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SAMPLING THEOREM FOR TWO DIMENSIONAL FRACTIONAL FOURIER TRANSFORM

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ABSTRACT

Sampling has a major role in filter, multirate digital signal processing, digital control, speech recognition technology and etc . There are many extensions and other contributions related to the sampling theorem like Band-pass sampling, Non-uniform sampling, Sampling theorems for stochastic processes, Reconstruction from past samples.

1. INTRODUCTION

Fractional calculus is a branch of mathematical analysis that studies the possibility of taking real number powers, real number fractional powers or complex number powers of the differentiation operator.

The Fractional Fourier transform (FRFT) is a mathematical generalization of the ordinary Fourier transform [1]. In 1980, Namias and Wiener's work, introduced the fractional Fourier transform (FRFT) as a way to solve certain classes of ordinary and partial differential equations arising in quantum mechanics from classical quadratic Hamiltonians.

The sampling theorem by C.E. Shannon in 1949 places restrictions on the frequency content of the time function signal, f(t), and can be simply stated as follows: In order to recover the signal function f(t) exactly, it is necessary to sample f(t) at a rate greater than twice its highest frequency component [2]. Due to additional degree of freedom Fractional Fourier transform applicable in security, robustness, payload capacity and visual transparence . Also it is used to communicate or store the watermarked image as erasure code, to reduce communication errors over a network with finite radon transform [3]. Also the Matrix Completion Method for Phase Retrieval is based on Fractional Fourier Transform Magnitudes [4]. Fractional Fourier Transform give best performance in the analysis of timing and carrier frequencyoffset estimation [5]. Sampling theory has applications in Compression, I Image Super-resolution [6, 7].

We developed this work using the work of Pei S.C., Ding J.J, Xia X. Zayed A.I., Garcia A.G [9, 10, 11]. In this work we have derived the sampling expressions for two dimensional fractional Fourier transform of types of functions 0-periodic, $\frac{\pi}{2}-$ compact support. Secondly the expression with $(\alpha-\frac{\pi}{2})$ periodic, $\alpha-$compact support and lastly 0-periodic, $\alpha-$compact support. It is observed that in all the cases, (2k+1, 2l+1) coefficients are required to reconstruct any two dimensional fractional Fourier transform domain of periodic function with compact support.

2. DEFINITION OF TWO DIMENSIONAL FRACTIONAL FOURIER TRANSFORM

2.1 Two Dimensional Fractional Fourier Transform

The two-dimensional fractional Fourier transform with parameters α of $f(x,y)$ denoted by FRFT{ $f(x,y)$} performs a linear operation, given by the integral transform.

$$\text{FRFT}\{f(x,y)\} = F_\alpha(\xi,\eta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)K_{\alpha,\theta}(x,y,\xi,\eta)dxdy \tag{2.1.1}$$

where $K_\alpha(x,y,\xi,\eta) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{1}{2sin\alpha}[(x^2+y^2+\xi^2+\eta^2)cos\alpha-2(x\xi+y\eta)}$

$$= C_{1\alpha}e^{iC_{2\alpha}[(x^2+y^2+\xi^2+\eta^2)cos\alpha-2(x\xi+y\eta)}$$

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where $C_{1\alpha} = \sqrt{\frac{1-icot\alpha}{2\pi}}$, $C_{2\alpha} = \frac{1}{2sin\alpha}$ $0 < \alpha < \frac{\pi}{2}$ (2.1.2)

2.2 Testing function space

An infinitely differentiable complex valued smooth function $\emptyset(x, y)$ on R^n belongs to $E(R^n)$, if for each compact set $I \subset S_{a,b}$, $J \subset S_{c,d}$ where

$S_{a,b} = \{x, y: x, y\epsilon R^n, |x| \le a, |y| \le b, a > 0, b > 0\}$

$\gamma_{E,m,n}[\emptyset(x,y)] = \sup_{x,y\epsilon I}|D_{x,y}^{m,n}\emptyset(x,y)| < \infty$ (2.1.3)

Thus $E(R^n)$ will denote the space of all $\emptyset(x, y)\epsilon E(R^n)$ with compact support contained in $S_{a,,b}$.

