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CHAOS SYNCRONIZATION OF THE PHOTOGRAVITATIONAL MAGNETIC BINARIES PROBLEM VIA NONLINEAR CONTROL

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ABSTRACT

In this article we have discussed the chaos synchronization of the photogravitational magnetic-binaries problem when the bigger primary is a source of radiation. We have designed a nonlinear controller based on the Lyapunov stability theory. Simulation studies are conducted to show the effectiveness of the proposed method.

Key words: *Space dynamics, photogravitational magnetic-binaries problem, synchronization, Lyapunov stability theory.*

1 INTRODUCTION

In the last several decades, much effort has been devoted to the study of nonlinear chaotic systems. Chaos control and synchronization are especially noteworthy and important research fields leveling to affect dynamics of chaotic systems in order to apply them for different kinds of applications that can be examined within many different scientific research. At present, there are different kinds of control methods and techniques that have been proposed for carrying out chaos control and synchronization of chaotic dynamical systems. Chaotic synchronization did not attract much attention until Pec-ora and Carroll introduced a method to synchronize two identical chaotic systems with deferent initial conditions in 1990 and they demonstrated that chaotic synchronization could be achieved by driving or replacing one of the variables of a chaotic system with a variable of another similar chaotic device. Various techniques have been proposed and implemented successfully for achieving stable synchronization between identical and non-identical systems notable among these methods, the active control scheme proposed by E. W. Bai & K. E. Lonngren 1997 has been received and successfully implemented in almost all the field of nonlinear sciences for synchronization for different systems with various techniques.

Nonlinear control is an effective method for making two different chaotic systems by synchronized. However, this method usually assumes that the Lyapunov function of error dynamic of synchronization is formed as $V = \frac{1}{2}\, e^t e$.

The synchronization problem via nonlinear control scheme is studied by Chen and Han in 2003, Chen 2005, Ju H. Park 2005, Amir Abbas Emadzadeh, and Mohammad Haeri 2005 and M. Mossa Al-sawalha, M.S.M. Noorani in 2009.

The different cases of the magnetic binaries problem have been studied by A. Mavragnais (1978, …… 1988) .

In 2015 Mohd Arif. have studied the equilibrium points of the photogravitational magnetic binaries problem.

In this article we have discussed the chaos synchronization of the photogravitational magnetic-binaries problem when the bigger primary is a source of radiation, here we have designed a nonlinear controller based on the Lyapunov stability. The system under consideration is chaotic for some values of parameter involved in the system. Here two systems (master and slave) are synchronized and start with deferent initial conditions. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time.

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2 EQUATION OF MOTION

In formulating the problem we shall assume that the two primaries (dipoles) in which the bigger primary is a source of radiation with magnetic fields move under the influence of gravitational force and a charged particle P (small body) of charge q and mass m moves in the vicinity of these primaries. The equation of motion and the integral of relative energy in the rotating coordinate system including the effect of the gravitational forces of the primaries on the small body written as:

$$\ddot{x} - \dot{y}\, f = U_x \tag{1}$$

$$\ddot{y} + \dot{x}\, f = U_y \tag{2}$$

$$\dot{x}^2 + \dot{y}^2 = 2\mathrm{U} - \mathrm{C} \tag{3}$$

Where

$$f = 2 - \left(\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{r_2^3}\right),\ U_x = \frac{\partial U}{\partial x} \text{ and } U_y = \frac{\partial U}{\partial y}$$

$$U = (x^2 + y^2)\left\{\frac{1}{2} + \frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{r_2^3}\right\} + x\left\{\frac{q_1\mu}{\mathrm{r}_1^3} - \frac{\lambda(1-\mu)}{r_2^3}\right\} + \frac{q_1(1-\mu)}{\mathrm{r}_1} + \frac{\mu}{\mathrm{r}_2} \tag{4}$$

$r_1^2 = (x-\mu)^2 + y^2$, $r_2^2 = (x+1-\mu)^2 + y^2$, $\lambda = \frac{M_2}{M_1}$ (M_1, M_2 are the magnetic moments of the primaries which lies perpendicular to the plane of the motion). q_1 is the source of radiation of bigger primary.

