting of a non empty set V and two element subset E of V x V is said to be a graph. Elements of V and E are respectively called vertices and edges.

Labeled Vertex and Labeled Edge [2]:
In the mathematical discipline of graph theory, a graph labeling is the assignment of labels, traditionally represented by integers, to the edges or vertices, or both, of a graph.

Formally, given a graph G = (V, E), a vertex labeling is a function on V to a set of labels. A graph with such a function defined is called a vertex-labeled graph. A Labeled Edge and Edge Labeled Graph are defined similarly.

Labeled Graph [2]:
A vertex/edge labeled graph is called a Labeled Graph. Special cases of vertex labeled and edge labeled graphs are called Colored Graphs, Weighted Graphs etc.

Example: Consider the graphs below whose vertices and edges are labeled.

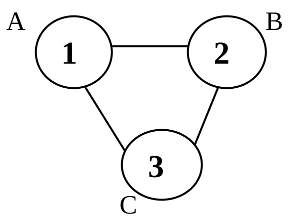

Figure-1: vertex labeled graph

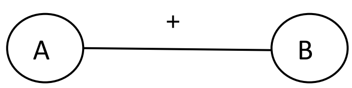

Figure-2: edge labeled graph

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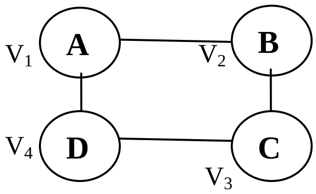


Figure-3: vertex labeled graph

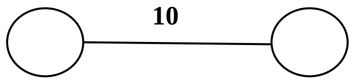


Figure-4: edge labeled graph

Vertex/Edge Colored Graph [1] [3]: A special vertex labeled graph in which no two adjacent vertices have the same label is called a Vertex Colored Graph. Edge colored graphs is defined similarly.

Colored Graphs [1] [3]: A special labeled graph in which no two adjacent vertices/edges have the same color (Label) is called a colored graph.

This definition of colored graph is not suitable for face book as will be seen later.

Example: In the example below no two adjacent vertices have the same color

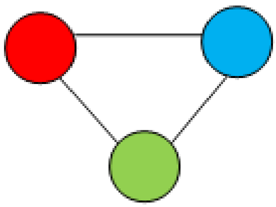


Figure-5: colored graph

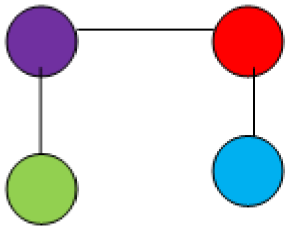


Figure-6: colored graph

In this definition every vertex Labeled graph is not a Colored graph.

Example: Consider a vertex labeled graph.

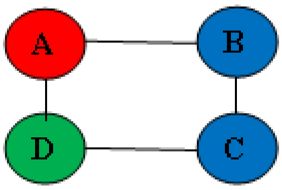


Figure-7: vertex labeled graph

Fig (7) is not a Colored graph as two adjacent vertices have the same color.

Example:

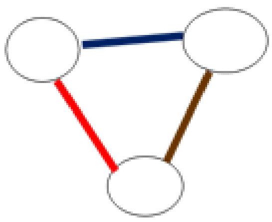


Figure-8: Here the edges are labeled with colors.

Example:

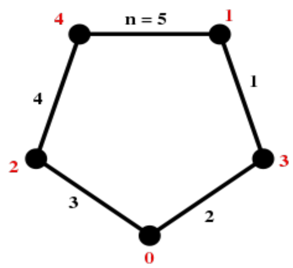


Figure-9: Here the edges are labeled with numbers.

Example:

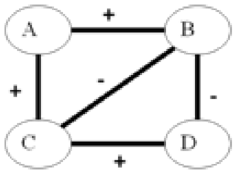


Figure-10: Here the edges are labeled with signs.

Example: Consider a Edge Labeled graph

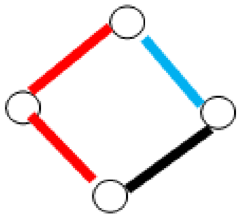


Figure-11: Edge labeled graph

Fig (11) is not a Colored graph as two adjacent edges have the same color.

Example: Consider a vertex labeled graph.

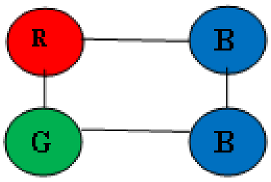


Figure-12: vertex labeled graph

Figure-12 is not a Colored graph as two adjacent vertices have the same color.

Here we define various concepts associated with the graph. Since some concepts like Incidence

Matrix, Adjacency Matrix are well known, we define only L-Matrices.

3. L-MATRIX OF A SINGLE LABELED VERTEX GRAPH DEFINED IN PAPER I [5]

We recall the definition of L-matrix of vertex labeled graphs and quote some results.