Note that the space E is complete and therefore a Frechet space. Moreover, we say that $f(x, y)$ is a two dimensional fractional Fourier transformable if it is a member of E .

2.3 Distributional Two Dimensional Fractional Fourier Transform (FRFT)

The two dimensional distributional Fractional Fourier transform of $f(x, y)\epsilon E^*(R^n)$ can be defined by

$FRFT\{f(x, y)\} = F_\alpha(\xi, \eta) = \langle f(x, y), K_\alpha(x, y, \xi, \eta)\rangle$ (2.1.3)

where, $K_\alpha(x, y, \xi, \eta) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{1}{2sin\alpha}[(x^2+y^2+\xi^2+\eta^2)cos\alpha - 2(x\xi+y\eta)}$

$= C_{1\alpha}e^{iC_{2\alpha}[(x^2+y^2+\xi^2+\eta^2)cos\alpha - 2(x\xi+y\eta)}$

where $C_{1\alpha} = \sqrt{\frac{1-icot\alpha}{2\pi}}$, $C_{2\alpha} = \frac{1}{2sin\alpha}, 0 < \alpha < \frac{\pi}{2},$ (2.3.2)

Right hand side of equation (2.3.1) has a meaning as the application of $f(x, y)\epsilon E^*(R^n)$ to $K_\alpha(x, y, \xi, \eta)\epsilon E.$

It can be extended to the complex space as an entire function given by

$FRFT\{f(x, y)\} = F_\alpha(\xi', \eta') = \langle f(x, y), K_\alpha(x, y, \xi', \eta')\rangle$ (2.3.3)

The right hand side is meaningful because for each $\xi', \eta'\epsilon C^n$, $K_{\alpha,\theta}(x, y, \xi', \eta')\epsilon E$ as a function of x, y.

3. SAMPLING THEOREM

3.1 Fractional Fourier Transform of 0-periodic Function with $\frac{\pi}{2}$ −Compact Support in Fractional Fourier Transform Domain:

If a function $f(x, y)$ is 0-periodic and of highest frequency satisfying the Dirichlet condition, where T is the period then $f(x, y)$ can be reconstructed from its (2k+1,2l+1) samples as per (Brown)

$f(x, y) = \sum_{m=0}^{2k}\sum_{n=0}^{2l} f(m\tau, n\Omega)g_{m,n}(x, y)$ (3.1.1)

$= f(0,0)g_{0,0}(x, y) + f(\tau\Omega)\, g_{1,1}(x, y) + \cdots \ldots \ldots + f(2k\tau, 2l\Omega)g_{2k,2l}(x, y)$

$g_{m,n}(x, y) = \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} \sin c\, \frac{1}{\tau\Omega}(x - m\tau - kT, y - n\Omega - lT$

$= \sum_{h=-k}^{k}\sum_{q=-l}^{l}(2k + 1)^{-1}\,(2l + 1)^{-1}e^{i\left(\frac{2\pi}{T}\right)h(x-m\tau)}e^{i\left(\frac{2\pi}{T}\right)q(y-n\Omega)}$ (3.1.2)

$= \sum_{h=-k}^{k}\sum_{q=-l}^{l}(2k + 1)^{-1}\,(2l + 1)^{-1}e^{i\left(\frac{2\pi hx}{T}\right)}e^{-i\left(\frac{2\pi h}{T}\right)m\tau}e^{i\left(\frac{2\pi qy}{T}\right)}e^{i\left(\frac{2\pi q}{T}\right)n\Omega}$

$= \sum_{h=-k}^{k}\sum_{q=-l}^{l}(2k + 1)^{-1}\,(2l + 1)^{-1}e^{i\left(\frac{2\pi hx}{T}\right)}e^{-i\left(\frac{2\pi h}{2k+1}\right)m}e^{i\left(\frac{2\pi qy}{T}\right)}$, where $\tau = \frac{T}{2k+1}$, $\Omega = \frac{T}{2l+1}$

