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# ON THE GENERALIZED abc - BLOCK EDGE TRANSFORMATION GRAPHS

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***ABSTRACT***

*Given a graph $G$ with vertex set $V(G)$, edge set $E(G)$ and block set $U(G)$, let $\overline{G}$ be the complement, $L(G)$ the line graph and $B(G)$ the block graph of $G$. Let $G^0$ be the graph with $V(G^0) = V(G)$ and with no edges, $G^1$ the complete graph with $V(G^1) = V(G)$, $G^+ = G$, and $G^- = \overline{G}$. Let $bq(G)$ ($\overline{bq}(G)$) be the graph whose vertices can be put in one to one correspondence with the set of edges and blocks of G in such a way that two vertices of $bq(G)$ ($resp.$, $\overline{bq}(G)$) are adjacent if and only if one corresponds to a block $B$ of $G$ and the other to an edge $e$ of $G$ and $e$ is in (resp., is not in) $B$. Given $a, b, c \in \{0,1,+,-\}$, the abc - block edge transformation graph $Q^{abc}(G)$ of $G$ is the graph with vertex set $V(Q^{abc}(G)) = E(G) \cup U(G)$ and the edge set $E(Q^{abc}(G)) = E((L(G))^a) \cup E((B(G))^b) \cup E(H)$ where $H = bq(G)$ if $c = +$, $H = \overline{bq}(G)$ if $c = -$, $H$ is the graph with $V(H) = E(G) \cup U(G)$ and with no edges if $c = 0$, $H$ is the complete bipartite graph with parts $E(G)$ and $U(G)$ if $c = 1$. In this paper, we investigate some basic properties such as order, size, vertex degree and connectedness of the these generalized abc - block edge transformation graphs $Q^{abc}(G)$.*

***2010 Mathematics Subject Classification:*** *05C12.*

***Keywords:*** *line graph, block graph, abc - block edge transformation graphs.*

## 1. INTRODUCTION

All the graphs considered here are finite, undirected without loops or multiple edges. We refer to [3] or [4] for unexplained terminology and notation. A *block* of a graph is a maximal nonseparable subgraph. Let $G = (V, E)$ be a graph with block set $U(G)=\{B_i;\ B_i$ is a block of $G\}$, $|V(G)| = n$, $|E(G)| = m$ and $|U(G)| = r$. The *degree* of a vertex $v_i$ in $G$ is the number of edges incident to $v_i$ and is denoted by $d_i = deg(v_i)$. As usual, $K_n$ be the complete graph of order $n$, $K_{p,q}$ the complete bipartite graph, $S_{p,q}$ the double star and $d_G(v)$ the degree of a vertex $v$ in $G$. If a block $B \in U(G)$ with the edge set $\{e_1, e_2, \ldots, e_s; s \geq 1\}$, then we say that the edge $e_i$ and block $B$ are incident with each other, where $1 \leq i \leq s$. If two distinct blocks are incident with a common cutvertex, then they are adjacent blocks. The *degree of a block* $B$ in $G$, denoted by $d_G(B)$, is the number of blocks adjacent to $B$ in $G$. We denote the number of edges incident with $B$ in $G$ by $D_G(B)$. The *line graph* $L(G)$ of a graph $G$ is the graph with vertex set as the edge set of $G$ and two vertices of $L(G)$ are adjacent whenever the corresponding edges in $G$ have a vertex in common. The complement of line graph is a jump graph [2]. Let $bq(G)$ ($\overline{bq}(G)$) be the graph whose vertices can be put in one to one correspondence with the set of edges and blocks of G in such a way that two vertices of $bq(G)$ ($resp.$, $\overline{bq}(G)$) are adjacent if and only if one corresponds to a block $B$ of $G$ and the other to an edge $e$ of $G$ and $e$ is in (resp., is not in) $B$.

## 2. GENERALIZED abc - BLOCK EDGE TRANSFORMATION GRAPHS

Inspired by the definition of total transformation graphs [5] and block-transformation graphs [1], we introduce the graph valued functions namely generalized $abc$ - block edge transformation graphs.

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For a graph $G = (V, E)$, let $G^0$ be the graph with $V(G^0) = V(G)$ and with no edges, $G^1$ the complete graph with $V(G^1) = V(G)$, $G^+ = G$, and $G^- = \overline{G}$.

In this paper, we consider some graph valued functions $Q^{abc}: \mathcal{G} \to \mathcal{G}$ from set of graphs $\mathcal{G}$ into $\mathcal{G}$, depending on parameters $a, b, c \in \{0, 1, +, -\}$ and call $Q^{abc}(G)$ the generalized $abc$ - block edge transformation graph of $G$.

**Definition:** Given a graph $G$ with edge set $E(G)$ and block set $U(G)$ and three variables $a, b, c \in \{0, 1, +, -\}$, the generalized abc - block edge transformation graph $Q^{abc}(G)$ of $G$ is the graph with vertex set $V(Q^{abc}(G)) = E(G) \cup U(G)$ and the edge set $E(Q^{abc}(G)) = E((L(G))^a) \cup E((B(G))^b) \cup E(H)$ where

(i) $H = bq(G)$ if $c = +$.
(ii) $H = \overline{bq}(G)$ if $c = -$.
(iii) $H$ is the graph with $V(H) = E(G) \cup U(G)$ and with no edges if $c = 0$.
(iv) $H$ is the complete bipartite graph with parts $E(G)$ and $U(G)$ if $c = 1$.

Thus we obtain 64 abc - block edge transformation graphs $Q^{abc}(G)$. Here note that $Q^{00+}(G) = bq(G)$ and $Q^{00-}(G) = \overline{bq}(G)$.

