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FG-COUPLED FIXED POINT THEOREMS INVOLVING CONTRACTIVE TYPE MAPPINGS

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ABSTRACT

Here, we prove some result on FG-coupled fixed point. Our result generalize some coupled fixed point results.

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1. INTRODUCTION

Fixed point has a large application in almost all fields like Biology, Computer science, Physics, Economics and many branches of engineering. In [1] Lakshmikantham *et al.* introduce the concept of coupled fixed points and proved some results satisfying mixed monotone property. Many authors proved many results on coupled fixed points [3-9]. In 2016 [2] Prajisha and Shaini Pulickkunnel introduced the notion of FG-coupled fixed point which is generalized form of coupled fixed.

2. PRELIMINARIES

In this section we gave some definitions which are very useful in proving the results.

Definition 2.1: Let X be partially ordered metric space. Let $F : X \times X \to X$ be a mapping. Then an element $(x, y) \in X \times X$ is a coupled fixed point of the mapping F if $F(x, y) = x$, $F(y, x) = y$.

Definition 2.2: Let $(X, \le)$ be a partially ordered set and $F : X \times X \to X$. Then F has the mixed monotone property if $F(x, y)$ is monotonically non decreasing in x and is monotonically non increasing in y, that is for any $x, y \in X$

$x_1, x_2 \in X$, $x_1 \le x_2 \in F(x_1, y) \le F(x_2, y)$ and
$y_1, y_2 \in X$, $y_1 \le y_2 \in F(x, y_1) \ge F(x, y_2)$

Definition 2.3: Let $(X, \le P_1)$ and $(Y, \le P_2)$ be two partially ordered metric spaces and $F : X \times Y \to X$ and $F : Y \times X \to X$ be two functions. An element $(x, y) \in X \times Y$ is called an FG-coupled fixed point if $F(x, y) = x$ and $G(y, x) = y$.

Definition 2.4: Let $(X, \le P_1)$ and $(Y, \le P_2)$ be two partially ordered sets and $F : X \times Y \to X$ and $F : Y \times X \to X$. Then F and G have mixed monotone property if F and G are monotone increasing in first variable and monotone decreasing in second variable, i.e., if for all $(x, y) \in X \times Y$,

$x_1, x_2 \in X$, $x_1 \le P_1 x_2 \Rightarrow F(x_1, y) \le P_1 F(x_2, y)$ and $G(y, x_1) \ge P_2 G(y, x_2)$
and $y_1, y_2 \in X$, $y_1 \le P_2 y_2 \Rightarrow \in F(x, y_1) \ge P_1 F(x, y_2)$ and $G(y_1, x) \le P_2 G(y_2, x)$.

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Some important Notes:

1. If $(x, y) \in X \times Y$ is an FG- coupled fixed point then the element $(y, x) \in Y \times X$ is GF- coupled fixed point.
2. The metric d on $X \times Y$ is defined as $d((x, y),(u,v)) = d_X(x,u) + d_Y(y,v)$ for all $(x, y),(u,v) \in X \times Y$.
3. Partial order relation $\leq$ on $X \times Y$ is defined as for any $(x, y),(u,v) \in X \times Y$;
   $(u,v) \leq (x, y) \Leftrightarrow x \geq P_1 u, y \leq P_2 v$.
4. $F_{n+1}(x, y) = F(F_n(x, y), G_n(y, x))$ and $G_{n+1}(y, x) = G(G_n(y, x), F_n(x, y))$ for every $n \in N$ and $(x, y) \in X \times Y$.

3. MAIN RESULT

Theorem: Let $(X, d_X, \leq_{p_1})$ and $(Y, d_Y, \leq_{p_2})$ be partially ordered complete metric spaces. Also let $F : X \times Y \to X$ and $G : Y \times X \to Y$ be any functions which have mixed monotone property. Assume that there exists non negative reals A, B, C with $2A + 3B + 3C < 2$ such that

$$\begin{aligned} d_X(F(x, y), F(u,v)) \leq \frac{A}{2}[d_X(x,u) + d_Y(y,v)] \\ + \frac{B}{2}[d_X(x, F(x, y)) + d_X(y, F(u,v)) + d_Y(y,v)] \\ + \frac{C}{2}[d_X(x, F(u,v)) + d_X(y, F(x, y)) + d_Y(y,v)] \\ \text{for all } x \geq p_1 u,\ y \leq p_2 v \end{aligned} \tag{1}$$

and

$$\begin{aligned} d_Y(G(y, x), G(v,u)) \leq \frac{A}{2}[d_X(x,u) + d_Y(y,v)] \\ + \frac{B}{2}[d_Y(y, F(y, x)) + d_Y(v, G(v,u)) + d_X(x, y)] \\ + \frac{C}{2}[d_Y(y, G(v,u)) + d_Y(v, G(y, x)) + d_X(u, y)] \\ \text{for all } x \leq p_1 u,\ y \geq p_2 v \end{aligned} \tag{2}$$