Here we have assumed that the distance between the primaries as the unit of distance and the coordinate of one primary is (μ, 0, 0) then the other is ($\mu-1$, 0, 0). We also assumed that the sum of their masses as the unit of mass. If mass of the one primaries μ then the mass of the other is ($1-\mu$). We choose the unit of time in such a way that the gravitational constant G has the value unity and $q = mc$ where c is the velocity of light.

3. CHAOS SYNCHRONIZATION VIA NONLINEAR CONTROL

Let

$$x = x_1,\ \dot{x} = x_2,\ y = x_3,\ \dot{y} = x_4$$

Then the equation (1) and (2) can be written as:

$$\dot{x_1} = x_2 \tag{5}$$

$$\dot{x_2} = 2\,x_4 + x_1 + A_1 \tag{6}$$

$$\dot{x_3} = x_4 \tag{7}$$

$$\dot{x_4} = -2\,x_2 + x_3 + A_2 \tag{8}$$

Where

$$A_1 = -3\,x_1\left\{\frac{q_1(x_1-\mu)^2}{\mathrm{r}_1^5} + \frac{\lambda\,(x_1+1-\mu)^2}{\mathrm{r}_2^5}\right\} - 3{x_3}^2\left\{\frac{q_1(x_1-\mu)}{\mathrm{r}_1^5} + \frac{\lambda\,(x_1+1-\mu)}{\mathrm{r}_2^5}\right\} + \left\{\frac{q_1(x_1-\mu)}{\mathrm{r}_1^3} + \frac{\lambda\,(x_1+1-\mu)}{\mathrm{r}_2^3}\right\}$$
$$+x_1\left\{\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{\mathrm{r}_2^3}\right\} - \left\{\frac{q_1(1-\mu)(x_1-\mu)}{\mathrm{r}_1^3} - \frac{\mu(x_1+1-\mu))}{\mathrm{r}_2^3}\right\} - x_4\left(\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{r_2^3}\right),$$

$$A_2 = -3\,x_1x_3\left\{\frac{q_1(x_1-\mu)}{\mathrm{r}_1^5} + \frac{\lambda\,(x_1+1-\mu)}{\mathrm{r}_2^5}\right\} - 3{x_3}^3\left\{\frac{q_1}{\mathrm{r}_1^5} + \frac{\lambda}{\mathrm{r}_2^5}\right\} + x_4\left(\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{r_2^3}\right) + 2\,x_3\left\{\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{\mathrm{r}_2^3}\right\} - x_3\left\{\frac{q_1(1-\mu)}{\mathrm{r}_1^3} - \frac{\mu}{\mathrm{r}_2^3}\right\},$$

$$r_1^2 = (x_1-\mu)^2 + {x_3}^2,\quad r_2^2 = (x_1+1-\mu)^2 + {x_3}^2,$$

Corresponding to master system (5, 6, 7 and 8), the identical slave system are:

$$\dot{y_1} = y_2 + u_1(t) \tag{9}$$

$$\dot{y_2} = 2\,y_4 + y_1 + A_3 + u_2(t) \tag{10}$$

$$\dot{y_3} = y_4 + u_3(t) \tag{11}$$

$$\dot{y_4} = -2\,y_2 + y_3 + A_4 + u_4(t) \tag{12}$$

Where

$$A_3 = -3\,y_1\left\{\frac{q_1(y_1-\mu)^2}{\mathrm{r}_1^5} + \frac{\lambda\,(y_1+1-\mu)^2}{\mathrm{r}_2^5}\right\} - 3{y_3}^2\left\{\frac{q_1(y_1-\mu)}{\mathrm{r}_1^5} + \frac{\lambda\,(y_1+1-\mu)}{\mathrm{r}_2^5}\right\} + \left\{\frac{q_1(y_1-\mu)}{\mathrm{r}_1^3} + \frac{\lambda\,(y_1+1-\mu)}{\mathrm{r}_2^3}\right\}$$
$$+y_1\left\{\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{\mathrm{r}_2^3}\right\} - \left\{\frac{q_1(1-\mu)(y_1-\mu)}{\mathrm{r}_1^3} - \frac{\mu(y_1+1-\mu))}{\mathrm{r}_2^3}\right\} - y_4\left(\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{r_2^3}\right),$$