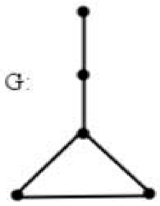


**Figure-1:** Graph G.

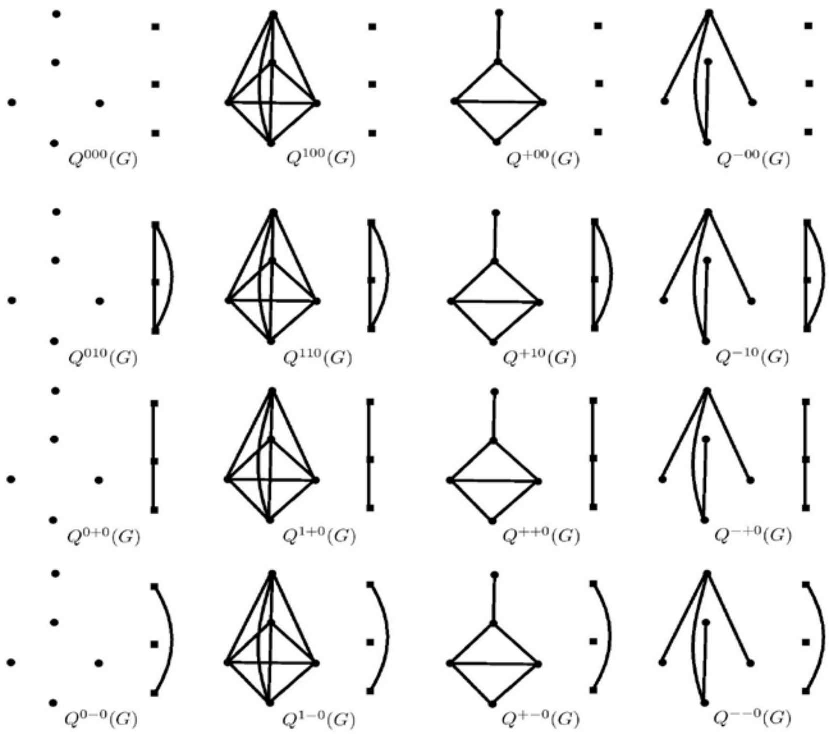


**Figure-2:** abc - block edge transformation graphs when $c = 0$.

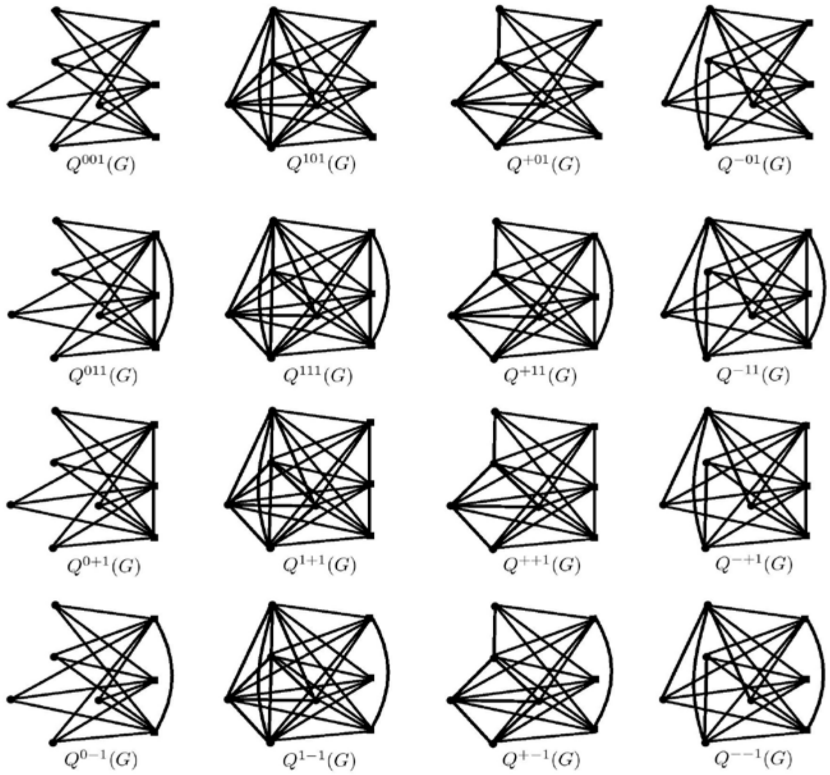


**Figure-3:** abc - block edge transformation graphs when $c = 1$.

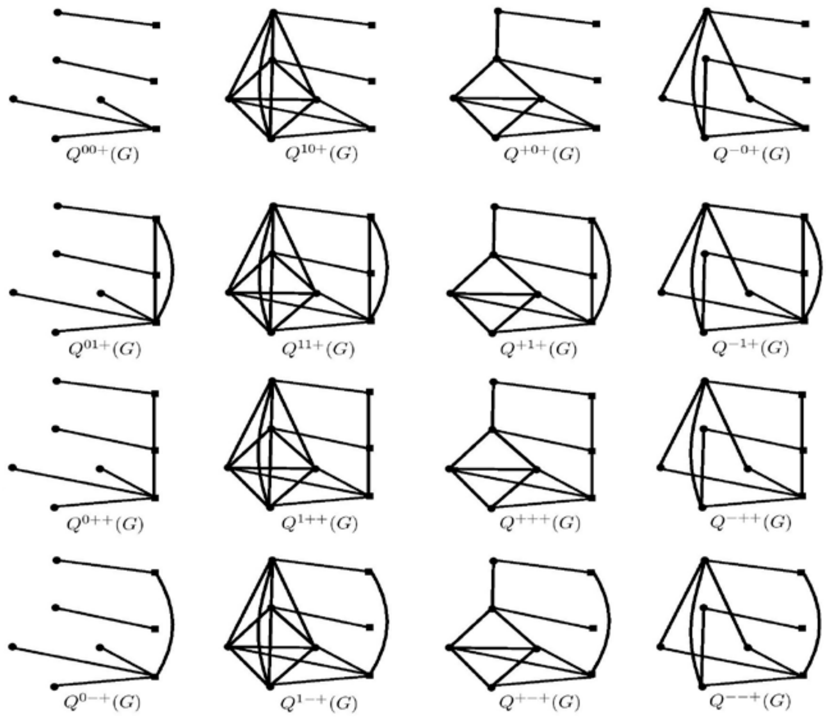


**Figure-4:** abc - block edge transformation graphs when $c = +$.

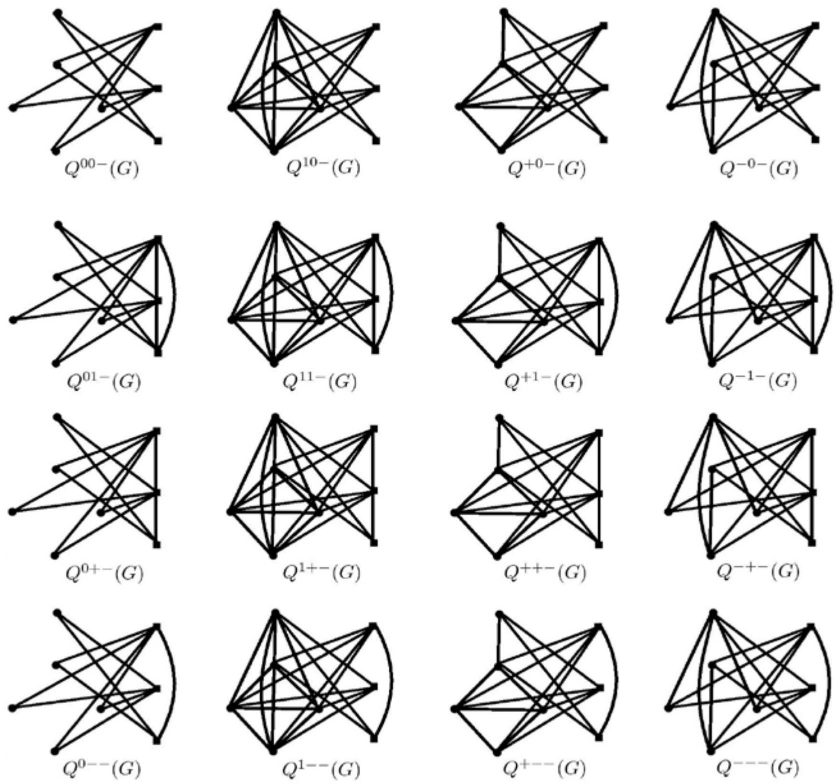


**Figure-5:** abc - block edge transformation graphs when $c = -$.

A graph $G$ and all its 64 abc - block edge transformation graphs are shown in Figures 1-5. The vertex $e_i'$ of $Q^{abc}(G)$ corresponding to edge $e_i$ of $G$ and is refereed as edge-vertex. The vertex $B_i'$ of $Q^{abc}(G)$ corresponding to block $B_i$ of G and is refereed as block-vertex. In Figure 2 to 5, the edge-vertices are denoted by circles and block-vertices are by squares.

The following remarks will be useful in the proof of our results.

**Remark 2.1:**
(i) $L(G)$ *is an induced subgraph of* $Q^{+bc}(G)$.
(ii) $\overline{L(G)}$ is an induced subgraph of $Q^{-bc}(G)$.
(iii) $K_m$ is an induced subgraph of $Q^{1bc}(G)$.

**Remark 2.2:**
(i) $B(G)$ *is an induced subgraph of* $Q^{a+c}(G)$.
(ii) $\overline{B(G)}$ is an induced subgraph of $Q^{a-c}(G)$.
(iii) $K_r$ is an induced subgraph of $Q^{a1c}(G)$.

