

ON STRONGLY EDGE IRREGULAR BIPOLAR FUZZY GRAPHS

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ABSTRACT

In this paper, strongly edge irregular bipolar fuzzy graphs and strongly edge totally irregular bipolar fuzzy graphs are introduced. A relation between strongly edge irregular bipolar fuzzy graph and strongly edge totally irregular bipolar fuzzy graph is studied. A necessary and sufficient condition under which they are equivalent is provided. Some properties of strongly edge irregular bipolar fuzzy graphs are studied and they are examined for strongly edge totally irregular bipolar fuzzy graphs.

Keywords: Degree and total degree of a vertex in bipolar fuzzy graphs, an edge degree in bipolar fuzzy graph, total edge degree in bipolar fuzzy graph, an edge irregular bipolar fuzzy graph, totally edge irregular bipolar fuzzy graph.

AMS Mathematics Subject Classification (2010): 05C12, 03E72, 05C72.

1. INTRODUCTION

Euler first introduced the concept of graph theory in 1736. Fuzzy set theory was first introduced by Zadeh in 1965[13]. The first definition of fuzzy graph was introduced by Haufmann in 1973 based on Zadeh's fuzzy relations in 1971. In 1975, Rosenfeld introduced the concept of fuzzy graphs [7]. Now, fuzzy graphs have many applications in branches of engineering and technology. A. Nagoorgani and K.Radha introduced the concept of degree, total degree, regular fuzzy graphs in 2008 [4]. A. Nagoorgani and S.R.Latha introduced the concept of irregular fuzzy graphs, neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs in 2008 [3].

Zhang initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. Bipolar fuzzy sets whose range of membership degree is $[-1, 1]$. In bipolar fuzzy sets, membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree within $(0, 1]$ of an element indicates that the element somewhat satisfies the property, and the membership degree within $[-1, 0)$ of an element indicates the element somewhat satisfies the implicit counter property. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible[2].

M.Akram and wieslaw A.Dudek introduced regular and totally regular bipolar fuzzy graphs. Also, they introduced the notion of bipolar fuzzy line graphs and presents some of their properties [2]. Sovan Samanta and Madhumangal Pal introduced Irregular Bipolar fuzzy graphs [12]. K. Radha and N. Kumaravel introduced the concept of an edge degree, total edge degree and edge regular fuzzy graphs and discussed about the degree of an edge in some fuzzy graphs [6]. N.R.Santhi Maheswari and C.Sekar introduced neighbourly edge irregular bipolar fuzzy graphs and discussed its properties [11]. N.R.Santhi Maheswari and C.Sekar introduced an edge irregular bipolar fuzzy graphs and discussed its properties. N.R.Santhi Maheswari and C.Sekar introduced strongly edge irregular fuzzy graphs and discussed its properties [8]. N.R.Santhi Maheswari and C.Sekar introduced neighbourly edge irregular fuzzy graphs and discussed its properties [9]. N.R.Santhi Maheswari and C.Sekar introduced an edge irregular fuzzy graphs and discussed its properties [10]. These motivates us to introduce strongly edge irregular bipolar fuzzy graphs and strongly edge totally irregular bipolar fuzzy graphs and discussed some of its properties. Throughout this paper, the vertices take the membership value $A = (\mu_A^P, \mu_A^N)$ and edges take the membership value $B = (\mu_B^P, \mu_B^N)$ where $\mu_A^P, \mu_B^P \in [0, 1]$ and $\mu_A^N, \mu_B^N \in [-1, 0]$.

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2. PRELIMINARIES

We present some known definitions related to fuzzy graphs and bipolar fuzzy graphs for ready reference to go through the work presented in this paper.

By a graph, we mean a pair $G = (V, E)$, where V is the set of vertices of a graph G and E is the set of edges of a graph G . Two vertices $u, v \in V$ are said to be neighbours if $uv \in E$. The set of all vertices adjacent to v is called the open neighbourhood of v and is denoted by $N(v) = \{u \in V / uv \in E\}$. When v is also included, it is called a closed neighbourhood and it is $N[v] = N(v) \cup \{v\}$. The degree of a vertex v is the number of edges incident at v or $\deg(v) = |N(v)|$ [2].

Definition 2.1: A fuzzy graph $G : (\sigma, \mu)$ is a pair of functions (σ, μ) , where $\sigma : V \rightarrow [0, 1]$ is a fuzzy subset of a non empty set V and $\mu : V \times V \rightarrow [0, 1]$ is a symmetric fuzzy relation on σ such that for all u, v in V , the relation $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ is satisfied. A fuzzy graph G is called complete fuzzy graph if the relation $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ is satisfied [1].