The matrix $A_L(G) = (a_{ij})$ of a vertex labeled graph G of order p called the L-Matrix of G is defined as follows:

The Matrix $A_l(G)$ is a square symmetric matrix of order p whose diagonal entries are all zero and whose other entries a_{ij} are given as follows:

$$a_{ij} = \begin{cases} 2 & \text{when } v_i \text{ and } v_j \text{ are adjacent with } l(v_i) = l(v_j) \\ -1 & \text{when } v_i \text{ and } v_j \text{ are non adjacent with } l(v_i) = l(v_j) \\ 1 & \text{when } v_i \text{ and } v_j \text{ are adjacent with } l(v_i) \neq l(v_j) \\ 0 & \text{when } v_i \text{ and } v_j \text{ are non adjacent with } l(v_i) \neq l(v_j) \end{cases}$$

Note: We can also define a_{ii} = -1 since v_1 and v_1 have same label but not adjacent, however this definition is not as convenient as taken $a_{ii} = 0$

Example 1: consider the L-matrix of a vertex labeled graph:

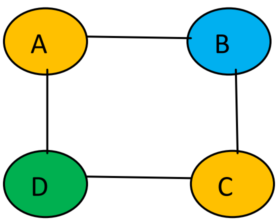


Figure-13

L-matrix of fig (13): $\begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

Example 2:

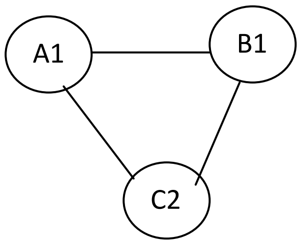


Figure-14

L-matrix of fig (14): $\begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Example: 3

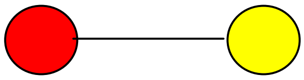


Figure-15

L-matrix of figure-15: $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Classically defined colored graph is a special case of a labeled graph, even the L-matrix of a colored graph is a special case of L-matrix of the labeled graph. A way of Coloring the vertices of a graph such that no two adjacent vertices share the same color is called Vertex Coloring. Similarly an edge coloring assigns a color to each edge so that no two adjacent edges share the same color.

Consider the L-matrix of a vertex labeled graph:

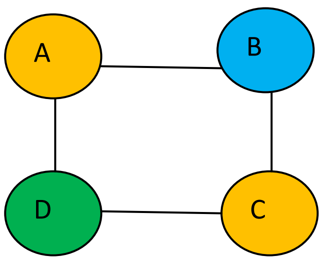


Figure-16

L-matrix of fig (16): $\begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

Multi Labeled Vertex/edge Graph [4]: A Multi Labeled Vertex is one in which a vertex/edge has *more than one label.* In face book every vertex is multi labeled. We define the L-matrix of the Multi Labeled Vertex as follows:

4. L-MATRIX OF A MULTI LABELED VERTEX GRAPH

We define L-matrix for edge labeled graphs(signed graphs) and obtain corresponding results.

In this case a vertex will have one or two labels i.e., l_{12} are two labels, a vertex may be assigned with one or both labels.

The matrix $A_L(G) = (a_{ij})$ of a multi labeled vertex graph G of order p called the L-Matrix of G is defined as follows:

The Matrix $A_l(G)$ is a square symmetric matrix of order p whose diagonal entries are all zero and whose other entries a_{ij} are given as follows:

$$a_{ij} = \begin{cases} 1 & if\, l_{1,2}(v_i) \neq l_{1,2}(v_j) when\ v_iv_j\ are\ adjacent \\ 0 & if\, l_{1,2}(v_i) \neq l_{1,2}(v_j) when\ v_iv_j\ are\ not\ adjacent \\ 2 & if\ l_1(v_i) = l_1(v_j) and\ l_2(v_i) \neq l_2(v_j) when v_iv_j\ are\ adjacent \\ -1 & if\ l_1(v_i) = l_1(v_j) and\ l_2(v_i) \neq l_2(v_j) when v_iv_j\ are\ not\ adjacent \\ 3 & if\ l_1(v_i) = l_1(v_j) and\ l_2(v_i) = l_2(v_j) when v_iv_j\ are\ adjacent \\ -2 & if\ l_1(v_i) = l_1(v_j) and\ l_2(v_i) = l_2(v_j) when v_iv_j\ are\ not\ adjacent \end{cases}$$

Example:

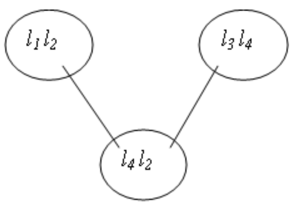


Figure-17:

The L-matrix of fig (17) = $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix}$

Note: We observe here that these matrices are not of any special kind in general. Also the computation of the L-Matrix for multi labeled vertex graph is too complicated we only consider 2-labeled graph.