$= \sum_{h=-k}^{k}\sum_{q=-l}^{l}\{(2k + 1)^{-1}\,(2l + 1)^{-1}e^{-ih\left(\frac{2\pi m}{2k+1}\right)}e^{-iq\left(\frac{2\pi n}{2l+1}\right)m}\}\{e^{i\left(\frac{2\pi}{T}\right)hx}e^{i\left(\frac{2\pi}{T}\right)qy}\}$

$g_{m,n}(x, y) = \sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}e^{i\left(\frac{2\pi}{T}\right)(hx+qy)}$ (3.1.3)

$C_{h,q} = (2k + 1)^{-1}(2l + 1)^{-1}e^{-ih\left(\frac{2\pi m}{2k+1}\right)}e^{-iq\left(\frac{2\pi n}{2l+1}\right)m}$, $h \le k, \quad q \le l$

$= 0$, otherwise

(3.1.1)⇒ $f(x, y) = \sum_{m=0}^{2k}\sum_{n=0}^{2l} f(m\tau, n\Omega)\left\{\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}e^{i\left(\frac{2\pi}{T}\right)(hx+qy)}\right\}$

Taking FRFT on both sides

$$FRFT\{f(x,y)\}(u,v) = \sum_{m=0}^{2k}\sum_{n=0}^{2l} f(m\tau,n\Omega)\{\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}\}\, FRFT\left\{e^{i\left(\frac{2\pi}{T}\right)(hx+qy)}\right\}(u,v)$$

$$= \sum_{m=0}^{2k}\sum_{n=0}^{2l} f(m\tau,n\Omega)\{\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}\}\, 2DFRFT\left\{e^{i\left[\left(\frac{2\pi h}{T}\right)x+\left(\frac{2\pi q}{T}\right)y\right]}\right\}(u,v)$$

$$= \sum_{m=0}^{2k}\sum_{n=0}^{2l} f(m\tau,n\Omega)\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}\sqrt{\frac{2\pi(1-icot\alpha)}{cot\alpha}}\, e^{\frac{i\pi}{2}} e^{\frac{i}{2cot\alpha}}$$
$$e^{\left[\left(\frac{4\pi^2h^2}{T^2}+\frac{4\pi^2q^2}{T^2}\right)-[2cosec\alpha\left(\frac{2\pi h}{T}u+\frac{2\pi q}{T}v\right)+\left(\frac{3+cos2\alpha}{sin2\alpha}\right)](u^2+v^2)\right]}$$

$$= \sqrt{\frac{2\pi(1-icot\alpha)}{cot\alpha}}\, e^{\frac{i\pi}{2}} e^{\frac{i}{2cot\alpha}\{-2cosec\alpha\left(\frac{2\pi h}{T}u+\frac{2\pi q}{T}v\right)+\left(\frac{3+cos2\alpha}{sin2\alpha}\right)](u^2+v^2)\}}$$
$$\sum_{m=0}^{2k}\sum_{n=0}^{2l} f(m\tau,n\Omega)\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}\, e^{\frac{i}{2cot\alpha}\frac{4\pi^2}{T^2}(h^2+q^2)} \tag{3.1.4}$$

Let $\varphi(u,v,\alpha,T) = \sum_{m=0}^{2k}\sum_{n=0}^{2l} f(m\tau,n\Omega)\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}\, e^{\frac{i}{2cot\alpha}\frac{4\pi^2}{T^2}(h^2+q^2)}$ (3.1.5)

$\varphi(u,v,\alpha,T)$ contains the main features of the transformed function

For the periodic function $f_1(T)$, when $T = 1$ & $\tau = \frac{T}{2k+1}$, $\Omega = \frac{T}{2l+1}$

Equation (3.1.4) and (3.1.5) becomes

$$FRFT\{f(x,y)\}(u,v) = \sqrt{\frac{2\pi(1-icot\alpha)}{cot\alpha}}\, e^{\frac{i\pi}{2}} e^{\frac{i}{2cot\alpha}\{-2cosec\alpha\left(\frac{2\pi h}{T}u+\frac{2\pi q}{T}v\right)+\left(\frac{3+cos2\alpha}{sin2\alpha}\right)](u^2+v^2)\}}$$
$$\sum_{m=0}^{2k}\sum_{n=0}^{2l} f\left(\frac{mT}{2k+1},\frac{nT}{2l+1}\right)\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}\, e^{\frac{i}{2cot\alpha}\frac{4\pi^2}{T^2}(h^2+q^2)} \tag{3.1.6}$$