If there is $(x_0, y_0) \in X \times Y$ with the condition $x_0 \leq_{p_1} F(x_0, y_0)$ and $y_0 \geq_{p_2} G(y_0, x_0)$, then there is an element $(x, y) \in X \times Y$ such that $x = F(x, y)$ and $y = G(y, x)$.

i.e. F and G have a unique FG-coupled fixed point.

From the hypothesis there is $(x_0, y_0) \in X \times Y$ such that $x_0 \leq_{p_1} F(x_0, y_0) = x_1$ (say) and $y_0 \geq_{p_2} G(y_0, x_0) = y_1$ (say).

Now for $n = 1, 2, 3, \ldots$ we define $x_{n+1} = F(x_n, y_n)$ and $y_{n+1} = G(y_n, x_n)$, then we get $x_{n+1} = F^{n+1}(x_0, y_0)$ and $y_{n+1} = G^{n+1}(y_0, x_0)$, since

$$\begin{aligned} x_{n+1} &= F(x_n, y_n) \\ &= F(F(x_{n-1}, y_{n-1}), G(y_{n-1}, x_{n-1})) \\ &= F^2(x_{n-1}, y_{n-1}) \\ &= F^3(x_{n-2}, y_{n-2}) \\ &\vdots \\ &= F^{n+1}(x_0, y_0). \end{aligned}$$

Similarly we have $y_{n+1} = G^{n+1}(y_0, x_0)$.

Now, by the principle of mathematical induction and mixed monotone property of F and G we can easily prove that $\{x_n\}$ is an increasing sequence in X and $\{y_n\}$ is a decreasing sequence in Y. For this, we have.

$$x_0 \leq_{p_1} x_1 \text{ and } y_0 \geq_{p_2} y_1.$$

We want to show that

$$x_n \leq_{p_1} x_{n+1} \text{ and } y_n \geq_{p_2} y_{n+1} \text{ for all } n \in N.$$

Suppose for $x=1$, $x_2 = F(x_1, y_1) \geq p_1F(x_0, y_1) \geq p_1F(x_0, y_0) = x_1$ and

$$y_2 = G(y_1, x_1) \leq p_2G(y_1, x_0) \leq p_2G(y_0, x_0) = y_1.$$

Assume that the result holds for $m = n$

i.e. $x_{m+1} \geq p_1x_m$ and $y_{m+1} \leq p_2y_m$

Now consider

$$x_{m+2} = F(x_{m+1}, y_{m+1}) \geq p_1F(x_m, y_{m+1}) \geq p_1F(x_m, y_m) = x_{m+1}$$
$$y_{m+2} = G(y_{m+1}, x_{m+1}) \leq p_2G(y_{m+1}, x_m) \leq p_2G(y_m, y_m) = y_{m+1}$$

Hence the result is true for all $x \in N$.

i.e. $\{x_n\}$ is an increasing sequence in X and $\{y_n\}$ is a decreasing sequence in Y.