Example:

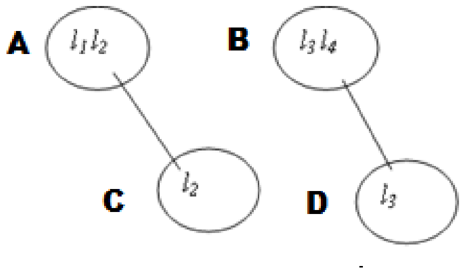


Figure-18

In fig (18) vertices A and B have two labels and vertices C and D have only one label.

The L-matrix of fig (18) = $\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}$

Example:

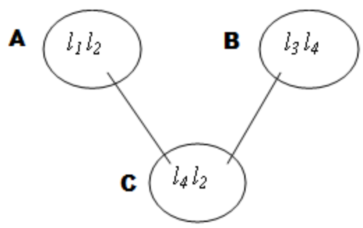


Figure-19

The L-matrix of fig (19) = $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix}$

5. L-MATRIX OF A SINGLE LABELED EDGE GRAPH

The Matrix $A_l(G)$ is a square symmetric matrix of order p whose diagonal entries are all zero and whose other entries a_{ij} are given as follows:

$$a_{ij} = \begin{cases} 2 & when\ l(e_i) = l(e_j) and\ e_i\ and\ e_j\ are\ incident\ with\ the\ same\ vertex \\ -1 & when\ l(e_i) = l(e_j) and\ e_i\ and\ e_j\ are\ not\ incident\ with\ the\ same\ vertex \\ 1 & when\ l(e_i) \neq l(e_j) and\ e_i\ and\ e_j\ are\ incident\ with\ the\ same\ vertex \\ 0 & when\ l(e_i) \neq l(e_j) and\ e_i\ and\ e_j\ are\ not\ incident\ with\ the\ same\ vertex \end{cases}$$

Here we consider graphs where edges are labeled not all and with definition of colored graph.

Example: Consider a graph:

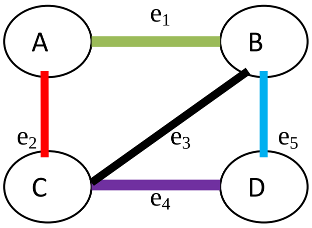


Figure-20

L-matrix of the above graph: fig (20) = $\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$

Example:

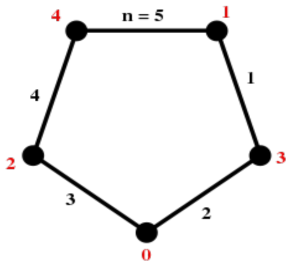


Figure-21

L-matrix of the above graph: fig (21) = $\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$

Example:

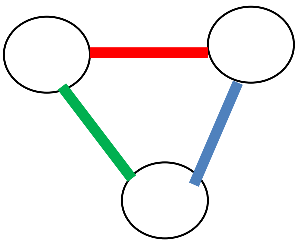


Figure-22

L-matrix of fig (22): $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Examples of Single Labeled Edge graph not necessarily colored graph (adjacent vertices may have the same color)
Consider a graph:

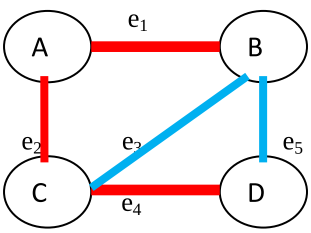


Figure-23

L-matrix of the above graph: fig (23) $\begin{bmatrix} 0 & 2 & 1 & -1 & 1 \\ 2 & 0 & 1 & 2 & 0 \\ 1 & 1 & 0 & 1 & 2 \\ -1 & 2 & 1 & 0 & 1 \\ 1 & 0 & 2 & 1 & 0 \end{bmatrix}$

6. L-MATRIX OF MULTI LABELED EDGE GRAPH

The matrix $A_L(G) = (a_{ij})$ of a multi labeled edge graph G of order p called the L-Matrix of G is defined as follows:

The Matrix $A_l(G)$ is a square symmetric matrix of order p whose diagonal entries are all zero and whose other entries a_{ij} are given as follows:

$$a_{ij} = \begin{cases} 1 & if\, l_{1,2}(e_i) \neq l_{1,2}(e_j) when\ e_ie_j\ are\ adjacent \\ 0 & if\, l_{1,2}(e_i) \neq l_{1,2}(e_j) when\ e_ie_j\ are\ not\ adjacent \\ 2 & if\ l_1(e_i) = l_1(e_j) and\ l_2(e_i) \neq l_2(e_j) when\ e_ie_j\ are\ adjacent \\ -1 & if\ l_1(e_i) = l_1(e_j) and\ l_2(e_i) \neq l_2(e_j) when\ e_ie_j\ are\ not\ adjacent \\ 3 & if\ l_1(e_i) = l_1(e_j) and\ l_2(e_i) = l_2(e_j) when\ e_ie_j\ are\ adjacent \\ -2 & if\ l_1(e_i) = l_1(e_j) and\ l_2(e_i) = l_2(e_j) when\ e_ie_j\ are\ not\ adjacent \end{cases}$$

