BLOOD CIRCULATORY SYSTEM WITH RECOVERY PERIOD DEPENDING ON DEPOSIT OF LOW DENSITY LIPOPROTEINS

NITHYAPRIYA.N,*
Department of Mathematics,
Sacred Heart College, Thirupattur, Tamil Nadu, India.

(Received On: 17-06-17; Revised & Accepted On: 07-07-17)

ABSTRACT
The human Blood Circulatory System (BCS) in which the deposit of Low Density Lipoproteins (LDL) during prophylactic period and recovering period after the deposit is treated. After the bulk deposit or buildup of low density lipoproteins medical attention starts. It is assumed that the time to prophylactic treatment has general distribution and deposit of LDL is proportional to time. Expected Recovery period is obtained.

Keywords: BCS, LDL Deposit, Joint Transform.

INTRODUCTION
Cholesterol is a waxy fat like substance that is found in all cells of the body. Our body needs some cholesterol to make hormones, vitamin D and substances that help us to digest foods. Our body itself makes all the cholesterol it needs, however cholesterol also is found in some of foods we eat. Cholesterol travels through blood stream in small packages called lipoproteins. These packages are made of fat (lipid) on the inside and proteins on the outside. Two kinds of lipoproteins carry cholesterol throughout the body namely, LDL and HDL. Healthy levels of both types of lipoproteins are important. LDL cholesterol is sometimes called “bad cholesterol”. A high LDL level leads to a buildup of cholesterol in arteries. The level of LDL cholesterol in blood is higher, the greater chance of getting heart disease. In this paper we consider a model in which the prophylactic time has general distribution and the deposit of LDL is proportional to time. After the bulk deposit or buildup of LDL medical attention starts. Two distinct distributions for the recovery periods depending on the quantity of deposit of LDL during life period are within or exceeding a random critical quantity is considered. The joint transform is obtained to derive the expected quantity of LDL deposit and recovery time. Numerical examples are also presented. The plaque constricts the lumen of the blood vessel thereby increasing the sheer force of blood flow [1, 2]. As the plaque continues to grow, the increased sheer force may cause rupture of the plaque, possibly resulting in the formation of thrombus (blood clot) [1], ischemic stroke, and heart attack [1]. Mathematical modelling of the atherosclerotic plaque formation has been studied by Calvez V, Ebde A, Meunier N, Raoult A [3]. Wenrui Hao1*, Avner Friedman [4] developed a mathematical model of the formation of a plaque, using a system of partial differential equations with in evolving plaque. For model assumption one may refer K.Usha, A.C.Tamil Selvi, R. Ramanarayanan and K.Usha and N.Nithya Priya [5, 6].

MODEL:
GENERAL PROPHYLACTIC TIME AND EXPONENTIAL DEPOSIT TIME OF LDL.

Model assumptions are,
1. The time to prophylactic treatment (L) of a person is a random variable with Cdf H(u) and pdf h(u).
2. The deposit quantity of LDL in the artery is proportional to time and the proportionality constant is c.
3. The recovering period R has general distribution with Cdf G1(v) and pdf g1(v) when deposit of LDL is less than the critical quantity ‘A’ and it has Cdf G2(v) and pdf g2(v) when the deposit of LDL is more than the critical quantity A to speed up the medical attention.
4. Critical quantity ‘A’ has exponential distribution with parameter.
   We may derive the joint distribution of L and R as follows.
The joint probability density function of \( L \) and \( R \) is,
\[
\frac{\partial}{\partial u} \left( \frac{\partial}{\partial v} p(L \leq u, R \leq v) \right) = f(u,v)
\]
\[
= h(u)e^{-\mu u}g_1(v) + h(u)(1-e^{-\mu u})g_2(v)
\]  
(1)

The first term of equation (1) is the part of the pdf that the deposit of LDL is less than the critical quantity. The second term of equation (1) is the part of the pdf that the deposit of LDL is more than the critical quantity.

Let us define the joint Laplace transform function as follows,
\[
E\left(e^{-st}e^{-tr}\right) = \int_0^\infty \int_0^\infty e^{-su}e^{-tv}f(u,v)dudv
\]
\[
= \int_0^\infty \int_0^\infty e^{-su}e^{-tv}\left(h(u)e^{-\mu u}g_1(v) + h(u)(1-e^{-\mu u})g_2(v)\right)dudv
\]  
(2)

This on simplification gives,
\[
E\left(e^{-st}e^{-tr}\right) = h^*(s + c\mu)g_1^*(t) + \left(h^*\left(s\right)-(s+c\mu)\right)g_2^*(t)
\]  
(3)

Here * indicates Laplace transform.

We may note \( E\left(e^{-st}\right) = h^*(s) \)
\[
E\left(e^{-tr}\right) = h^*(c\mu)g_1^*(t) + \left(1-h^*(c\mu)g_2^*(t)\right)
\]  
(4)

Using differentiations and setting \( p = h^*(c\mu) \) and \( q = 1-p \)
\[
E(R) = pE(R_1) + qE(R_2)
\]  
(5)

If \( h(u) \) has exponential distribution with parameter ‘a’ then,
\[
p = \frac{\alpha}{\mu c + \alpha}
\]

**NUMERICAL ILLUSTRATION**

Using (5) we assume the fixed values for \( \mu = 25, R_1 = 0.1 \) and \( R_2 = 0.5 \) so that \( E(R_1) = 10 \) and \( E(R_2) = 2 \). We provide the different values for the parameter of exponential distribution of time to prophylactic treatment (L) of the person and the deposit quantity of LDL in the artery.

\( c = 10 \text{ mg}, 20 \text{ mg}, 30 \text{ mg}, 40 \text{ mg} \) and

\( a = 35\text{ yrs}, 45\text{ yrs}, 55\text{ yrs}, 65\text{ yrs} \).

**The Effect of variation of E(R)**

<table>
<thead>
<tr>
<th>( a )</th>
<th>35</th>
<th>45</th>
<th>55</th>
<th>65</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c = 10 )</td>
<td>2.9825</td>
<td>3.2203</td>
<td>3.4426</td>
<td>3.6508</td>
</tr>
<tr>
<td>( c = 20 )</td>
<td>2.5234</td>
<td>2.6606</td>
<td>2.7928</td>
<td>2.9204</td>
</tr>
<tr>
<td>( c = 30 )</td>
<td>2.3567</td>
<td>2.4528</td>
<td>2.5466</td>
<td>2.6380</td>
</tr>
<tr>
<td>( c = 40 )</td>
<td>2.2705</td>
<td>2.3445</td>
<td>2.4171</td>
<td>2.4883</td>
</tr>
</tbody>
</table>
CONCLUSION

The constant deposit of LDL leads to the threat of cardiac arrest as the age increases. After the bulk deposit or buildup of low density lipoproteins the chart value reveals when to start medical care for a person with the risk and the Expected Recovery period is obtained.

ACKNOWLEDGEMENTS

The author thank Sacred Heart College for providing necessary facilities for research. The author thanks Dr.R.Ramanarayanan, Dr .Usha, and Dr.A.C.Tamil Selvi for their guidance and support in bringing our research paper successfully.

REFERENCES

5. K.Usha, A.C.Tamil Selvi, R. Ramanarayanan, Manpower Systems with Recruitment period depending on Attrition during the busy period. Mathematical and computational models, copy right © 2012, Narosa publishing House, New Delhi, India (271-275).
6. K.Usha , N.Nithiya Priya, R. Ramanarayanan, Stochastic Analysis of Inventory system with sales time depending on Production time, IJAM, Volume 24 No.4 2011, 573-582.