**Remark 2.3:**
(i) $bq(G)$ *is a spanning subgraph of* $Q^{ab+}(G)$.
(ii) $\overline{bq}(G)$ is a spanning subgraph of $Q^{ab-}(G)$.
(iii) $K_{m,r}$ is a spanning subgraph of $Q^{ab1}(G)$.

**Theorem 2.1 [6]:** *Let* $G$ *be a graph of size* $q \geq 1$. *Then* $\overline{L(G)}$ *is connected if and only if* $G$ *contains no edge that is adjacent to every other edge of* $G$ *unless* $G = K_4$ *or* $C_4$.

Since abc - block edge transformation graphs $Q^{abc}(G)$ are defined on the edge set and block set of a graph $G$. Isolated vertices of $G$ (if $G$ has) play no rule in $Q^{abc}(G)$, we assume that the graph $G$ under consideration is nonempty and has no isolated vertices. In this paper, we investigate some basic properties such as order, size, vertex degree and connectedness of the these generalized abc - block edge transformation graphs $Q^{abc}(G)$.

## 3. ORDER, SIZE AND VERTEX DEGREE OF $Q^{abc}(G)$

It is shown in [3] that let $b_G(v)$ be the number of blocks to which vertex $v$ belongs in a connected graph $G$. Then the number of blocks of $G$ is given by $r = b(G) = 1 - n + \sum_{v\in V(G)} b_G(v)$.

**Theorem 3.1:** *Let $G$ be an $(n, m)$ connected graph with $r$ blocks and let $b_G(v)$ be the number of blocks to which vertex $v$ belongs in $G$. Then*

(i) The order of $Q^{abc}(G) = m + r$.