7. NEW MATRIX

1. Matrix whose edges are labeled with **TWO LABELS** together and whose entries a_{ij} are given as follows:

$$\boldsymbol{a}ij = \begin{cases} \mathbf{1} & h\ only\ labeled\ edge \\ \mathbf{2} & wo\ labeled\ \ edge \\ \mathbf{0} & otherwise \end{cases}$$

Example: Consider the following graph in which edges are labeled with TWO labels.

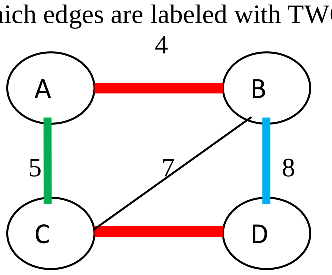


Figure-24

The new matrix of the above graph is: $\begin{bmatrix} 0 & 2 & 2 & 0 \\ 2 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$

Energy of the above matrix is: 10.0

Example: Consider the following graph in which edges are labeled with TWO labels.

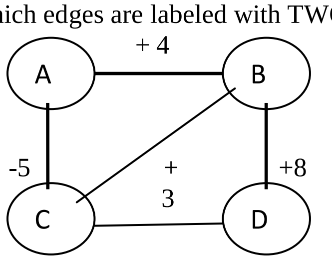


Figure-25

The new matrix of the above graph is: $\begin{bmatrix} 0 & 2 & 2 & 0 \\ 2 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$

Energy of the above matrix: 10.0

Example: Consider the following graph in which edges are labeled with TWO labels:

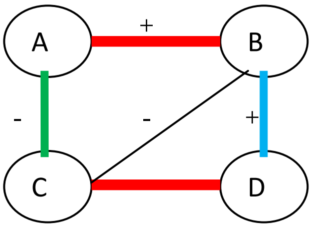


Figure-25

The new matrix of the above graph is: $\begin{bmatrix} 0 & 2 & 2 & 0 \\ 2 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$

Energy of the above matrix is: 10.0

8. BASIC DEFINITIONS OF A VERTEX AND AN EDGE IN FACEBOOK THEORY

Vertex: A vertex in a face book network stands for an individual/group that has an account in face book. It is possible that a few people operate the same account. Hence this group represents a single vertex.

Figure-27

In the above figure A and B vertices indicates persons/individuals who have an account in face book.

Labeled Vertex: A vertex in a face book is labeled as *l* where *l* may stand for online, offline, language, religion, school...etc., Therefore a vertex will have more than one label but it always has one label either '+' or '-' indicating online or offline.

Note: 1.All vertices in Face Book are labeled.

Example:

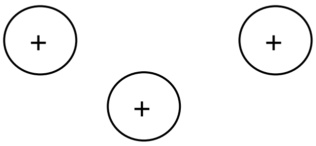


Figure-28

SIGNED Labeled Edge:

Example: Consider the following graph representing a network of 4 people (A, B, C, D) where all 4 people are online. Physically this means that A can view all vertices with same label (+) but A may not converse with any of the vertices.

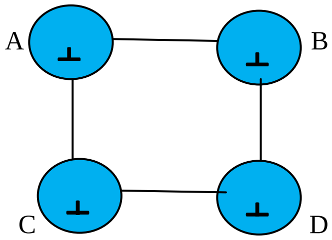


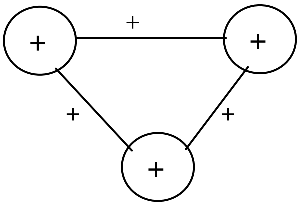
Figure-29

AB is an edge if vertices have COMMON SIGN. If A and B are online at a given instant of time then the edge AB gets a label '+', if A and B converse then AB gets the label '+ +'. Hence there is a complete graph of online people.

Similarly an edge AB is labeled '-'if A and B are offline.

Note: 1. AB is an edge with a '+' sign if A and B are online.
2. If A and B are online and they converse then the edge AB is labeled'++'

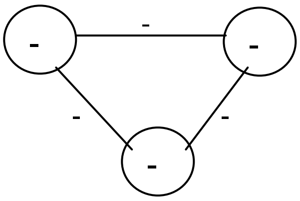
Example:



Therefore the above network results in a complete graph.