(3.1.5) ⇒

$$\varphi(u,v,\alpha,T) = \sum_{m=0}^{2k}\sum_{n=0}^{2l} f\left(\frac{mT}{2k+1},\frac{nT}{2l+1}\right)\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}\, e^{\frac{i}{2cot\alpha}\frac{4\pi^2}{T^2}(h^2+q^2)}$$

$$\varphi(u,v,\alpha,1) = \sum_{m=0}^{2k}\sum_{n=0}^{2l} f\left(\frac{m}{2k+1},\frac{n}{2l+1}\right)\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}\, e^{\frac{2i\pi^2}{cot\alpha}(h^2+q^2)} \tag{3.1.7}$$

Sampling theorem in FRFT of the function with 0-periodic & $\frac{\pi}{2}$- compact support can be represented by equation (4.1.6) & $f(x,y)$ can be calculated from (2k+1,2l+1) samples of it in the time domain when $\alpha = \pi(h-\frac{1}{2})$ and $\alpha = \pi(q-\frac{1}{2})$ then $cot\alpha = 0 \Rightarrow tan\alpha = \infty$, $\varphi(u,v,\alpha,1)$ is periodic for $\alpha \neq \pi(h-\frac{1}{2})$ and $\alpha \neq \pi(q-\frac{1}{2})$.

3.2 Sampling theorem for for $(\alpha-\frac{\pi}{2})$ periodic function with compact support α in FRFT domain:

We have discussed the sampling theorem in FRFT of 0-periodic function with $\frac{\pi}{2}$- compact support and it is to be reconstructing the function or its FRFT from the samples of the function. Here we consider the sampling relation for the compact support α and the function of $(\alpha-\frac{\pi}{2})$ period & compact support α.

Theorem: Any function $f(x,y)$ of $(\alpha-\frac{\pi}{2})$ period with compact support α with fundamental frequency $(u_{\alpha_0}, v_{\alpha_0})$, can be reconstructed of order α, i.e. by

Proof: Using the inversion formula in FRFT

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_\alpha(u,v)\overline{K_\alpha}\,(x,y,u,v)\,dudv$$

where, $\overline{K_\alpha}(x,y,u,v) = \frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{1-icot\alpha}}\frac{1}{sin^2\alpha}\quad e^{-c_{2\alpha}[(x^2+y^2+u^2+v^2)cos\alpha-2(xu+yv)]}$

Replace α by $\alpha-\frac{\pi}{2}$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{(\alpha-\frac{\pi}{2})}(u,v)\frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{1-icot(\alpha-\frac{\pi}{2})}}\frac{1}{sin^2(\alpha-\frac{\pi}{2})}\quad dudv$$
$$e^{-\frac{1}{2sin(\alpha-\frac{\pi}{2})}[(x^2+y^2+u^2+v^2)cos(\alpha-\frac{\pi}{2})-2(xu+yv)]}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{\left(\alpha-\frac{\pi}{2}\right)}(u,v)\,(2\pi)^{-3/2}(1+itan\alpha)^{-1/2}cosec^2(\alpha-\frac{\pi}{2})$$
$$e^{-\frac{1}{2}[(x^2+y^2+u^2+v^2)cot(\alpha-\frac{\pi}{2})-2(xu+yv)cosec(\alpha-\frac{\pi}{2})]}dudv$$