Now,

$$\begin{aligned}
d_X(x_n, x_{n+1}) &= d_X(F^n(x_0, y_0), F^{n+1}(x_0, y_0)) \\
&= d_X[F(F^{n-1}(x_0, y_0), G^{n-1}(y_0, x_0)), F(F^n(x_0, y_0), G^n(y_0, x_0))] \\
&\leq \frac{A}{2}[d_X(F^{n-1}(x_0, y_0), F^n(x_0, y_0)) + d_Y(G^n(y_0, x_0), G^n(y_0, x_0))] \\
&\quad + \frac{B}{2}[d_X(F^{n-1}(x_0, y_0), F(F^{n-1}(x_0, y_0)), G^{n-1}(y_0, x_0)) \\
&\quad + d_X(F^n(x_0, y_0), F(F^n(x_0, y_0)), G^n(y_0, x_0)) + d_Y(G^{n-1}(y_0, x_0), G^n(y_0, x_0))] \\
&\quad + \frac{C}{2}[d_X(F^{n-1}(x_0, y_0), F(F^n(x_0, y_0), G^n(y_0, x_0)) \\
&\quad + d_X(F^n(x_0, y_0), F(F^{n-1}(x_0, y_0), G^{n-1}(y_0, x_0)) + d_Y(G^{n-1}(y_0, x_0), G^n(y_0, x_0))] \\
&= \frac{A}{2}[d_X(x_{n-1}, x_n) + d_Y(y_{n-1}, y_n)] + \frac{B}{2}[d_X(x_{n-1}, x_n) + d_X(x_n, x_{n+1}) + d_Y(y_{n-1}, y_n)] \\
&\quad + \frac{C}{2}[d_X(x_{n-1}, x_{n+1}) + d_X(x_n, x_n) + d_Y(y_{n-1}, y_n)] \\
&= \left(\frac{A}{2} + \frac{B}{2}\right)d_X(x_{n-1}, x_n) + \left(\frac{A+B+C}{2}\right)d_Y(y_{n-1}, y_n) \\
&\quad + \frac{B}{2}d_X(x_n, x_{n+1}) + \frac{C}{2}d_X(x_{n-1}, x_{n+1}) \\
&= \left(\frac{A}{2} + \frac{B}{2}\right)d_X(x_{n-1}, x_n) + \left(\frac{A+B+C}{2}\right)d_Y(y_{n-1}, y_n) \\
&\quad + \frac{B}{2}d_X(x_n, x_{n+1}) + \frac{C}{2}d_X(x_{n-1}, x_n) + \frac{C}{2}d_X(x_n, x_{n+1})
\end{aligned}$$

$$\Rightarrow \left(\frac{2-B-C}{2}\right)d_X(x_n, x_{n+1}) \leq \left(\frac{A+B+C}{2}\right)[d_X(x_{n-1}, x_n) + d_Y(y_{n-1}, y_n)]$$

$$\Rightarrow d_X(x_n, x_{n+1}) \leq \left(\frac{A+B+C}{2-B-C}\right)[d_X(x_{n-1}, x_n) + d_Y(y_{n-1}, y_n)]$$

Similarly we obtain

$$d_Y(y_n, y_{n+1}) \leq \frac{A+B+C}{2-B-C}[d_X(x_{n-1}, x_n) + d_Y(y_{n-1}, y_m)]$$

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Adding (3) and (4), we get

$$d_X(x_n, x_{n+1}) + d_Y(y_n, y_{n+1}) \le 2\left(\frac{A+B+C}{2-B-C}\right)[d_X(x_{n-1}, x_n) + d_Y(y_{n-1}, y_n)]$$

Assume $\frac{2(A+B+C)}{(2-B-C)} = \lambda < 1$ as $2A+3B+3C<1$

$$\begin{aligned}\Rightarrow d_X(x_n, x_{n+1}) + d_Y(y_n, y_{n+1}) &\le \lambda[d_X(x_{n-1}, x_n) + d_Y(y_{n-1}, y_n)] \\ &\le \lambda^2[d_X(x_{n-2}, x_{n-1}) + d_Y(y_{n-2}, y_{n-1})] \\ &\vdots \quad \vdots \\ &\le \lambda^n[d_X(x_0, x_1) + d_Y(y_0, y_1)]\end{aligned}$$

Let us consider $m > n$, as $0 < \lambda < 1$, we get

$$\begin{aligned}\Rightarrow d_X(x_n, x_m) + d_Y(y_n, y_m) &\le d_X(x_n, x_{n+1}) + d_Y(y_n, y_{n+1})] \\ &+ d_X(x_{n+1}, x_{n+2}) + d_Y(y_{n+1}, y_{n+2}) \\ &\vdots \quad \vdots \\ &+ d_X(x_{m-1}, x_m) + d_Y(y_{m-1}, y_m)] \\ &\le \lambda^n(d_X(x_0, x_1) + d_Y(y_0, y_1)] \\ &+ \lambda^{n+1}[d_X(x_0, x_1) + d_Y(y_0, y_1)] \\ &\vdots \quad \vdots \\ &+ \lambda^{m-1}[d_X(x_0, x_1) + d_Y(y_0, y_1)] \\ &= (\lambda^n + \lambda^{n+1} + \ldots + \lambda^{m-1})[d_X(x_0, x_1) + d_Y(y_0, y_1)] \\ &= \frac{\lambda^n}{1-\lambda}(d_X(x_0, x_1) + d_Y(y_0, y_1)) \\ &\to 0 \text{ as } n \to \infty, \text{ since } \lambda < 1\end{aligned}$$

Thus we get that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in X and Y respectively. Since X and Y are complete metric spaces there exists $x \in X$ and $y \in Y$ such that $x_n \to x$ and $y_n \to y$ as $n \to \infty$.

i.e. $\lim_{n\to\infty} F^n(x_0, y_0) = x$, $\lim_{n\to\infty} G^n(y_0, x_0) = y$.