$$A_2 = -3\,y_1y_3\left\{\frac{q_1(y_1-\mu)}{\mathrm{r}_1^5} + \frac{\lambda\,(y_1+1-\mu)}{\mathrm{r}_2^5}\right\} - 3{y_3}^3\left\{\frac{q_1}{\mathrm{r}_1^5} + \frac{\lambda}{\mathrm{r}_2^5}\right\} + y_4\left(\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{r_2^3}\right) + 2\,y_3\left\{\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{\mathrm{r}_2^3}\right\} - y_3\left\{\frac{q_1(1-\mu)}{\mathrm{r}_1^3} - \frac{\mu}{\mathrm{r}_2^3}\right\},$$

$$r_1^2 = (y_1-\mu)^2 + {y_3}^2,\quad r_2^2 = (y_1+1-\mu)^2 + {y_3}^2,$$

where $u_i(t)$; i =1 ,2,3,4 are control functions to be determined. Let $e_i = y_i - x_i$; i = 1, 2, 3, 4 be the synchronization errors. From (5) to (12), we obtain the error dynamics as follows:

$$\dot{e_1} = e_2 + u_1(t) \tag{13}$$

$$\dot{e_2} = 2e_4 + e_1 + A_3 - A_1 + u_2(t) \tag{14}$$

$$\dot{e_3} = e_4 + u_3(t) \tag{15}$$

$$\dot{e_4} = -2e_2 + e_3 + A_4 - A_2 + u_4(t) \tag{16}$$

According to the Lyapunov stability theory, when controller satisfies the assumption with $V(e) = \frac{1}{2}\, e^t e$ a positive definite function and the first derivative of this function $\dot{V} < 0$ the chaos synchronization of two identical systems (master and slave) for different initial conditions is achived.

Let a positive definite Lyapunov function

$$V(e) = \frac{1}{2}\, e^t e$$

Then we have the first derivative of $V(e)$:

$$\dot{V} = e_1[e_2 + u_1(t)] + e_2[2e_4 + e_1 + A_3 - A_1 + u_2(t)] + e_3[e_4 + u_3(t)] + e_4[-2e_2 + e_3 + A_4 - A_2 + u_4(t)] \tag{17}$$

Therefore, if we choose the controller u as follows,

$$u_1 = -2e_2 - e_1 \tag{18}$$
$$u_2 = -e_2 - A_3 + A_1 \tag{19}$$
$$u_3 = -e_3 - 2e_4 \tag{20}$$
$$u_4 = -A_4 + A_2 - e_4 \tag{21}$$

Then

$$\dot{V} = -{e_1}^2 - {e_2}^2 - {e_3}^2 - {e_4}^2 < 0 \tag{22}$$

Hence the error state

$$\lim_{t\to\infty} \|e(t)\| = 0$$

which gives asymptotic stability of the system. This means that the controlled chaotic systems (5, 6, 7, 8) and (9, 10, 11, 12) is synchronized for any initial conditions.

4. NUMERICAL SIMULATION

We select the parameters $\mu = .0001$ $q_1 = 1.5$ and $\lambda = 1$, the state orbits of the chaotic system are shown in Figure 1, with the different initial conditions $[x_1(0) = 0.0, x_2(0) = 12.0, x_3(0) = 1.0, x_4(0) = 4.0]$ for master systems and $[y_1(0) = 10.0, y_2(0) = 2.0, y_3(0) = 4.0, y_4(0) = 0.0]$ for slave systems Simulation results for uncoupled system are presented in figures.2,4,6,8 and that of controlled system are shown in figures.3,5,7 and 9 respectively. Figures (10, 11, 12, 13) display the chaos-synchronization errors of systems.

It can be seen from the figures that the chaos-synchronization errors converge to zero rapidly.