Definition 2.2: A bipolar fuzzy graph with an underlying set V is defined to be the pair (A, B) , where $A = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy set on V and $B = (\mu_B^P, \mu_B^N)$ is a bipolar fuzzy set on E such that $\mu_B^P(x, y) \leq \min\{\mu_A^P(x), \mu_A^P(y)\}$ and $\mu_B^N(x, y) \geq \max\{\mu_A^N(x), \mu_A^N(y)\}$ for all $(x, y) \in E$. Here, A is called bipolar fuzzy vertex set on V and B is called bipolar fuzzy edge set on E [2].

Definition 2.3: The positive degree of a vertex $u \in G$ is defined as $d_G^P(u) = \sum \mu_B^P(u, v)$, for $uv \in E$. The negative degree of a vertex $u \in G$ is defined as $d_G^N(u) = \sum \mu_B^N(u, v)$, for $uv \in E$ and $\mu_B^P(uv) = \mu_B^N(uv) = 0$ if uv not in E . The degree of a vertex u is defined as $d_G(u) = (d_G^P(u), d_G^N(u))$ [2].

Definition 2.4: Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*(V, E)$, where $A = (\mu_A^P, \mu_A^N)$ and $B = (\mu_B^P, \mu_B^N)$ be two bipolar fuzzy sets on a non empty set V and $E \subseteq V \times V$ respectively. Then, G is said to be regular bipolar fuzzy graph if all the vertices of G have same degree (c_1, c_2) [2].

Definition 2.5: The total degree of a vertex $u \in V$ is denoted by $td(u)$ and is defined as $td_G(u) = (td_G^P(u), td_G^N(u))$, where $td_G^P(u) = \sum \mu_B^P(u, v) + \mu_A^P(u)$, $td_G^N(u) = \sum \mu_B^N(u, v) + \mu_A^N(u)$ [2].

Definition 2.6: Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*(V, E)$. Then, G is said to be totally regular bipolar fuzzy graph if all the vertices of G have same total degree (c_1, c_2) [2].

Definition 2.7: Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*(V, E)$. Then, G is said to be an irregular bipolar fuzzy graph if there exists a vertex which is adjacent to a vertices with distinct degrees [17].

Definition 2.8: Let $G: (A, B)$ be a bipolar fuzzy graph $G^*(V, E)$. Then, G is said to be a neighbourly irregular fuzzy graph if every pair of adjacent vertices have distinct degrees [7].

Definition 2.9: Let $G: (A, B)$ be a bipolar fuzzy graph $G^*(V, E)$. Then, G is said to be a highly irregular fuzzy graph if every vertex in G is adjacent to the vertices have distinct degrees [4].

Definition 2.10: Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*(V, E)$. The positive degree of an edge is defined as $d_G^P(uv) = d_G^P(u) + d_G^P(v) - 2\mu_B^P(uv)$ and the negative degree of an edge is defined as $d_G^N(uv) = d_G^N(u) + d_G^N(v) - 2\mu_B^N(uv)$. The degree of an edge is defined as $d_G(uv) = (d_G^P(uv), d_G^N(uv))$. The minimum degree of an edge is $\delta_E(G) = \wedge \{d_G(uv) : uv \in E\}$.

The maximum degree of an edge is $\Delta_E(G) = \vee \{d_G(uv) : uv \in E\}$ [6].

Definition 2.11: Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*(V, E)$. The total positive degree of an edge is defined as $td_G^P(uv) = d_G^P(u) + d_G^P(v) - \mu_B^P(uv)$ and the negative degree of an edge is defined as $td_G^N(uv) = d_G^N(u) + d_G^N(v) - \mu_B^N(uv)$. The total edge degree is defined as $td_G(uv) = (td_G^P(uv), td_G^N(uv))$. It can also be defined as $td_G(uv) = d_G(uv) + B(uv)$.

The minimum total degree of an edge is $\delta_{tE}(G) = \wedge \{td_G(uv) : uv \in E\}$.

The maximum total degree of an edge is $\Delta_{tE}(G) = \vee \{td_G(uv) : uv \in E\}$ [6].

Definition 2.12: Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*(V, E)$. Then, G is said to be neighbourly edge irregular bipolar fuzzy graph if every pair of adjacent edges have distinct degrees [8].