Figure-30

Example:



Therefore the above network results in a complete graph.

Figure-31

We observe here that signed edges occur naturally in SNS. If A and B are online/offline then the edge AB has a label '+' and AB is a positive edge or edge AB has a label '-' and AB is a negative edge. Even though AB is an edge only when it is positive we can further continue. A and B is a positive edge and need not converse, if they converse then AB has label '++'.

Single Signed Edge: If an edge AB has a label '+' then A and B are online. If AB has a label '-' then one of them is offline.

Example:

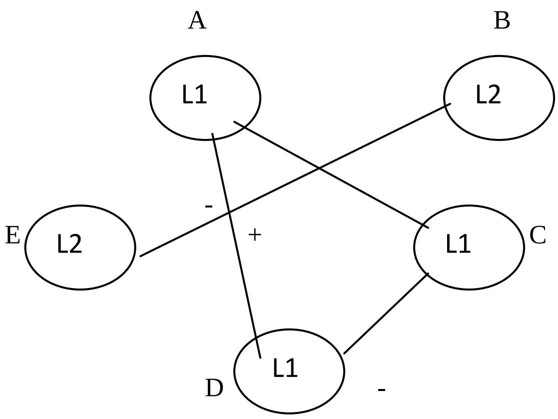


Figure-32

In the figure above vertices A, D, C has the same label (L1), therefore there are edges between A, D, C whereas the vertices B and E have the same label (L2) as a result there is an edge between them.

Multi Signed Edges: [6]: (2, d) sigraphs]

1) We can also define multi labeled edges as given below with two signs together + and –
+ + indicates that both are online and both are under conversation
+ - indicates that A has sent the request and B has not accepted the request.
- + indicates that A has not accepted the request sent by B.
- - indicates that neither has sent the requests.

Example 1: Consider a particular network of 10 people who have an account in face book.

Out of these 10 people we consider 6 to be online i.e., 6 vertices have the label '+' sign.

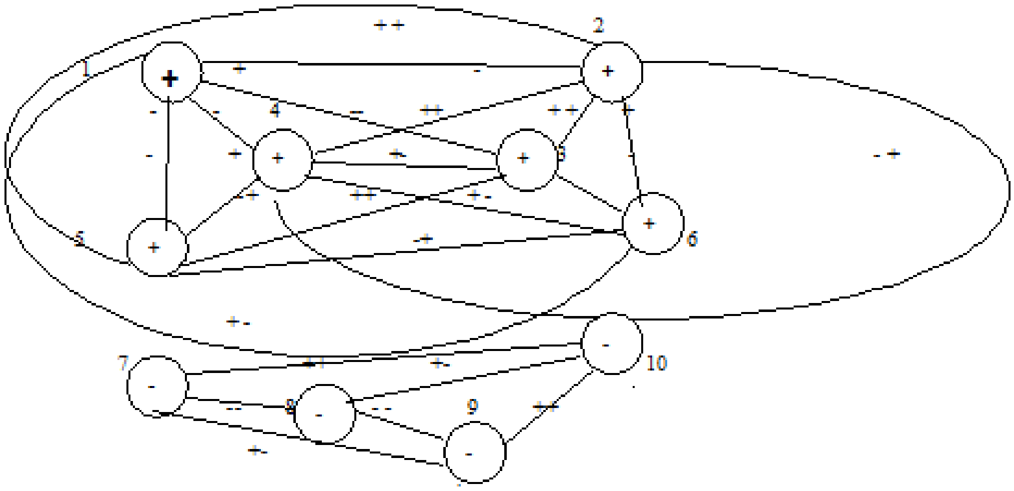


Figure-33

From the definition of an edge, there is an edge between 2 vertices with the same label i.e. vertices having the label '+' will all be connected by an edge.

The edges labeled with + + shows that they are under conversation i.e., vertices 1, 2, 3 are under conversation as the edges between 1, 2, 3 have labeled edge '+ +'. Vertices 1 and 4 have a labeled edge '- +' i.e., 4 has sent the request but 1 has not accepted it. Vertices 1 and 6 have '+ -'labeled edge i.e., 1 has sent the request but 6 has not accepted. Vertices 1 and 5 have '- -'labeled edge i.e., neither 1 nor 5 have sent the request.

2) Say if A and B are the vertices with the same label then the edge between them may have the following labels:
+ + : Indicates both A and B is online, and there is a mutual request between them.
+ - : Indicates both are online and there is no conversation between them.
- + : Indicates both are online and there is no conversation between them.
- - : Indicates both are offline.

The matrix of a Single labeled Edge differs from the matrix of Multi Labeled Edges.

We now consider edges with labels other than signs'+' and '-'

Edge for Multi Labeled vertices: AB is an edge if at least one of the labels of A is same as that of B.