Now, we prove $F(x, y) \ne x$ and $G(y, x) = y$.

If F and G are continuous functions. Then

$$x = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} F(x_n, y_n) = F(\lim_{n\to\infty} x_n, \lim_{n\to\infty} y_n) = F(x, y)$$

and

$$y = \lim_{n\to\infty} y_{n+1} = \lim_{n\to\infty} G(y_n, x_n) = G(\lim_{n\to\infty} y_n, \lim_{n\to\infty} x_n) = G(y, x)$$

Thus (x, y) is an FG-coupled fixed point of F and G.

If F and G are not continuous mappings then prove that they have a FG-coupled fixed point.

For this suppose $F(x, y) \ne x$ and $G(y, x) \ne y$.

$$d_X(F(x, y), x) > 0 \text{ and } d_Y(G(y, x), y) > 0.$$

Now,

$$
\begin{aligned}
d_X(F(x,y),x) &\le d_X(F(x,y),x_{n+2}) + d_X(x_{n+2},x)\\
&= \lim_{n\to\infty}\{d_X(F(F^n(x_0,y_0),G^n(y_0,x_0)),F^{n+2}(x_0,y_0)) + d_X(F^{n+2}(x_0,y_0),F^n(x_0,y_0))\}\\
&= \lim_{n\to\infty}\{d_X(F(F^n(x_0,y_0),G^n(y_0,x_0)),F(F^{n+1}(x_0,y_0),G^{n+1}(y_0,x_0))\\
&\quad + d_X(F(F^{n+1}(x_0,y_0),G^{n+1}(y_0,x_0)),F(F^{n-1}(x_0,y_0),G^{n-1}(y_0,x_0)))\}\\
&\le \lim_{n\to\infty}\{\frac{A}{2}[d_X(F^n(x_0,y_0),F^{n+1}(x_0,y_0)) + d_Y(G^n(y_0,x_0),G^{n+1}(y_0,x_0))]
\end{aligned}
$$

$$
y = \lim_{n\to\infty} y_{n+1} = \lim_{n\to\infty} G(y_n,x_n) = G(\lim_{n\to\infty} y_n, \lim_{n\to\infty} x_n) = G(y,x)
$$

$$
\begin{aligned}
&+d_X(F^{n+1}(x_0,y_0),F(F^n(x_0,y_0),G^n(y_0,x_0))) + d_Y(G^n(y_0,x_0),G^{n+1}(y_0,x_0))]\\
&+\frac{A}{2}[d_X(F^{n+1}(x_0,y_0),F^{n-1}(x_0,y_0)) + d_Y(G^{n+1}(y_0,x_0),G^{n-1}(y_0,x_0))]\\
&+\frac{B}{2}[d_X(F^{n+1}(x_0,y_0)),F(F^{n+1}(x_0,y_0),G^{n+1}(y_0,x_0))\\
&+d_X(F^{n-1}(x_0,y_0)),F(F^{n-1}(x_0,y_0),G^{n-1}(y_0,x_0)) + d_Y(G^{n+1}(y_0,x_0),G^{n-1}(y_0,x_0))]\\
&+\frac{C}{2}[d_X(F^{n+1}(x_0,y_0)),F(F^{n-1}(x_0,y_0),G^{n-1}(y_0,x_0))\\
&+d_X(F^{n-1}(x_0,y_0)),F(F^{n+1}(x_0,y_0),G^{n+1}(y_0,x_0)) + d_Y(G^{n+1}(y_0,x_0),G^{n-1}(y_0,x_0))]\\
&= \lim_{n\to\infty}\{\frac{A}{2}[d_X(x_n,x_{n+1}) + d_Y(y_n,y_{n+1})] + \frac{B}{2}[d_X(x_n,x_{n+1}) + d_X(x_{n+1},x_{n+2}) + d_Y(y_n,y_{n+1})]\\
&+\frac{C}{2}[d_X(x_n,x_{n+2}) + d_X(x_{n+1},x_{n+1}) + d_Y(y_n,y_{n+1})]\\
&+\frac{A}{2}[d_X(x_{n+1},x_{n-1}) + d_Y(y_{n+1},y_{n-1})] + \frac{B}{2}[d_X(x_{n+1},x_{n+2}) + d_X(x_{n-1},x_n) + d_Y(y_{n+1},y_{n-1})]\\
&+\frac{C}{2}[d_X(x_{n+1},x_n) + d_X(x_{n-1},x_{n+2}) + d_Y(y_{n+1},y_{n-1})]\\
&\to 0 \text{ as } n\to\infty
\end{aligned}
$$