$$= (2\pi)^{-3/2}(1+itan\alpha)^{-1/2}(-sec^2\alpha)$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{1}{2}[(x^2+y^2+u^2+v^2)(-tan\alpha)+2(xu+yv)(sec\alpha)]}\ F_{\left(\alpha-\frac{\pi}{2}\right)}(u,v)dudv$$
$$= (2\pi)^{-3/2}\left(1+icot\left(\frac{\pi}{2}-\alpha\right)\right)^{-1/2} cosec^2\left(\frac{\pi}{2}-\alpha\right)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{\left(\alpha-\frac{\pi}{2}\right)}(u,v)$$
$$e^{-\frac{1}{2}[(x^2+y^2+u^2+v^2)cot\left(\frac{\pi}{2}-\alpha\right)+2(xu+yv)cosec\left(\frac{\pi}{2}-\alpha\right)]}dudv$$
$$= (2\pi)^{-3/2}(1+icot\alpha')^{-1/2}cosec^2\alpha'\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{\left(\alpha-\frac{\pi}{2}\right)}(u,v)$$
$$e^{-\frac{1}{2}[(x^2+y^2+u^2+v^2)cot\alpha'+2(xu+yv)cosec\alpha']}dudv$$
$$= (2\pi)^{-3/2}(1+icot\alpha')^{-1/2}cosec^2\alpha'\, e^{-\frac{1}{2}[(x^2+y^2)cot\alpha']}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{\left(\alpha-\frac{\pi}{2}\right)}(u,v)$$
$$e^{-\frac{1}{2}[(u^2+v^2)cot\alpha'+2(xu+yv)cosec\alpha']}dudv \qquad (4.2.1)$$

As $F_{\left(\alpha-\frac{\pi}{2}\right)}(u,v)$ is periodic in $\left(\alpha-\frac{\pi}{2}\right)^{th}$ domain, we can express it using the conventional two dimensional Fourier series
$$F_{\left(\alpha-\frac{\pi}{2}\right)}(u,v) = \sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}e^{i\,(hu_{\alpha_0}u+qv_{\alpha_0}v)} \qquad (4.2.2)$$

where $C_{h,q}$ are the Fourier series coefficients & k & l are the order of the highest nonzero harmonic components in the α^{th} domain.

Now from (4.2.1)
$$f(x,y) = (2\pi)^{-3/2}(1+icot\alpha')^{-1/2}cosec^2\alpha'\, e^{-\frac{1}{2}[(x^2+y^2)cot\alpha']}$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{1}{2}[(u^2+v^2)cot\alpha'+2(xu+yv)cosec\alpha']}\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}e^{i\,(hu_{\alpha_0}u+qv_{\alpha_0}v)}\,dudv$$

Interchanging summation and integration as per Pie & Ding
$$f(x,y) = (2\pi)^{-3/2}(1+icot\alpha')^{-1/2}cosec^2\alpha'\, e^{-\frac{1}{2}[(x^2+y^2)cot\alpha']}$$
$$\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{1}{2}[(u^2+v^2)cot\alpha']}\, e^{i[(-ixcosec\alpha'+hu_{\alpha_0})u+(-iycosec\alpha'+qv_{\alpha_0})v]}dudv$$
$$= (2\pi)^{-3/2}(1+icot\alpha')^{-1/2}cosec^2\alpha'\, e^{-\frac{1}{2}[(x^2+y^2)cot\alpha']}\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\pi i[(u^2+v^2)\frac{cot\alpha'}{2\pi i}]}\, e^{i\pi[2(\frac{(-i)x}{2\pi}cosec\alpha'+\frac{h}{2\pi}u_{\alpha_0})u+2(\frac{(-i)y}{2\pi}cosec\alpha'+\frac{q}{2\pi}v_{\alpha_0})v]}dudv$$
$$= (2\pi)^{-3/2}(1+icot\alpha')^{-1/2}cosec^2\alpha'\, e^{-\frac{1}{2}[(x^2+y^2)cot\alpha']}\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}$$
$$\int_{-\infty}^{\infty} e^{\pi i\left(u^2\frac{cot\alpha'}{2\pi i}+2(\frac{(-i)x}{2\pi}cosec\alpha'+\frac{h}{2\pi}u_{\alpha_0})u\right)}du\quad \int_{-\infty}^{\infty} e^{\pi i\left(v^2\frac{cot\alpha'}{2\pi i}+2(\frac{(-i)y}{2\pi}cosec\alpha'+\frac{q}{2\pi}v_{\alpha_0})v\right)}dv$$
$$= (2\pi)^{-3/2}(1+icot\alpha')^{-1/2}cosec^2\alpha'\, e^{-\frac{1}{2}[(x^2+y^2)cot\alpha']}\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}\int_{-\infty}^{\infty} e^{\pi i(Au^2+2Bu)}du$$
$$\int_{-\infty}^{\infty} e^{\pi i(Av^2+2Cv)}dv$$