Example:

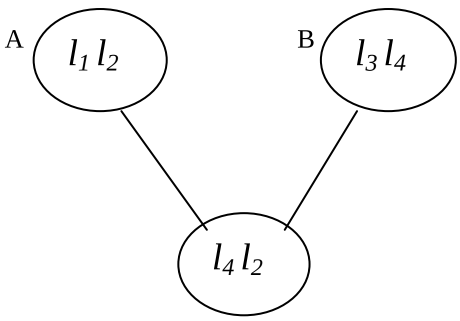


Figure-34

The L-matrix of fig (34) = $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix}$

9. PROPOSITION 4 [5]

Let G = (V,E) be a colored graph with at least one edge. Then there exists a unique signed graph H on V with at least one positive edge such that the L-matrix of G is the same as the adjacency matrix of the signed graph H.

Proposition 4[5]: in terms of face book: (Single Labeled edges)
Let G:(V, E) be a labeled graph of a face book network with labeled vertices and edges and with at least two vertices with the same label then there exists a unique labeled graph H on V with at least two of the vertices being online such that the L-matrix of the labeled graph of the face book network is twice as the adjacent matrix of the labeled graph H of the same face book network.

Proposition 4[5]: in terms of face book: (Two Labeled edges)
Let G:(V, E) be a labeled graph of a face book network with labeled vertices and edges and with at least two vertices with the same label then there exists a unique labeled graph H on V with at least two of the vertices being online and under conversation or online not conversing or both offline such that the L-matrix of the labeled graph of the face book network is not equal to the adjacent matrix of the labeled graph H of the same face book network.

We now apply the basic definition of Single labeled edge of face book theory to the proposition 4 [reference: 5] by an example.

Consider a graph G consisting of 6 vertices (people) i.e., A, B, C, D, E, F out of which 4 people (A, B, C, D) are online. Therefore A, B, C, D vertices are labeled with '+' sign (from the definition of labeled vertex)

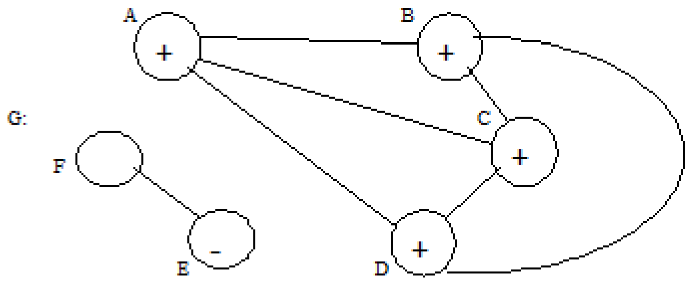


Figure-35

The graph is constructed as follows: for any vertex v in G l(v) denote the label of v.

In H v_iv_j is a '+' edge iff v_iv_j is an edge in G and $l(v_i) = l(v_j)$ ='+'

In H v_iv_j is a '-' edge iff v_iv_j is an edge in G and $l(v_i) = l(v_j)$ ='-'

Further v_iv_j is a '-'edge in H iff v_i v_j are not adjacent in G and $l(v_i) \neq l(v_j)$.

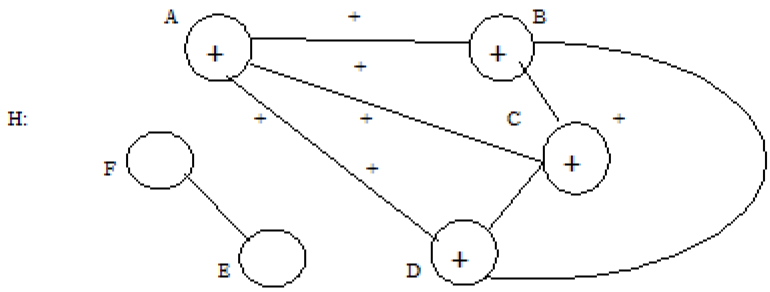


Figure-36

$$\text{L-Matrix of G} = \begin{bmatrix} 0 & 2 & 2 & 2 & 0 & 0 \\ 2 & 0 & 2 & 2 & 0 & 0 \\ 2 & 2 & 0 & 2 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 0 \end{bmatrix}$$

$$\text{Adjacent matrix of H} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \frac{\textbf{\textit{The L-Matrix of G}}}{\mathbf{2}} = \textbf{Adjacent Matrix of H}$$

We define the Adjacency matrix of a (2, d) Sigraph as follows:

$$aij = \begin{cases} 2 & \textit{if there exists a} + +\textit{sign associated with the edge } i, j \\ -2 & \textit{if there exists a} - -\textit{sign associated with the edge } i, j \\ -1 & \textit{if there exists a} - +\textit{sign associated with the edge } i, j \\ 1 & \textit{if there exists a} + -\textit{sign associated with the edge } i, j \\ 0 & \textit{otherwise} \end{cases}$$