Definition 2.13: Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*(V, E)$. Then, G is said to be neighbourly edge totally irregular bipolar fuzzy graph if every pair of adjacent edges have distinct total degrees [8].

Definition 2.14: Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*(V, E)$. Then, G is said to be an edge irregular bipolar fuzzy graph if there exists at least one edge which is adjacent to the edges having distinct degrees [10].

Definition 2.15: Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*(V, E)$. Then, G is said to be an edge totally irregular bipolar fuzzy graph if there exists at least one edge which is adjacent to the edges having total distinct degrees [10].

Definition 2.16: Star $K_{1,n}$ with n spokes (having $n + 1$ vertices with n pendant edges) [12].

Definition 2.17: Barbell graph $B_{n,m}$ is defined by n pendant edges attached with one end of K_2 and m pendant edges attached with other end of K_2 [8].

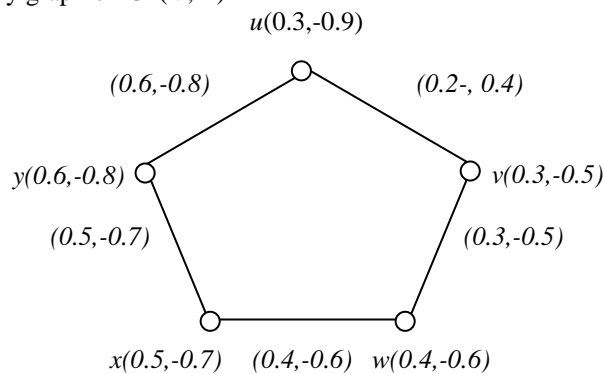
3. STRONGLY EDGE IRREGULAR BIPOLAR FUZZY GRAPHS

Definition 3.1: Let $G: (A, B)$ be a bipolar fuzzy graph, where $A = (\mu_A^P, \mu_A^N)$ and $B = (\mu_B^P, \mu_B^N)$ be two bipolar fuzzy sets on a non empty set V and $E \subseteq V \times V$ respectively. Then, G is said to be strongly edge irregular bipolar fuzzy graph if every pair of edges in G have distinct degrees [16].

Definition 3.2: Let $G: (A, B)$ be a bipolar fuzzy graph, where $A = (\mu_A^P, \mu_A^N)$ and $B = (\mu_B^P, \mu_B^N)$ be two bipolar fuzzy sets on a non empty set V and $E \subseteq V \times V$ respectively. Then, G is said to be strongly edge totally irregular bipolar fuzzy graph if every pair of edges in G have distinct total degrees [16].

Example 3.3: Graph which is both strongly edge irregular bipolar fuzzy graph and strongly edge totally irregular bipolar fuzzy graph.

Consider a bipolar fuzzy graph on $G^*(V, E)$



Here, $d_G(u) = (0.8, -1.2)$, $d_G(v) = (0.5, -0.9)$, $d_G(w) = (0.7, -1.1)$, $d_G(x) = (0.9, -1.3)$ and $d_G(y) = (1.1, -1.5)$

$$d_G^P(uv) = d_G^P(u) + d_G^P(v) - 2\mu_B^P(uv) = (0.8) + (0.5) - 2(0.2) = 0.9$$

$$d_G^N(uv) = d_G^N(u) + d_G^N(v) - 2\mu_B^N(uv) = (-1.2) + (-0.9) - 2(-0.4) = -1.3$$

$$d_G(uv) = (d_G^P(uv), d_G^N(uv)) = (0.9, -1.3).$$

$$d_G^P(vw) = d_G^P(v) + d_G^P(w) - 2\mu_B^P(vw) = (0.5) + (0.7) - 2(0.5) = 0.6$$

$$d_G^N(vw) = d_G^N(v) + d_G^N(w) - 2\mu_B^N(vw) = (-0.9) + (-1.1) - 2(-0.5) = -1.$$

$$d_G(vw) = (d_G^P(vw), d_G^N(vw)) = (0.6, -1).$$

$$d_G^P(wx) = d_G^P(w) + d_G^P(x) - 2\mu_B^P(wx) = (0.7) + (0.9) - 2(0.4) = 0.8.$$

$$d_G^N(wx) = d_G^N(w) + d_G^N(x) - 2\mu_B^N(wx) = (-1.1) + (-1.3) - 2(-0.6) = -1.2.$$