$$
\Rightarrow\ d_X(F(x,y),x) \le\ 0
$$

Hence

$$
d_X(F(x,y),x) = 0
$$

$$
\Rightarrow\ F(x,y) = x
$$

Similarly $G(y,x) = y$.

Thus we get that (x,y) is a FG-coupled fixed point of the functions F and G.

Now we shall prove the uniqueness part of the theorem.

Let us suppose that there are two FG-coupled fixed points of F and G say (x,y) and (x',y')

i.e. $F(x,y) = x$, $G(y,x) = y$ and $F(x',y') = x'$, $G(y',x') = y'$.

Case-I: If (x,y) and (x',y') are comparable.

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Then

$$\begin{aligned} d_X(x,x') &= d_X[F(x,y),F(x',y')] \\ &\le \frac{A}{2}[d_X(x,x')+d_Y(y,y')]+\frac{B}{2}[d_X(x,F(x,y))+d_X(x',F(x',y'))+d_Y(y,y')] \\ &\quad +\frac{C}{2}[d_X(x,F(x',y'))+d_X(x',F(x,y))+d_Y(y,y')] \\ &= \frac{A}{2}[d_X(x,x')+d_Y(y,y')]+\frac{B}{2}[d_X(x,x)+d_X(x',x')+d_Y(y,y')] \\ &\quad +\frac{C}{2}[d_X(x,x')+d_X(x',x)+d_Y(y,y')] \end{aligned}$$

$$d_X(x,x') \le \left(\frac{A+2C}{2}\right)d_X(x,x')+\left(\frac{A+B+C}{2}\right)d_Y(y,y')$$

Similarly, we have

$$\Rightarrow\ d_Y(y,y') \le \frac{A+2C}{2}d_Y(y,y')+\frac{A+B+C}{2}d_X(x,x')$$

Adding (6) and (7) we obtain

$$d_X(x,x')+d_Y(y,y') \le \frac{A+B+C}{2-A-2C}[d_X(x,x')+d_Y(y,y')]$$

which is a contradiction as $\frac{A+B+C}{2A-2C}<1.$

Hence,

$$\begin{aligned} & d_X(x,x')+d_Y(y,y')=0 \\ \Rightarrow\ & d_X(x,x')=0 \text{ and } d_Y(y,y')=0 \\ \Rightarrow\ & x=x' \text{ and } y=y' \end{aligned}$$

Case-II: If (x,y) and $(x'y')$ are not comparable. Then $\exists\ (u,v)\in X\times Y$ such that (u,v) is comparable to both (x,y) and (x',y').

We define two sequences $\{u_n\}$ and $\{v_n\}$ such that $u_0=u$, $v_0=v$ and $u_{n+1}=F(u_n,u_n)$, $v_{n+1}=G(v_n,u_n)$

Since, $(u,v)/$ is comparable with (x,y).

We may choose $(x,y)\ge(u,v)=(u_0,u_0)$.

By the Principle of mathematical induction, it is easy to prove that

$(x,y)\ge(u_n,v_n)$ for all n.

Now

$$\begin{aligned} d_X(x,u_{n+1}) &= d_X(F(x,y),F(u_n,v_n)) \\ &\le \frac{A}{2}[d_X(x,u_n)+d_Y(y,v_n)]+\frac{B}{2}[d_X(x,F(x,y))+d_X(u_n,F(u_n,v_n))+d_Y(y,v_n)] \\ &\quad +\frac{C}{2}[d_X(x,F(u_n,v_n))+d_X(u_n,F(x,y))+d_Y(y,v_n)] \end{aligned}$$

$$= \frac{A}{2}[d_X(x,u_n) + d_Y(y,v_n)] + \frac{B}{2}[d_X(x,x) + d_X(u_n,u_{n+1}) + d_Y(y,v_n)]$$
$$+ \frac{C}{2}[d_X(x,u_{n+1})) + d_X(x,u_n) + d_Y(y,v_n)]$$