We now apply the basic definition (2) of labeled edge of face book theory to the proposition 4 [reference: 5] Consider a graph G which is a network of 6 people i.e., A, B, C, D, E, F

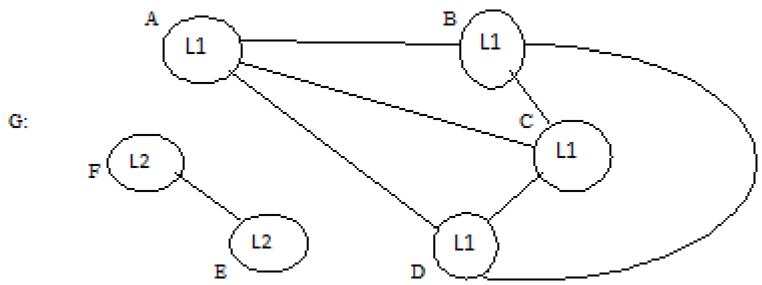


Figure-37

In H v_iv_j is a '+ + 'edge iff A and B both are online and both are under conversation. i.e.,

+ + : Indicates both A and B is online, and there is a mutual request between them.
+ - : Indicates both are online and there is no conversation between them.
- + : Indicates both are online and there is no conversation between them.
- - : Indicates both are offline.

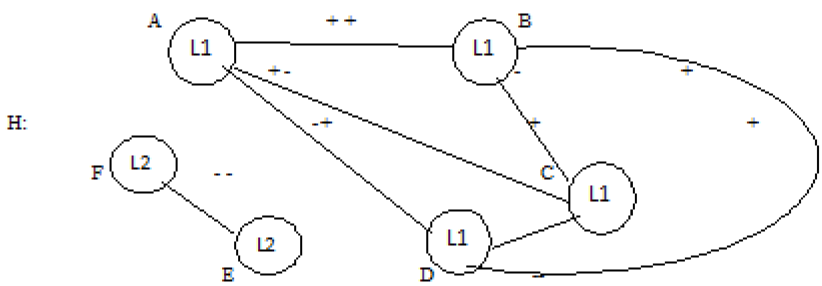


Figure-38

L-Matrix of $G = \begin{bmatrix} 0 & 2 & 2 & 2 & 0 & 0 \\ 2 & 0 & 2 & 2 & 0 & 0 \\ 2 & 2 & 0 & 2 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 0 \end{bmatrix}$

Adjacent matrix of $H = \begin{bmatrix} 0 & 2 & 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 2 & 0 & 0 \\ -1 & 1 & 0 & -2 & 0 & 0 \\ 1 & 2 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

L-Matrix of G ≠ Adjacent Matrix of H

We observe that L-matrix of a colored graph G is again ≠ Adjacency matrix of a (2,d) sigraph Therefore we conclude that proposition 4 [reference:5] is a consequence of the altered definition.

Why we have altered proposition 4 [reference: 5]:

Consider the following graph:

G:

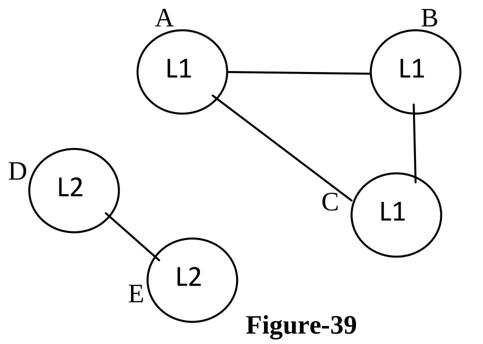


Figure-39

Applying proposition 4 [5]: we get the following graph H:

H:

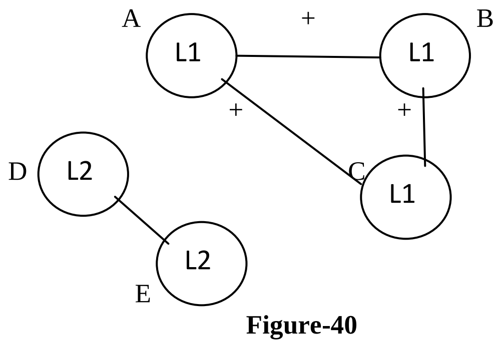


Figure-40

-

In the above labeled graph G even though there is an edge in G there is a '-' edge in H which is a contradiction , i.e., a '-' edge in H indicates that vertices D and E are offline which is a contradiction. Therefore we have changed it to a labeled edge in H.(it can be either '+' or '-')

When two vertices in graph G have same label (say some character) according to proposition 4 there should be '+' edge (in case of single labeled edge) or '+ +' or'+ -' edge (in case of two labeled edge) in H but in the above face book network there is a '-' edge in H which is a contradiction, so we alter proposition 4 [reference:5] and change to if there is an edge in G there is a labeled edge in H i.e., we reduce a two labeled edge to a single labeled edge and get our result.