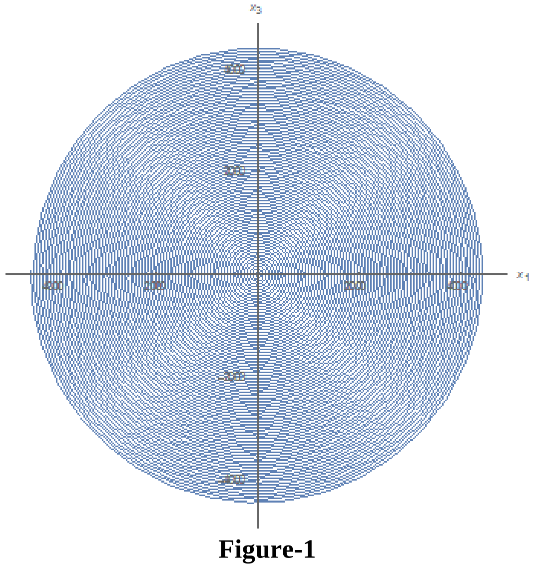

Figure-1

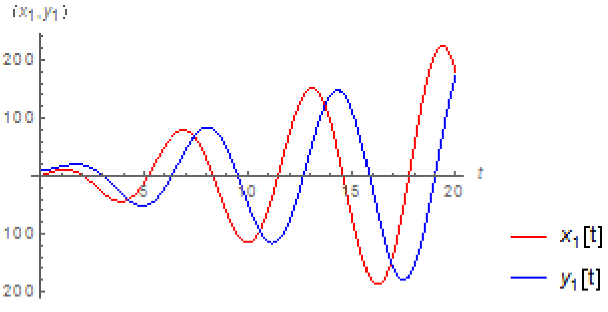

Figure-2

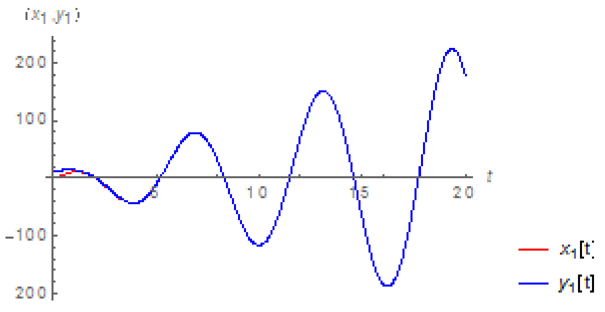

Figure-3

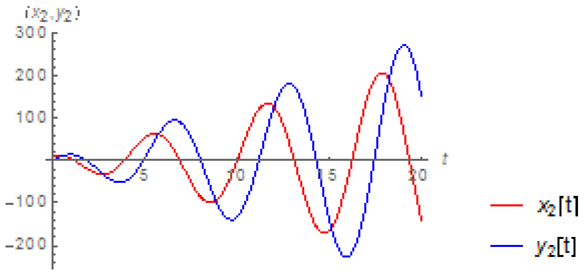

Figure-4

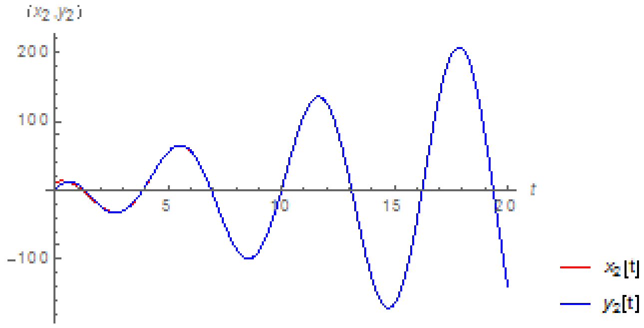

Figure-5

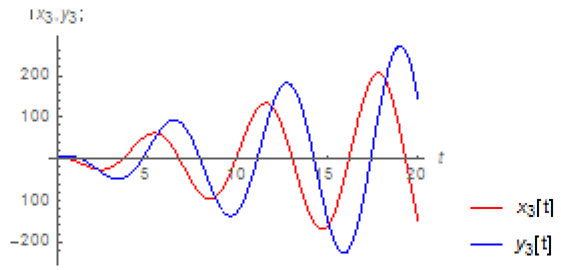

Figure-6

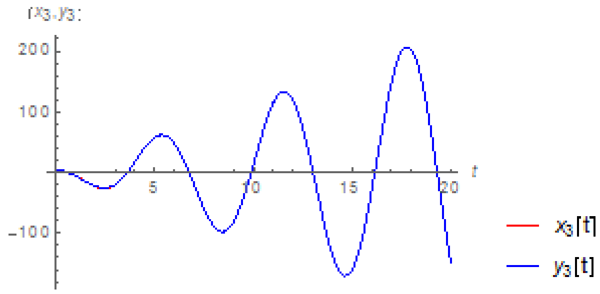

Figure-7

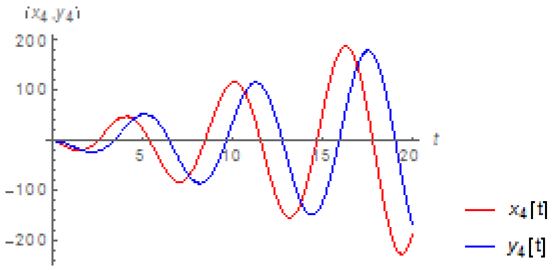

Figure-8

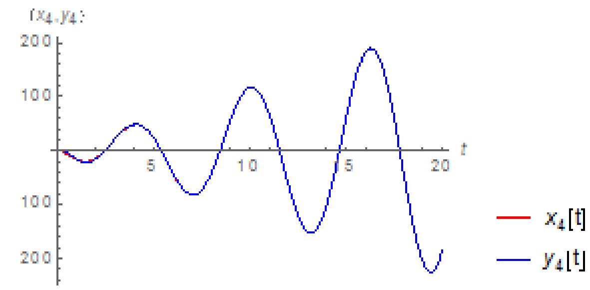

Figure-9

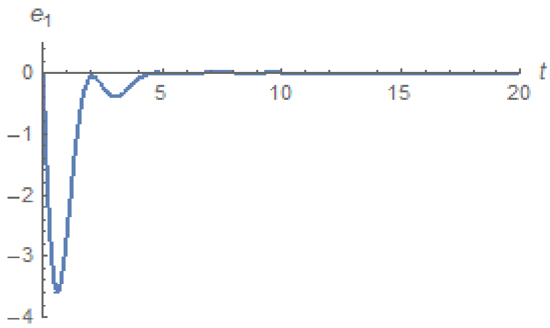

Figure-10

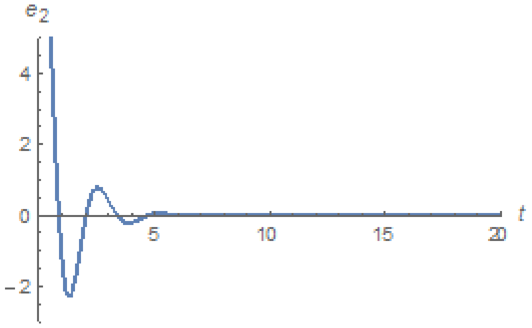

Figure-11

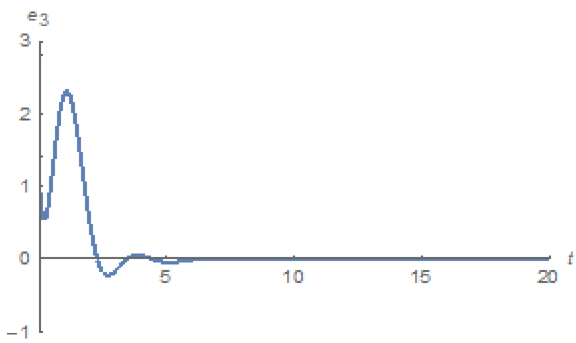

Figure-12

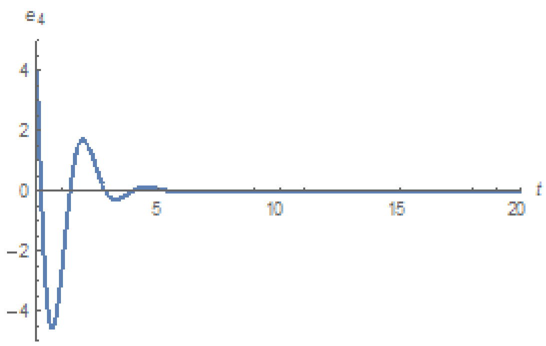

Figure-13

5. CONCLUSION

An investigation on chaos synchronization in the photogravitational magnetic-binaries problem when the bigger primary is a source of radiation including the effect of the gravitational forces of the primaries on the small body, via nonlinear controller based on the Lyapunov stability theory have been made. Here two systems (master and slave) are synchronized and start with deferent initial conditions. This problem may be treated as the design of control laws for chaotic slave system using the known information of the master system so as to ensure that the controlled receiver synchronizes with master system. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time. The results were validated by numerical simulations using Mathematica.

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