$$\Rightarrow \left(1 - \frac{C}{2}\right) d_X(x,u_{n+1}) \le \left(\frac{A+C}{2}\right) d_X(x,u_n) + \frac{A+B+C}{2} d_Y(y,v_n) + \frac{B}{2} d_X(u_n,u_{n+1})$$

$$d_X(x,u_{n+1}) \le \frac{A+C}{2-C} d_X(x,u_n) + \frac{A+B+C}{2-C} d_Y(y,v_n) + \frac{B}{2} d_X(u_n,u_{n+1})$$

Similarly, we get

$$d_Y(y,v_{n+1}) \le \frac{A+C}{2-C} d_Y(y,v_n) + \frac{A+B+C}{2-C} d_X(x,u_n) + \frac{B}{2} d_Y(v_n,v_{n+1})$$

Adding (8) and (9), we obtain

$$d_X(x,u_{n+1}) + d_Y(y,v_{n+1}) \le \frac{2A+B+2C}{2-C}[d_X(x,u_n) + d_Y(y,v_n)] + \frac{B}{2}[d_X(u_n,u_{n+1}) + d_Y(v_n,v_{n+1})]$$

Let $h = \frac{2A+B+2C}{n} < 1$,

$$\Rightarrow d_X(x,u_{n+1}) + d_Y(y,y_{n+1}) \le h[d_X(x,u_n) + d_Y(y,v_n)] + \frac{B}{2}[d_X(u_n,u_{n+1}) + d_Y(v_n,v_{n+1})]$$
$$\le h^2[d_X(x,u_{n-1}) + d_Y(y,v_{n-1})] + \frac{B}{2}[d_X(u_n,u_{n+1}) + d_Y(v_n,v_{n+1})]$$
$$\vdots$$
$$\le h^n[d_X(x,u_0) + d_Y(y,u_0)] + \frac{B}{2}[d_X(u_n,u_{n+1}) + d_Y(v_n,v_{n+1})]$$
$$\to 0 \text{ as } n \to \infty, \text{ since } h < 1.$$

$$\Rightarrow d_X(x,u_{n+1}) + d_Y(y,v_{n+1}) = 0$$
$$\Rightarrow d_X(x,u_{n+1}) = 0 \text{ and } d_Y(y,v_{n+1}) = 0$$
$$\Rightarrow x = u_{n+1} \text{ and } y = v_{n+1}$$

Similarly, we can get

$$x' = u_{n+1} \text{ and } y' = v_{n+1}$$

Hence $x = x'$ and $y = y'$.

This proves the uniqueness of the result.

Corollary: In the hypothesis of last theorem, if we take $F = G$ and $X = Y$. Then we have a unique coupled fixed point of F instead of FG-coupled fixed point.

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REFERENCES

1. T. Gnana Bhasakar, V. Lakshmikantham; Fixed point theorems in partially ordered metric spaces and applications, Nonlinear Anal. 65 (7) (2006) 1379-1393.
2. E. Prajisha, P. Shaini; FG- coupled fixed point theorems for various contractions in partially ordered metric spaces, http://arxiv.org/abs/1602.00543
3. V. Lakshmikantham, L. Ciric; Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces; Nonlinear Analysis 70 (2009) 4341-4349.
4. E. Karapinar, Couple fixed point theorems for nonlinear contractions in cone metric spaces, Comput. Math. Appl. 59 (2010), 3656-3668.
5. D. Guo, V. Lakshmikantham; Coupled fixed points of non-linear operators with applications; Non-linear Analysis Theory, Methods and Applications; Vol.11; No.5; 623-632; 1987.
6. V. Berinde; Generalized coupled fixed point theorems for mixed monotone mappings in partially ordered metric spaces; Nonlinear Analysis 74 (2011) 7347-7355.
7. F. Sabetghadam, H.P. Masiha, A.H. Sanatpour, Some coupled fixed point theorems in cone metric spaces, Fixed Point Theory. Appl.(2009); doi:10.1155/2009/125426. Art. ID 125426.
8. N. V. Luong, N. X. Thuan, Coupled fixed points in partially ordered metric spaces and application, Nonlinear Anal. 74 (2011), 983-992.
9. W. Shatanawi, Partially ordered cone metric spaces and coupled fixed point results, Comput. Math. Appl. 60 (2010), 2508- 2515.

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