Proposition 4 (Altered)[reference:5]

Let G =(V,E) be a vertex labeled graph with at least one edge then there exists '-' labeled edge graph H on V with at least one '+' labeled edge such that L-Matrix of G = 2 (Adjacency Matrix of H).

Now we apply proposition 4 [reference: 5] (Altered) to face book

Consider a network of face book represented by a graph G consisting of 5 people A,B,C,D,E out of which 3 people(vertices)A,B,C have the same label i.e., l_1 and the remaining 2 ,D,E have the same label i.e.,l_2.

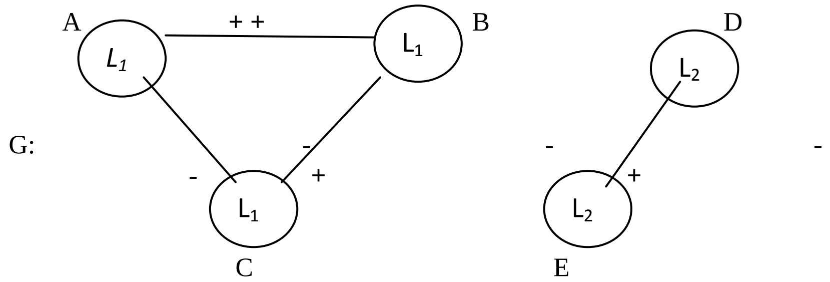


Figure-41

NOTE: In the graph above labeling of edges is done according to definition 2 of Labeled Edge. Graph H is constructed as follows:

There is a labeled edge in H if there is an edge in G .i.e. There is a '+' edge in H if there exists a '+ -'or '- +' and a '+ +' edge in G, and there is a '-'edge in H if there is a '- -'edge in G.

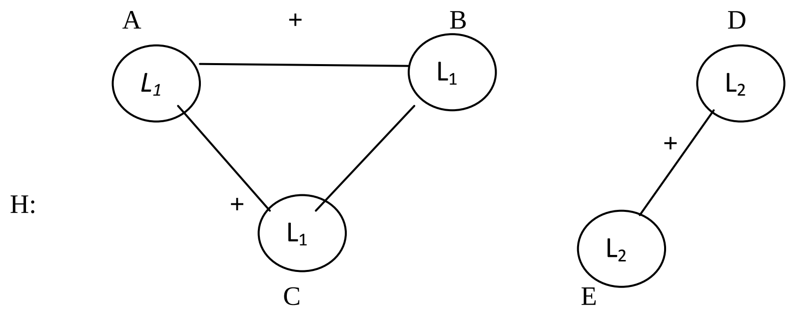


Figure-42

$$\text{L-Matrix of } H = \begin{bmatrix} 0 & 2 & 2 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}$$

$$\text{Adjacency Matrix of } H = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Therefore L-**Matrix of G = 2 (Adjacency Matrix of H)**

10. CONCLUSION

Our main aim is to apply graph theory to SNS and for this we need to define vertices and edges which is the reason why we have extensively studied labeled graphs .Since in SNS edges and vertices are labeled naturally. We further observe the following facts.

In face book we define vertices as individual who have an account in face book and a labeled vertex is one with some characteristics of the person.

AB is an edge if and only if A and B have the same label. In this definition both the vertices and edges may have multi labels. However AB is defined as a labeled edge with label '+ +' if they are online and communicated, whereas edges with labels '+ -"or'- +' indicate '+' edge but one of them not communicating with the other. A labeled edge '- - 'indicates '-'edge.

In face book at least one edge is '+' and one edge is '-'.

This is practically true since if there are 'n' vertices and 'k' of them have the same labels, then the probability that there is a '+' edge is $\frac{k}{n}$ .Since 'n' is quite large and 'k' smaller than 'n' (k<n) we see that $0 \leq \frac{k}{n} \leq 1$ .Therefore there is a '+' edge.

The result we got is only in example; we will examine in future whether the result is true in general. However we have at least one situation where the result is true.

We believe that preposition 4 (reference: 4) could be so designed that under the condition of the proposition, the L-Matrix of G =k times the adjacency Matrix of H. This will have to be examined in future work.

While defining the l-matrix for the sake of convenience we have taken the diagonal elements as zero, but strictly speaking we should have taken the diagonal elements as -1, however to retain the validity of some theorems we have taken the diagonal elements as zero.

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6. (2,d)-Sigraph (journal) by] E. Sampathkumar Department of Studies in Mathematics, University of Mysore Mysore-570 006, India.

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