(ii) The size of $Q^{abc}(G)$= $\begin{cases} |E((L(G))^a)| + |E((B(G))^b)| & if\ \ c = 0. \\ |E((L(G))^a)| + |E((B(G))^b)| + mr & if\ \ c = 1. \\ |E((L(G))^a)| + |E((B(G))^b)| + m & if\ \ c = +. \\ |E((L(G))^a)| + |E((B(G))^b)| + mr - m & if\ \ c = -. \end{cases}$

where

$$E((L(G))^a) = \begin{cases} 0 & if\ \ a = 0. \\ \frac{m(m-1)}{2} & if\ \ a = 1. \\ -m + \frac{1}{2}\sum_{v\in V(G)} d_G^2(v) & if\ \ a = +. \\ \frac{m(m+1)}{2} - \frac{1}{2}\sum_{v\in V(G)} d_G^2(v) & if\ \ a = -. \end{cases}$$

and

$$E((B(G))^b) = \begin{cases} 0 & if\ \ b = 0. \\ \frac{r(r-1)}{2} & if\ \ b = 1. \\ \sum_{v\in V(G)} \frac{b_G(v)(b_G(v)-1)}{2} & if\ \ b = +. \\ \frac{r(r-1)}{2} - \sum_{v\in V(G)} \frac{b_G(v)(b_G(v)-1)}{2} & if\ \ b = -. \end{cases}$$

**Theorem 3.2:** *Let $G$ be an $(n, m)$-graph with $r$ blocks. Then the degree of edge-vertex $e'$ $(e = uv$ in $G)$ and block-vertex $B'$ in $Q^{abc}(G)$ when $c = 0$ are*

(i) $d_{Q^{ab0}(G)}(e') = \begin{cases} 0 & if\ \ a = 0\ \&\ b \in \{0,1,+,-\}. \\ m - 1 & if\ \ a = 1\ \&\ b \in \{0,1,+,-\}. \\ d_G(u) + d_G(v) - 2 & if\ \ a = +\ \&\ b \in \{0,1,+,-\}. \\ m + 1 - d_G(u) - d_G(v) & if\ \ a = -\ \&\ b \in \{0,1,+,-\}. \end{cases}$

(ii) $d_{Q^{ab0}(G)}(B') = \begin{cases} 0 & if\ \ b = 0\ \&\ a \in \{0,1,+,-\}. \\ r - 1 & if\ \ b = 1\ \&\ a \in \{0,1,+,-\}. \\ d_G(B) & if\ \ b = +\ \&\ a \in \{0,1,+,-\}. \\ r - 1 - d_G(B) & if\ \ b = -\ \&\ a \in \{0,1,+,-\}. \end{cases}$

**Theorem 3.3:** *Let $G$ be an $(n, m)$-graph with $r$ blocks. Then the degree of edge-vertex $e'$ $(e = uv$ in $G)$ and block-vertex $B'$ in $Q^{abc}(G)$ when $c = 1$ are*

(i) $d_{Q^{ab1}(G)}(e') = \begin{cases} r & if\ \ a = 0\ \&\ b \in \{0,1,+,-\}. \\ r + m - 1 & if\ \ a = 1\ \&\ b \in \{0,1,+,-\}. \\ r + d_G(u) + d_G(v) - 2 & if\ \ a = +\ \&\ b \in \{0,1,+,-\}. \\ r + m + 1 - d_G(u) - d_G(v) & if\ \ a = -\ \&\ b \in \{0,1,+,-\}. \end{cases}$

(ii) $d_{Q^{ab1}(G)}(B') = \begin{cases} m & if\ \ b = 0\ \&\ a \in \{0,1,+,-\}. \\ m + r - 1 & if\ \ b = 1\ \&\ a \in \{0,1,+,-\}. \\ m + d_G(B) & if\ \ b = +\ \&\ a \in \{0,1,+,-\}. \\ m + r - 1 - d_G(B) & if\ \ b = -\ \&\ a \in \{0,1,+,-\}. \end{cases}$

**Theorem 3.4:** *Let $G$ be an $(n, m)$-graph with $r$ blocks. Then the degree of edge-vertex $e'$ $(e = uv$ in $G)$ and block-vertex $B'$ in $Q^{abc}(G)$ when $c = +$ are*

(i) $d_{Q^{ab+}(G)}(e') = \begin{cases} 1 & if\ \ a = 0\ \&\ b \in \{0,1,+,-\}. \\ m & if\ \ a = 1\ \&\ b \in \{0,1,+,-\}. \\ d_G(u) + d_G(v) - 1 & if\ \ a = +\ \&\ b \in \{0,1,+,-\}. \\ m + 2 - d_G(u) - d_G(v) & if\ \ a = -\ \&\ b \in \{0,1,+,-\}. \end{cases}$

 

(ii) $d_{Q^{ab+}(G)}(B') = \begin{cases} D_G(B) & if\ \ b = 0\ \ \&\ \ a \in \{0,1,+,-\}. \\ D_G(B) + r - 1 & if\ \ b = 1\ \ \&\ \ a \in \{0,1,+,-\}. \\ D_G(B) + d_G(B) & if\ \ b = +\ \ \&\ \ a \in \{0,1,+,-\}. \\ D_G(B) + r - 1 - d_G(B) & if\ \ b = -\ \ \&\ \ a \in \{0,1,+,-\}. \end{cases}$

**Theorem 3.5:** *Let $G$ be an $(n, m)$-graph with $r$ blocks. Then the degree of edge-vertex $e'$ $(e = uv$ in $G)$ and block-vertex $B'$ in $Q^{abc}(G)$ when $c = -$ are*

