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A FINITE POPULATION DISCRETE-TIME INVENTORY SYSTEM WITH POSTPONED DEMANDS

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ABSTRACT

In this paper, we consider the discrete time inventory system in which the demands are originated from the finite number of homogeneous population. The inter demand times of the primary arrival follows geometric distribution. The inventory is replenished according to $(\boldsymbol{s}, \boldsymbol{S})$ policy. According to the policy the inventory is reaches a level $\boldsymbol{s}$, we place an order so that the inventory level is up to S and the lead times are assumed to follow a geometric distribution. The demands that occur during stock out period is permitted to enter into the pool. These pooled demands are satisfied only when the inventory level is above $\boldsymbol{s}$. The pooled demands are selected one by one according to the first come first serve basis and the inter-selection time is distributed as geometric. The joint probability distribution of the number of demands in the pool and the inventory level is obtained in steady state case and some important system performance measures are derived in the steady state case.

AMS Subject Classification: *90B05, 60J27.*

Keywords: *Discrete time review Inventory Systems, (S, s) ordering policy, Finite population, Postponed demands.*

1. INTRODUCTION

Inventory models have been considered under continuous review as well as periodic review. In the recent past discrete time models have started receiving attention of researchers in the areas of queueing and telecommunications. In discrete time setting, it is assumed that the time axis is calibrated into epochs by small units and that all the events are deemed to occur only at these epochs. With the advent of fast computing devices and efficient transaction reporting facilities, such epochs with small gaps can be conveniently assumed so that events can occur at these epochs.

In most of the continuous review inventory systems considered in the literature, the demanded items are directly delivered from the stock (if available) and the demands that occurred during the stock-out period are either lost (lost sales case) or satisfied only after the arrival of ordered items (backlogging case). In the latter case, it is assumed that either all (full backlogging case) or any prefixed number of demands (partial backlogging case) that occurred during the stock out period are satisfied. Continuous review inventory system with postponed demands has received considerable attention in the last few decades. Further there are researcher developed the inventory model with postponed demands under discrete time review as well. Berman *et al.* [3] introduced the concept of postponement of demand in the inventory system. They assumed that both demand and service rates are deterministic. The often quoted review articles Radhamani *et al.* [8], and Sivakumar *et al.*, [9] provide excellent summaries of many of these modeling efforts.

In the case of inventory modeling under discrete times, the first paper was by Bar-Lev and Perry [2], who assumed that demands are non-negative integer value random variables and items have constant life times. In this paper, we consider the discrete time inventory system in which the demands are originated from the finite number of homogeneous population. The inter demand times of the primary arrival follows geometric distribution. The inventory is replenished according to $(\boldsymbol{s}, \boldsymbol{S})$ policy. According to the policy the inventory is reaches a level $\boldsymbol{s}$, we place an order so that the inventory level is up to S and the lead times are assumed to follow a geometric distribution. The demands that occur during stock out period are permitted to enter into the pool. These pooled demands are satisfied only when the inventory level is above $\boldsymbol{s}$. The pooled demands are selected one by one according to the first come first serve basis and the inter-selection time is distributed as geometric.

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The rest of the paper is organized as follows. In Section 2, we describe the problem under consideration and in the next section we model the problem mathematically. The steady-state analysis of the model is presented in Section 4 and some important system performance measures are derived in Section 5.

Notation:
- $A(i,j)$: entry at (i,j) th position of A
- 0 : zero vector of appropriate dimension
- e : a column vector of 1's of appropriate dimension
- I_n: identity matrix of order n
- $\delta_{i,j}$: Kronecker delta function

2. PROBLEM FORMULATIONS

We consider a discrete time inventory system where the time axis is divided into intervals of equal length, called slots (epochs). It is assumed that all system activities (arrivals, postponed demand selection and replenishment) occur at the slot boundaries, and therefore they may occur at the same time. The maximum capacity of the inventory is $\boldsymbol{S}$.

- The arrival of demands generated from a homogeneous population of finite size (M). Each demand generates the arrival according to a Bernoulli stream with rate $1-a$, thus $1-a$ is the probability that a demand arrives at a slot and a is the probability that an arrival does not take place in a slot. When the on-hand inventory level is more than one then the arriving demand is satisfied immediately.
- We consider (s,S) ordering policy, according to the policy when the inventory reaches a level s, we place an order $Q(=\ (S\ -\ s)\ >\ s\ +\ 1)$ units so that the inventory level is up to S. The lead time is assumed to follow geometric distribution with success probability $b(>\ 0)$. The condition $S-s\ >\ s+1$ is assumed so that when the supply of an order is received during the stock out period, the inventory level would be brought above the reorder level. Otherwise the inventory will have perpetual stock out.
- If the arrival finds the inventory level is zero, he is permitted to enter into the pool of his own choice. These pooled demands are satisfied only when the inventory level is above s. The pooled demands are selected one by one according to the first come first serve basis and the inter-selection time is distributed as geometric distribution with success probability c.