$$d_G(wx) = (d_G^P(wx), d_G^N(wx)) = (0.8, -1.2).$$

$$d_G^P(xy) = d_G^P(x) + d_G^P(y) - 2\mu_B^P(xy) = (0.9) + (1.1) - 2(0.5) = 1$$

$$d_G^N(xy) = d_G^N(x) + d_G^N(y) - 2\mu_B^N(xy) = (-1.3) + (-1.5) - 2(-0.7) = -1.4$$

$$d_G(xy) = (d_G^P(xy), d_G^N(xy)) = (1, -1.4).$$

$$d_G^P(yu) = d_G^P(y) + d_G^P(u) - 2\mu_B^P(yu) = (1.1) + (0.8) - 2(0.6) = 0.7.$$

$$d_G^N(yu) = d_G^N(y) + d_G^N(u) - 2\mu_B^N(yu) = (-1.5) + (-1.2) - 2(-0.8) = -1.1$$

$$d_G(yu) = (d_G^P(yu), d_G^N(yu)) = (0.7, -1.1).$$

Here, $d_G(uv) = (0.9, -1.3)$, $d_G(vw) = (0.6, -1)$, $d_G(wx) = (0.8, -1.2)$, $d_G(xy) = (1, -1.4)$, $d_G(yu) = (0.7, -1.1)$. It is noted that G is strongly edge irregular bipolar fuzzy graph. Also, $td_G(uv) = (1.1, -1.7)$, $td_G(vw) = (0.9, -1.5)$, $td_G(wx) = (1.2, -1.8)$, $td_G(xy) = (1.5, -2.1)$, $td_G(yu) = (1.3, -1.9)$. It is noted that G is strongly edge totally irregular bipolar fuzzy graph.

Remark 3.4: A strongly edge irregular bipolar fuzzy graph need not be strongly edge totally irregular bipolar fuzzy graph

Remark 3.5: An strongly edge totally irregular bipolar fuzzy graph need not be strongly edge irregular bipolar fuzzy graph.

Theorem 3.6: Let $G: (\sigma, \mu)$ be a connected bipolar fuzzy graph on $G^*(V, E)$ and B is a constant function. If G is strongly edge irregular bipolar fuzzy graph, then G is strongly edge totally irregular bipolar fuzzy graph.

Proof: Assume that B is a constant function, let $B(uv) = (c_1, c_2)$ for all $uv \in E$, where c_1 and c_2 are constant. Let uv and xy be any pair of edges in E . Suppose that G is strongly edge irregular bipolar fuzzy graph. Then $d_G(uv) \neq d_G(xy)$, where uv and xy are pair edges in E .

Then $d_G(uv) \neq d_G(xy)$, where uv and xy are edges in E .

$$\text{Consider } d_G(uv) \neq d_G(xy) \Rightarrow (d_G^P(uv), d_G^N(uv)) \neq (d_G^P(xy), d_G^N(xy))$$

$$\Rightarrow (d_G^P(uv), d_G^N(uv)) + (c_1, c_2) \neq (d_G^P(xy), d_G^N(xy)) + (c_1, c_2).$$

$$\Rightarrow (d_G^P(uv), d_G^N(uv)) + (B(uv)) \neq (d_G^P(xy), d_G^N(xy)) + B(xy).$$

$$\Rightarrow td_G(uv) \neq td_G(xy), \text{ where } uv \text{ and } xy \text{ are any pair of edges in } E.$$

Hence G is strongly edge totally irregular bipolar fuzzy graph.

Theorem 3.7: Let $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*(V, E)$ and B is a constant function. If G is strongly edge totally irregular bipolar fuzzy graph, then G is strongly edge irregular bipolar fuzzy graph.

Proof: Proof is similar to the above theorem 3.6.

Remark 3.8: Theorems 3.6 and 3.7 jointly yield the following result. Let $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*(V, E)$. If B is a constant function, then G is strongly edge irregular bipolar fuzzy graph if and only if G is strongly edge totally irregular bipolar fuzzy graph.

Remark 3.9: Let $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*(V, E)$. If G is both strongly edge irregular bipolar fuzzy graph and strongly edge totally irregular bipolar fuzzy graph. Then B need not be a constant function.

Theorem 3.10: Let $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*(V, E)$. If G is strongly edge irregular bipolar fuzzy graph, then G is neighbourly edge irregular bipolar fuzzy graph.

Proof: Let $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*(V, E)$.