(i) $d_{Q^{ab-}(G)}(e') = \begin{cases} r - 1 & if\ \ a = 0\ \ \&\ \ b \in \{0,1,+,-\}. \\ m + r - 2 & if\ \ a = 1\ \ \&\ \ b \in \{0,1,+,-\}. \\ d_G(u) + d_G(v) + r - 3 & if\ \ a = +\ \ \&\ \ b \in \{0,1,+,-\}. \\ m + r - d_G(u) - d_G(v) & if\ \ a = -\ \ \&\ \ b \in \{0,1,+,-\}. \end{cases}$

(ii) $d_{Q^{ab-}(G)}(B') = \begin{cases} m - D_G(B) & if\ \ b = 0\ \ \&\ \ a \in \{0,1,+,-\}. \\ m + r - D_G(B) - 1 & if\ \ b = 1\ \ \&\ \ a \in \{0,1,+,-\}. \\ d_G(B) + m - D_G(B) & if\ \ b = +\ \ \&\ \ a \in \{0,1,+,-\}. \\ m + r - 1 - D_G(B) - d_G(B) & if\ \ b = -\ \ \&\ \ a \in \{0,1,+,-\}. \end{cases}$

## 4. CONNECTEDNESS OF $Q^{abc}(G)$

The first theorem is well-known.

**Theorem 4.1:** *For a given graph $G$, $Q^{ab0}(G)$ is not connected.*

**Theorem 4.2:** *For a given graph $G$, $Q^{ab1}(G)$ is connected.*

**Proof:** The result follows from the fact of Remark 2.3 (iii) i. e., $K_{m,r}$ is a spanning subgraph of $Q^{ab1}(G)$ with parts $E(G)$ and $U(G)$. Therefore $Q^{ab1}(G)$ is connected
When $c = +$, we have the following theorems:

**Theorem 4.3:** *For a given graph $G$, $Q^{00+}(G)$ is connected if and only if $G$ is a block.*

**Proof:** Suppose $G$ is a block with $m$ edges. Then $Q^{00+}(G) = K_{1,m}$ and which is connected.

Conversely, if $G$ has at least two blocks, then $Q^{00+}(G)$ has at least two disjoint stars. Therefore $Q^{00+}(G)$ is disconnected, a contradiction.

**Theorem 4.4:** *For a given graph $G$, $Q^{1b+}(G)$ is connected.*

**Proof:** From Remark 2.1 (iii), we have $K_m$ is an induced subgraph of $Q^{1b+}(G)$ with vertex set $E(G)$ and each block-vertex $B'$ is adjacent to at least one edge-vertex $e'$ where $e$ is incident with a block $B$ in $G$. Therefore $Q^{1b+}(G)$ is connected.

**Theorem 4.5:** *For a given graph $G$, $Q^{+0+}(G)$ is connected if and only if $G$ is connected.*

**Proof:** Suppose $G$ is connected. Then $L(G)$ is connected. By Remark 2.1 (i), $L(G)$ is a connected induced subgraph of $Q^{+0+}(G)$ and each block-vertex $B'$ is adjacent to at least one edge-vertex $e'$ where $e$ is incident with a block $B$ in $G$. Therefore $Q^{+0+}(G)$ is connected.

Conversely, suppose $Q^{+0+}(G)$ is connected. If $G$ is a disconnected graph with at least two components $G_1$ and $G_2$, then $Q^{+0+}(G) = Q^{+0+}(G_1) \cup Q^{+0+}(G_2)$ is disconnected, a contradiction.

**Theorem 4.6:** *For a given graph $G$, $Q^{-0+}(G)$ is connected if and only if $G$ contains no block $K_2$ that is adjacent to every other edge of $G$.*

**Proof:** Suppose a graph $G$ contains no block $K_2$ that is adjacent to every other edge of $G$. If $G$ is a block, then $Q^{-0+}(G) = \overline{L(G)} + K_1$ is connected. If $G$ has more than one block, then we consider the following two cases:

**Case-1:** If $G$ contains no edge that is adjacent to every other edge of $G$, then by Remark 2.1 and Theorem 2.1, $\overline{L(G)}$ is a connected subgraph of $Q^{-0+}(G)$, and in $Q^{-0+}(G)$, each block-vertex $B_i'$ is adjacent to at least one edge-vertex $e_j'$, where $e_j$ is incident with $B_i$ in $G$. Thus $Q^{-0+}(G)$ is connected.

**Case-2:** If $G$ contains an edge $e$ that is adjacent to every other edge of $G$, then clearly $e$ is incident with a block $B$ of size more than 2. And $Q^{-0+}(G-e)$ is a connected subgraph of $Q^{-0+}(G)$ and $e', B', e_1'$ is a path in $Q^{-0+}(G)$, where $e_1$ is incident with $B$, and each block-vertex $B_i'$ in $Q^{-0+}(G)$ is adjacent to at least one edge-vertex $e_j'$, where $e_j$ is incident with $B_i$ in $G$. Hence $Q^{-0+}(G)$ is connected.

Conversely, suppose $Q^{-0+}(G)$ is connected. Assume $G$ contains a block $K_2$, say $e$, that is adjacent to every other edge of $G$, then it is easy to see that $Q^{-0+}(G) = Q^{-0+}(G-e) \cup K_2$ is disconnected, a contradiction.

**Theorem 4.7:** *For a given graph $G$, $Q^{a1+}(G)$ is connected.*

**Proof:** From Remark 2.2 (iii), we have $K_r$ is an induced subgraph of $Q^{a1+}(G)$ with vertex set $U(G)$ and each edge-vertex $e'$ is adjacent to exactly one block-vertex $B'$ where $e$ is incident with a block $B$ in $G$. Therefore $Q^{a1+}(G)$ is connected.