where , A= $\frac{cot\alpha'}{2\pi i}$, B = $\frac{-ix}{2\pi}cosec\alpha'+\frac{h}{2\pi}u_{\alpha_0}$ C = $\frac{-iy}{2\pi}cosec\alpha'+\frac{q}{2\pi}v_{\alpha_0}$
$$= (2\pi)^{-3/2}(1+icot\alpha')^{-1/2}cosec^2\alpha'\, e^{-\frac{1}{2}[(x^2+y^2)cot\alpha']}\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}\ \frac{1}{\sqrt{A}}e^{\frac{i\pi}{4}}e^{\frac{-i\pi B^2}{A}}\ \frac{1}{\sqrt{A}}e^{\frac{i\pi}{4}}e^{\frac{-i\pi C^2}{A}}$$
$$= (2\pi)^{-3/2}(1+icot\alpha')^{-1/2}cosec^2\alpha'\, e^{-\frac{1}{2}[(x^2+y^2)cot\alpha']}\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}\ \frac{1}{A}e^{\frac{i\pi}{2}}e^{\frac{-i\pi}{A}}(\frac{B^2+C^2}{A})$$
$$= (2\pi)^{-3/2}(1+icot\alpha')^{-1/2}cosec^2\alpha'\, e^{-\frac{1}{2}[(x^2+y^2)cot\alpha']}\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}$$
$$\frac{2i}{cot\alpha'}e^{\frac{i\pi}{2}}e^{-i\pi\{\frac{\left(\frac{-ix}{2\pi}cosec^2\alpha'+\frac{h}{2\pi}u_{\alpha_0}\right)^2+\left(\frac{-iy}{2\pi}cosec^2\alpha'+\frac{h}{2\pi}v_{\alpha_0}\right)^2}{\frac{cot\alpha'}{2\pi i}}}$$
$$= (2\pi)^{-3/2}(1+icot\alpha')^{-1/2}cosec^2\alpha' tan\alpha'\, e^{-\frac{1}{2}[(x^2+y^2)cot\alpha']}\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}$$
$$(2\pi i)\,e^{\frac{i\pi}{2}}e^{-i\pi\{\frac{\left(-ixcosec^2\alpha'+hu_{\alpha_0}\right)^2+\left(-iycosec^2\alpha'+qv_{\alpha_0}\right)^2}{\frac{cot\alpha'}{2\pi i}4\pi^2}\}}$$
$$= (2\pi)^{\frac{-3}{2}+1}(1+icot\alpha')^{-1/2}e^{\frac{i\pi}{2}}\ \frac{1}{sin\alpha' cos\alpha'}\ e^{-\frac{1}{2}[(x^2+y^2)cot\alpha']}\sum_{h=-k}^{k}\sum_{q=-l}^{l} C_{h,q}$$
$$e^{\{\frac{\left(-ixcosec^2\alpha'(x)+hu_{\alpha_0}\right)^2+\left(-iycosec^2\alpha'+qv_{\alpha_0}\right)^2}{2cot\alpha'}\}} \qquad (4.2.3)$$

It is clear that from (4.2.3) that we can reconstruct the time domain function f(x,y) which is an compact support α of $(\alpha - \frac{\pi}{2})$ period form (2k+1,2l+1) Fourier series coff. $C_{h,q}$ Of $F_{\left(\alpha-\frac{\pi}{2}\right)}(u,v)$ in $\left(\alpha - \frac{\pi}{2}\right)$ domain.

CONCLUSION

Sampling theorem of two dimensional fractional Fourier transform, we observed that two dimensional fractional Fourier transform of 0-periodic with $(\alpha - \frac{\pi}{2})$ compact support, can be calculated using (2k+1) samples of the function in time domain, where k is the order of positive highest nonzero harmonic component in conventional Fourier domain.

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