Unlike continuous review inventory systems, multiple events such as demand, supply and pooled demand selection may occur between epochs n and n+1, n = 0, 1, 2, . . . Hence we adopt the following convention: If the events such as demand for an item, pooled demand selection and supply of an order take place at $n\ (n\ =\ 1,2,3,\dots)$, it is assumed that first supply is received then demand occurs and finally the pooled demand selection takes place.

3. MODEL DESCRIPTIONS

Let X_n denote the number of demands in the pool, and Y_n denote the inventory level at time n. From the assumptions made on the input and output processes, it can be shown that the stochastic process $(X,Y\ )\ =\ \{(X_n,Y_n),n\in\ N\}$ is a Discrete Time Markov Chain with state space given by, $E\ =\ \{(i,j)\ :\ i\ =\ 0,1,\dots,M,j\ =\ 0,1,\dots,S\}$.

The transition probability function is defined as for $(i,j),(k,l)\in E$,

$$p((i,j),(k,l))\ =\ Pr[X_{n+1}\ =\ k,Y_{n+1}\ =\ l|X_n\ =\ i,Y_n\ =\ j]$$

The transition probability matrix P of this process,

$$P\ =\ \Big(\Big(\ p\big((i,j),(k,l)\big)\Big)\Big),(i,j),(k,l)\in\ E$$

Then the transition probability matrix P can be viewed as,

$$P=\begin{array}{c} \\ 0 \\ 1 \\ 2 \\ \dots \\ M-1 \\ M \end{array}\begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & \dots & M-1 & M \end{array} \\ \begin{pmatrix} A_0 & B_0 & & & & \\ C_1 & A_1 & B_1 & & & \\ & C_2 & A_2 & B_2 & & \\ & & \dots & \dots & \dots & \\ & & & C_{M-1} & A_{M-1} & B_{M-1} \\ & & & & C_M & A_M \end{pmatrix}\end{array}$$

Where,

$$[A_0]_{ij} = \begin{cases} ba^M & j = Q+i & i = 0,1,2,...,s \\ b(1-a^M) & j = Q+i-1 & i = 0,1,2,...,s \\ \bar{b}a^M & j = i & i = 0,1,2,...,s \\ \bar{b}(1-a^M) & j = i-1 & i = 1,2,...,s \\ a^M & j = i & i = s+1, s+2,...,S \\ 1-a^M & j = i-1 & i = s+1, s+2,...,S \\ 0 & 0therwise & \end{cases}$$

For $k = 0,1,...,M-1,$

$$[B_k]_{ij} = \begin{cases} b(1-a^{M-k}) & j = 0 & i = 1 \\ 0 & otherwise & \end{cases}$$

For $k = 1,2,...,M,$

$$[A_k]_{ij} = \begin{cases} a^{M-k}b\bar{c} & j = Q+i & i = 0,1,2,...,s \\ (1-a^{M-k})b\bar{c} & j = Q+i-1 & i = 0,1,2,...,s \\ (1-\delta_{k,M})a^{M-k}\bar{b} + \delta_{k,M}\bar{b} & j = 0 & i = 0 \\ (1-a^{M-k})\bar{b} & j = 0 & i = 1 \\ a^{M-k}\bar{b} & j = i & i = 1,2,...,s \\ (1-a^{M-k})\bar{b} & j = i-1 & i = 2,3,...,s \\ a^{M-k}\bar{c} & j = i & i = s+1,...,S \\ (1-a^{M-k})\bar{c} & j = i-1 & i = s+1,...,S \\ 0 & otherwise & \end{cases}$$

$$[c_k]_{ij} = \begin{cases} a^{M-k}bc & j = Q+i-1 & i = 0,1,2,...,s \\ (1-a^{M-k})bc & j = Q+i-+2 & i = 0,1,2,...,s \\ a^{M-k}c & j = i-1 & i = s+1, s+2,...,S \\ (1-a^{M-k})c & j = i-2 & i = s+1, s+2,...,S \\ 0 & otherwise & \end{cases}$$