**Theorem 4.8:** *For a given graph $G$, $Q^{0++}(G)$ is connected if and only if $G$ is connected..*

**Proof:** Suppose $G$ is connected. Then $B(G)$ is connected. By Remark 2.2 (i), $B(G)$ is a connected induced subgraph of $Q^{0++}(G)$ and each edge-vertex $e'$ is adjacent to exactly one block-vertex $B'$ where $e$ is incident with a block $B$ in $G$. Therefore $Q^{0++}(G)$ is connected.

Conversely, suppose $Q^{0++}(G)$ is connected. If $G$ is disconnected graph with at least two component $G_1$ and $G_2$, then $Q^{0++}(G) = Q^{0++}(G_1) \cup Q^{0++}(G_2)$ is disconnected, a contradiction.

**Theorem 4.9:** *For a given graph $G$, $Q^{+++}(G)$ is connected if and only if $G$ is connected.*

**Proof:** Suppose $G$ is connected. Then by Theorem 4.8, $Q^{0++}(G)$ is connected, and we have $Q^{+++}(G)$ is spanning subgraph of $Q^{+++}(G)$. Therefore, $Q^{+++}(G)$ is connected.

Conversely, suppose $Q^{+++}(G)$ is connected. If $G$ is disconnected graph with at least two component $G_1$ and $G_2$, then $Q^{+++}(G) = Q^{+++}(G_1) \cup Q^{+++}(G_2)$ is disconnected, a contradiction.

**Theorem 4.10:** *$Q^{-++}(G)$ is connected for any graph $G$.*

**Proof:** If $G$ is connected, then by Remark 2.2 (i), $B(G)$ is a connected induced subgraph of $Q^{-++}(G)$, and each edge-vertex $e_i'$ in $Q^{-++}(G)$ is adjacent to exactly one block-vertex $B_x'$, where $B_x$ is incident with $e_i$ in $G$. Thus $Q^{-++}(G)$ is connected.

If $G$ is disconnected, then by Remark 2.1 (ii) and Theorem 2.1, $\overline{L(G)}$ is a connected induced subgraph of $Q^{-++}(G)$, and each block-vertex $B_x'$ in $Q^{-++}(G)$ is adjacent to at least one edge-vertex $e_i'$, where $e_i$ is incident with $B_x$ in $G$. Thus $Q^{-++}(G)$ is connected.

**Theorem 4.11:** *For a given graph $G$, $Q^{0-+}(G)$ is connected if and only if $G$ contains no block that is adjacent to every other block of $G$.*

**Proof:** Suppose a graph $G$ contains no block that is adjacent to every other block of $G$. Then $\overline{B(G)}$ is a connected induced subgraph of $Q^{0-+}(G)$ with vertex set $U(G)$, and each edge-vertex $e'$ is adjacent to exactly one block-vertex $B'$ where $e$ is incident with a block $B$ in $G$. Therefore $Q^{0-+}(G)$ is connected.

Conversely, suppose $Q^{0-+}(G)$ is connected. Assume $G$ contains a block $B$ that is adjacent to every other blocks of $G$, and $B$ is incident with $e_1, e_2, \ldots, e_s$ edges. Then it is easy to see that $Q^{0-+}(G) = Q^{0-+}(G - \{e_1, e_2, \ldots, e_s\}) \cup K_{1,s}$ is disconnected, a contradiction.

**Theorem 4.12:** *$Q^{+-+}(G)$ is connected for any graph $G$.*

**Proof:** If $G$ is connected, then by Remark 2.1 (i), $L(G)$ is a connected induced subgraph of $Q^{+-+}(G)$, and each block-vertex $B_x'$ in $Q^{+-+}(G)$ is adjacent to at least one edge-vertex $e_i'$, where $e_i$ is incident with $B_x$ in $G$. Thus $Q^{+-+}(G)$ is connected.

If $G$ is disconnected, then by Remark 2.2 (ii), $\overline{B(G)}$ is a connected induced subgraph of $Q^{+-+}(G)$, and each edge-vertex $e_i'$ in $G^{+-+}$ is adjacent to exactly one block-vertex $B_x'$, where $B_x$ is incident with $e_i$ in $G$. Thus $Q^{+-+}(G)$ is connected.

**Theorem 4.13:** *For a given graph $G$, $Q^{--+}(G)$ is connected if and only if $G$ contains no block $K_2$ that is adjacent to every other edge of $G$.*

**Proof:** Suppose a graph $G$ contains no block $K_2$ that is adjacent to every other edge of $G$. Then by Theorem 4.6, $Q^{-0+}(G)$ is connected, and we have $Q^{-0+}(G)$ is a spanning subgraph of $Q^{--+}(G)$. Therefore, $Q^{--+}(G)$ is connected. Conversely, suppose $Q^{--+}(G)$ is connected. Assume $G$ contains a block $K_2$, say $e$, that is adjacent to every other edge of $G$. Then it is easy to see that $Q^{--+}(G) = Q^{--+}(G-e) \cup K_2$ is disconnected, a contradiction.

When $c = -$, we have the following theorems:

**Theorem 4.14:** *For a given graph $G$, $Q^{00-}(G)$ is connected if and only if $G$ has at least three blocks.*

**Proof:** Suppose $G$ contains at least three blocks. Then each edge-vertex is adjacent to at least two block-vertex in $Q^{00-}(G)$. Therefore it is sufficient to prove every pair of edge-vertices are connected. Let $e'_1$ and $e'_2$ be the edge-vertices of $Q^{00-}(G)$. Then there exist a block $B'$ which is not incident with $e_1$ and $e_2$ in $G$ such that $e'_1$ and $e'_2$ are connected through $B'$ in $Q^{00-}(G)$. Therefore, every pair of vertices in $Q^{00-}(G)$ are connected. Hence $Q^{00-}(G)$ is connected.

Conversely, suppose $Q^{00-}(G)$ is connected. Assume $G$ is a block. Then $Q^{00-}(G) = (m+1)K_1$ is disconnected, a contradiction. Assume $G$ has two blocks $B_1$ and $B_2$ with $x$ and $y$ number of incident edges respectively. Then $Q^{00-}(G) = K_{1,x} \cup K_{1,y}$ is disconnected, a contradiction.

**Theorem 4.15:** *For a given graph $G$, $Q^{1b-}(G)$ is connected if and only if $G$ is not a block.*

**Proof:** Suppose $G$ is not a block. By Remark 2.1 (iii), we have $K_m$ is an induced subgraph of $Q^{1b-}(G)$ with vertex set $E(G)$, and each block-vertex $B'$ is adjacent with at least one edge-vertex $e'$ where $e$ is not incident with block $B$ in $G$. Therefore $Q^{1b-}(G)$ is connected.

Conversely, suppose $Q^{1b-}(G)$ is connected. Assume $G$ is a block. Then $Q^{1b-}(G) = K_m \cup K_1$ is disconnected, a contradiction.

**Theorem 4.16:** *For a given graph $G$, $Q^{+0-}(G)$ is connected if and only if $G$ is neither a block nor a union of two blocks.*

**Proof:** Suppose $G$ is neither a block nor a union of two blocks. Then we consider the following cases:

**Case-1:** Suppose $G$ is connected. Then it has at least two blocks. Hence by Remark 2.1 (i), $L(G)$ is a connected subgraph of $Q^{+0-}(G)$, and also each block-vertex $B'_i$ in $Q^{+0-}(G)$ is adjacent to at least one edge-vertex $e'_j$, where $e_j$ is not incident with $B_i$ in $G$. Thus $Q^{+0-}(G)$ is connected.

**Case-2:** Suppose $G$ is disconnected. Then it has at least three blocks. We see that in $Q^{+0-}(G)$, each block-vertex $B'_i$ is adjacent at least two edge-vertices $e'_j$, where $e_j$ is not incident with $B_i$ in $G$, and each edge-vertex $e'_j$ is adjacent to edge-vertex $e'_k$ and at least two block-vertices $B'_i$ in $Q^{+0-}(G)$, where $e_k$ is adjacent to $e_j$, and $B_i$ is not incident with $e_j$ in $G$.

Since in such a case, there is a path between any two vertices of $Q^{+0-}(G)$. Hence $Q^{+0-}(G)$ is connected.

Conversely, suppose $Q^{+0-}(G)$ is connected. If $G$ is a block, then $Q^{+0-}(G) = L(G) \cup K_1$ is disconnected, a contradiction. If $G = B_1 \cup B_2$ is a union of two blocks, then $Q^{+0-}(G)$ is a disconnected graph having two components namely $L(B_1) + K_1$ and $L(B_1) + K_1$, a contradiction.

**Theorem 4.17:** *For a given graph $G$, $Q^{-0-}(G)$ is connected if and only if $G \neq P_3$ is not a block.*

**Proof:** Suppose $G \neq P_3$ is not a block. We consider the following two cases:

**Case-1:** Suppose $G$ contains no edge that is adjacent to every other edge of $G$. Then by Remark 2.1 (ii) and Theorem 2.1, $\overline{L(G)}$ is a connected subgraph of $Q^{-0-}(G)$, and each block-vertex $B'_i$ is adjacent to at least one edge-vertex $e'_j$ in $Q^{-0-}(G)$, where $e_j$ is not incident with $B_i$ in $G$. Thus $Q^{-0-}(G)$ is connected.

 

**Case-2:** Suppose $G$ contains an edge $e$ that is adjacent to all other edge of $G$. Then by definition of $Q^{-0-}(G)$, each edge-vertex $e_i'$ is adjacent to edge-vertex $e_k'$ and at least one block-vertex $B_j'$, where $B_j$ is not incident with $e_i$, and $e_k$ is not adjacent to $e_i$ in G. And also each block-vertex $B_j'$ is adjacent to at least one edge-vertex $e_i'$, where $e_i$ is not incident with $B_j$ in $G$. Hence there is a path between any two vertices of $Q^{-0-}(G)$. Therefore $Q^{-0-}(G)$ is connected.

Conversely, suppose $Q^{-0-}(G)$ is connected. If $G$ is a block, then $Q^{-0-}(G) = \overline{L(G)} \cup K_1$ is disconnected, a contradiction. If $G = P_3$, then $Q^{-0-}(G) = 2K_2$ is disconnected, a contradiction.

**Theorem 4.18:** *For a given graph $G$, $Q^{a1-}(G)$ is connected if and only if $G$ is not a block.*

**Proof:** Suppose $G$ is not a block. By Remark 2.2 (iii), we have $K_r$ is an induced subgraph of $Q^{a1-}(G)$ with vertex set $U(G)$, and each edge-vertex $e'$ is adjacent to at least one block-vertex $B'$ where $e$ is not incident with block $B$ in $G$. Therefore, $Q^{a1-}(G)$ is connected.

Conversely, suppose $Q^{a1-}(G)$ is connected. Assume $G$ is a block. Then $Q^{a1-}(G) = (L(G))^a \cup K_1$ is disconnected, a contradiction.

**Theorem 4.19:** *For a given graph $G$, $Q^{0+-}(G)$ is connected if and only if $G$ is neither a block nor a union of two blocks.*

**Proof:** Suppose $G$ is neither a block nor a union of two blocks. Then we consider the following cases:

**Case-1:** Suppose $G$ is connected. Then it has at least two blocks. By Remark 2.2 (i), $B(G)$ is a connected induced subgraph of $Q^{0+-}(G)$ with vertex set $U(G)$, and also each edge-vertex $e_i'$ in $Q^{0+-}(G)$ is adjacent to at least one block-vertex $B_j'$, where $e_i$ is not incident with $B_j$ in $G$. Therefore $Q^{0+-}(G)$ is connected.