Here we notice that, all the matrices $\boldsymbol{A_k}, \boldsymbol{B_k}$ and $\boldsymbol{C_k}$ are square matrix of order $\boldsymbol{S + 1}$

4. CALCULATING LIMITING PROBABILITIES

It can be seen from the structure of P, the homogeneous Markov chain $\{(X_n, L_n), n \in N\}$ on the finite state space is irreducible. Hence the limiting probability distribution

$$\pi^{(i,j)} = \lim_{n\to\infty} \Pr\,[\mathrm{X_n = i, L_n = j \,|X_0 = k, L_0 = l}],$$

Where $\pi^{(i,j)}$ is the steady state probability for the state (i, j) exists and is independent of the initial state (k, l). Let Π be the steady state limiting probability vector of P. That is, Π satisfies

$$\Pi P = 0, \Pi e = 1.$$

The vector Π can be represented by, $\Pi = (\Pi^{<0>}, \Pi^{<1>}, \Pi^{<2>}, \dots, \Pi^{<M>})$,
and $\Pi^{<i>} = \left(\pi^{(i,0)}, \pi^{(i,1)}, \dots, \pi^{(i,S)}\right)$, for $i = 0,1,2, \dots, M$.

Now the structure of P shows, the model under study is a finite birth death model in the Markovian environment. Hence we use the algorithm discussed by Gaver *et al.* [7] for computing the limiting probability vector. For the sake of completeness we provide the algorithm here.

Algorithm:

1. Determine recursively the matrix D_n, $0 \leq n \leq M$ by using
$$D_0 = A_0, D_i = A_i + C_i (I - D_{i-1})^{-1} B_0, \quad i = 1,2, \dots, M.$$
2. Solve the system $\Pi^{<M>}(I - D_M) = 0$
3. Compute recursively the vector $\Pi^{<i>}$, $i = M-1, \dots, 0$, using
$$\Pi^{<i>} = \Pi^{<i+1>} C_{i+1} (I - D_i)^{-1}, \quad i = M-1, \dots, 0$$
4. Normalize the vector Π, by using $\Pi e = 1$

5. SYSTEM PERFORMANCE MEASURES

In this section, we derive some system performance measures in the steady-state case.

Expected inventory level

Let ζ_i denote the expected inventory level in the steady-state. Then ζ_i is given by

$$\zeta_i = \sum_{i=0}^{M} \sum_{j=1}^{S} j\pi^{(i,j)}$$

Expected reorder rate

Let ζ_r denote the expected reorder level in the steady-state. Then ζ_r is given by

$$\zeta_r = (1 - a^M)\pi^{(0,s+1)} + \sum_{i=1}^{M} [\,(\bar{c}(1 - a^{M-i}) + ca^{M-i})\pi^{(i,s+1)} + (1 - a^{M-i})c\pi^{i-s+2}]$$

Expected number of customer in the Pool

Let ζ_p denote the expected number of customer in the pool in the steady-state. Then ζ_p is given by

$$\zeta_p = \sum_{i=1}^{M} i\Pi^{<i>} e$$

Long-run expected cost rate function

The long-run expected cost rate for this model is defined to be

$$C(S, s, M) = c_h \zeta_i + c_s \zeta_r + c_w \zeta_p$$

$$C(S, s, M) = c_h \sum_{i=0}^{M} \sum_{j=1}^{S} j\pi^{(i,j)} + c_w \sum_{i=1}^{M} i\Pi^{<i>} e + c_s [\sum_{i=1}^{M} [\,(\bar{c}(1 - a^{M-i}) + ca^{M-i})\pi^{(i,s+1)} + (1 - a^{M-i})c\pi^{i-s+2}]) + (1 - a^M)\pi^{(0,s+1)}]$$

Where,

$\boldsymbol{c_s}$: Setup cost per order

$\boldsymbol{c_h}$: The inventory carrying cost per unit item per unit time

$\boldsymbol{c_w}$: Waiting cost of a customer in the orbit per unit time

Due to the complex form of the limiting distribution, it is difficult to discuss the properties of the cost function $C(S, s, M)$ analytically. Although we have not established analytically, our experience with considerable numerical examples indicates the function, $\mathrm{C(S, s, M)}$ to be convex. A typical 3-dimensional plot of $\mathrm{C(S, s, M)}$ is presented in Figure 1.