**Case-2:** Suppose $G$ is disconnected. Then it has at least three blocks and we have $Q^{00-}(G)$ is a spanning subgraph of $Q^{0+-}(G)$. Therefore by Theorem 4.14, $Q^{0+-}(G)$ is connected.

Conversely, suppose $Q^{0+-}(G)$ is connected. If $G$ is a block, then $Q^{0+-}(G) = (m+1)K_1$ is disconnected, a contradiction. If $G = B_1 \cup B_2$ is not a union of blocks, where $B_1$ and $B_2$ are blocks incident with $x$ and $y$ number of edges respectively, then $Q^{0+-}(G) = K_{1,x} \cup K_{1,y}$ is disconnected, a contradiction.

**Theorem 4.20:** *For a given graph $G$, $G^{++-}$ is connected if and only if $G$ is neither a block nor a union of two blocks.*

**Proof:** Suppose $G$ is neither a block nor a union of two blocks. Then by Theorem 4.19, $Q^{0+-}(G)$ is connected, and we have $Q^{0+-}(G)$ is spanning subgraph of $Q^{++-}(G)$. Therefore, $Q^{++-}(G)$ is connected.

Conversely, suppose $Q^{++-}(G)$ is connected. If $G$ is a block, then $Q^{++-}(G) = L(G) \cup K_1$ is disconnected, a contradiction. If $G = B_1 \cup B_2$ is not a union of blocks, then $Q^{++-}(G) = (L(B_1) + K_1) \cup (L(B_2) + K_1)$ is disconnected, a contradiction.

**Theorem 4.21:** *For a given graph $G$, $Q^{-+-}(G)$ is connected if and only if $G$ is not a block.*

**Proof:** Suppose $G \neq P_3$ is not a block. Then by Theorem 4.17, $Q^{-0-}(G)$ is connected, and we have $Q^{-0-}(G)$ is spanning subgraph of $Q^{-+-}(G)$. Therefore, $Q^{-+-}(G)$ is connected. If $G = P_3$, then $Q^{-+-}(G) = P_4$ is connected.
Conversely, if $G$ is a block, then $G^{-+-} = \overline{L(G)} \cup K_1$ is disconnected, a contradiction.

**Theorem 4.22:** *For a given graph $G$, $Q^{0--}(G)$ is connected if and only if $G$ is not a connected graph with one or two blocks.*

**Proof:** Suppose $G$ is not a connected graph with one or two blocks. We consider the following cases:

**Case-1:** If $G$ is a connected graph. Then $G$ contains at least three blocks and we have $Q^{00-}(G)$ is a spanning subgraph of $Q^{0--}(G)$. Hence by Theorem 4.14, $Q^{0--}(G)$ is connected.

**Case-2:** If $G$ is not a connected graph, then $G$ contains at least two blocks. Suppose $G$ contains at least three blocks. Then result is oblivious by Theorem 4.14. If $G = B_i \cup B_j$ where $B_i$ and $B_j$ are blocks with $x$ and $y$ number of incident edges respectively, then $Q^{0--}(G) = S_{x,y}$ is connected.

 

Conversely, suppose $Q^{0--}(G)$ is connected. Assume $G$ is a block. Then $Q^{0--}(G) = \overline{B(G)} \cup K_1$ is disconnected, a contradiction. Assume $G$ is a connected graph with two blocks $B_1$ and $B_2$ having $x$ and $y$ number of incident edges respectively, then $Q^{0--}(G) = K_{1,x} \cup K_{1,y}$ is disconnected, a contradiction.

**Theorem 4.23:** *For a given graph* $G$, $Q^{+--}(G)$ *is connected if and only if* $G$ *is not a block.*

**Proof:** Suppose $G$ is not a block. Then by Theorem 4.22, $Q^{0--}(G)$ is connected, and we have $Q^{0--}(G)$ is spanning subgraph of $Q^{+--}(G)$. Therefore, $Q^{+--}(G)$ is connected. If $G$ is connected graph with two blocks, then $Q^{+--}(G)$ is connected.

Converse is obvious.

**Theorem 4.24:** *For a given graph* $G$, $Q^{---}(G)$ *is connected if and only if* $G \neq P_3$ *is not a block.*

**Proof:** Suppose $G \neq P_3$ is not a block. Then by Theorem 4.17, $Q^{-0-}(G)$ is connected, and we have $Q^{0--}(G)$ is spanning subgraph of $Q^{---}(G)$. Therefore, $Q^{---}(G)$ is connected.

Conversely, suppose $Q^{---}(G)$ is connected. If $G$ is a block, then $Q^{---}(G) = \overline{L(G)} \cup K_1$ is disconnected, a contradiction. If $G = P_3$, then $Q^{---}(G) = 2K_2$ is disconnected, a contradiction.

## 5. CONCLUSION

In this paper, we have introduced 64 abc - block edge transformation graphs and studied their order, size, vertex degree and connectedness of these 64 abc-block edge transformation graphs. The study of diameter, traversability, planarity, chromatic number, domination number, spectra, energy and topological indices of these new graphs can be interesting. Characterization of these 64 abc - block edge transformation graphs can be quite challenging, (i.e., to prove that: A graph $G$ is a generalized $abc$ - block edge transformation graph if and only if it is isomorphic to the generalized $abc$ - block edge transformation graph $Q^{abc}(H)$ of some graph $H$).

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