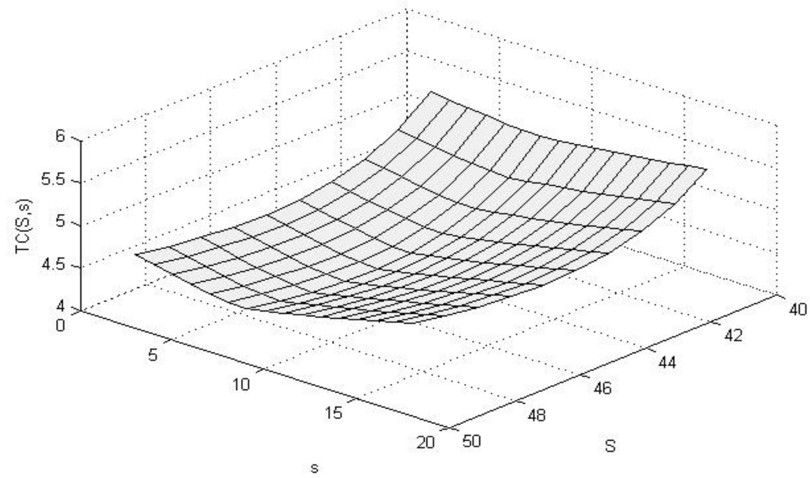

Figure-1: 3-dimensional plot of C(S, s, M)

We have studied the effect of varying the cost and other system parameters on the optimal values and the results agreed with what one would expect. That is, Figure 2 shows that, the cost function $C(S,s,M)$ decreases when the probability of choosing the postponed demand from the pool increases and also the cost function $C(S,s,M)$ decreases when the replenishment probability increases.

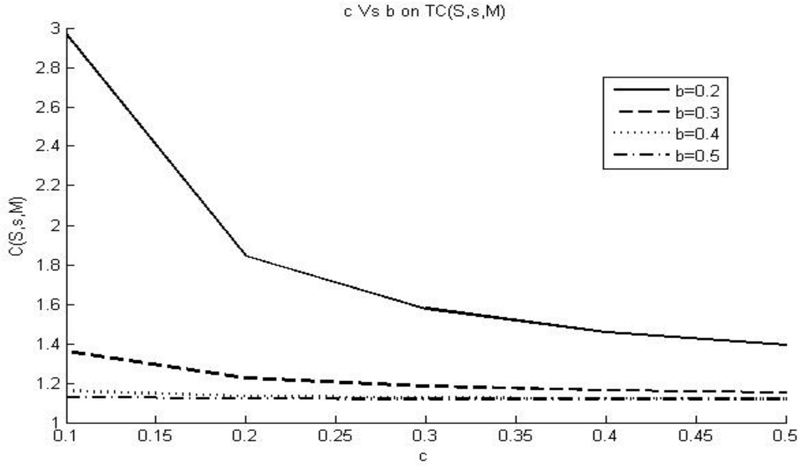

Figure-2: c Vs b on C(S, s, M)

REFERENCES

[1] Abboud, N. E., (2001), A discrete-time Markov production-inventory model with machine breakdowns, Computers & Industrial Engineering, 39, 95 - 107.

[2] Bar-Lev,S.K. and Perry,D. (1989), A discrete time Markovian inventory model for perishable commodities, Stochastic Analysis and Applications, 7, 243 - 259.

[3] O. Berman, E.H. Kaplan, D.G. Shimshak, Deterministic approximations for inventory management at service facilities, IIE Transactions 25 (1993) 98-104.

[4] Gaver, D.P., Jacobs, P.A. and Latouche, G., (1984), Finite birth-and-death models in randomly changing environments, Advances in Applied Probability, 16, 715 - 731

[5] Latouche, G. and Ramaswami, V., (1999), Introduction to matrix analytic methods in stochastic modeling,. ASA SIAM, Philadelphia.

[6] Minh D.L., (1977), A Discrete Time Single Server Queue from a Finite Population, Management Science, 23(7), 756-767

[7] Periyasamy, C., (2013), A Discrete Time Inventory System With Retrial Demands, International Journal of Engineering Research & Technology, 2[7], 165 - 170

[8] Radhamani, V., Chitra Devi, P., Sivakumar, B., (2014), A Stochastic Inventory System with Postponed Demands and Infinite Pool in Discrete-Time Setup, Journal of the Operations Research Society of China, 2(4), 455-480

[9] Sivakumar, B., Jayaraman, R., Arivarignan, G., (2012) A discrete time inventory system with postponed demands, Journal of Computational and Applied Mathematics 236(13), 3073-3083

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