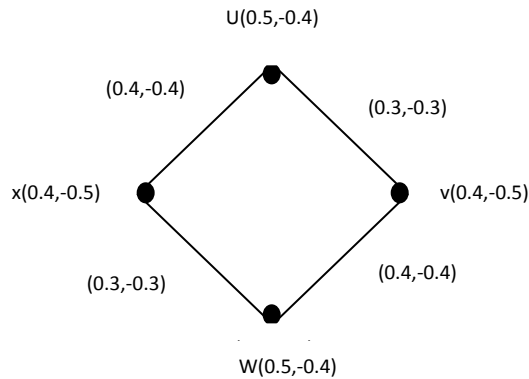
Let us assume that G is strongly edge irregular bipolar fuzzy graph \Rightarrow every pair of edges in G have distinct degrees \Rightarrow every pair of adjacent edges having distinct degrees. Hence G is neighbourly edge irregular bipolar fuzzy graph.

Theorem 3.11: Let $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*(V, E)$. If G is strongly edge totally irregular bipolar fuzzy graph, then G is neighbourly edge totally irregular bipolar fuzzy graph.

Proof: Proof is similar to the above theorem 3.10.

Remark 3.12: Converse of the above theorems 3.10 and 3.11 need not be true.

Example 3.13: Graph which is both neighbourly edge irregular bipolar fuzzy graph and neighbourly edge totally irregular bipolar fuzzy graph. Consider $G^*: (V, E)$ where $V = \{u, v, w, x\}$ and $E = \{uv, vw, wx, xu\}$.



$$d_G(u) = (0.7, -0.7) \quad d_G(v) = (0.7, -0.7) \quad d_G(w) = (0.7, -0.7) \quad d_G(x) = (0.7, -0.7)$$

$$d_G(uv) = (d_G^+(uv), d_G^-(uv)) = (0.8, -0.8), \quad d_G(vw) = (d_G^+(vw), d_G^-(vw)) = (0.6, -0.6)$$

$$d_G(wx) = (d_G^+(wx), d_G^-(wx)) = (0.8, -0.8), \quad d_G(xu) = (d_G^+(xu), d_G^-(xu)) = (0.6, -0.6).$$

Here, $d_G(uv) = (0.6, -0.6)$, $d_G(vw) = (0.8, -0.8)$, $d_G(wx) = (0.6, -0.6)$, $d_G(xu) = (0.8, -0.8)$. It is noted that every pair of adjacent edges have distinct degrees. Hence G is neighbourly edge irregular bipolar fuzzy graph, but not strongly edge irregular bipolar fuzzy graph.

$$td_G(xu) = (1.1, -1.1), \quad td_G(vw) = (1, -1).$$

$$td_G(wx) = (1.1, -1.1), \quad td_G(xu) = (1, -1).$$

It is observed that every pair of adjacent edges have distinct total degrees. So, G is neighbourly edge totally irregular bipolar fuzzy graph, but not strongly edge totally irregular bipolar fuzzy graphs.

Theorem 3.14: Let $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*(V, E)$ and B is a constant function. If G is strongly edge irregular bipolar fuzzy graph, then G is an irregular bipolar fuzzy graph.

Proof: Let $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*(V, E)$. Assume that B is a constant function, let $B(uv) = (c_1, c_2)$ for all $uv \in E$, where c_1 and c_2 are constant. Let us suppose that G is strongly edge irregular bipolar fuzzy graph. Let the edges uw and ux which are incident at the vertex u .

$$\text{Then } d_G(uw) \neq d_G(ux) \neq d_G(vy) \Rightarrow (d_G^P(uw), d_G^N(uw)) \neq (d_G^P(ux), d_G^N(ux))$$

$$\Rightarrow (d_G^P(u) + d_G^P(w) - 2c_1, d_G^N(u) + d_G^N(w) - 2c_2) \neq (d_G^P(u) + d_G^P(x) - 2c_1, d_G^N(u) + d_G^N(x) - 2c_2)$$

$$\Rightarrow d_G^P(u) + d_G^P(w) - 2c_1 \neq d_G^P(u) + d_G^P(x) - 2c_1 \quad (\text{or}) \quad d_G^N(u) + d_G^N(w) - 2c_2 \neq d_G^N(u) + d_G^N(x) - 2c_2$$

$$\Rightarrow d_G^P(u) + d_G^P(w) \neq d_G^P(u) + d_G^P(x) \quad (\text{or}) \quad d_G^N(u) + d_G^N(w) \neq d_G^N(u) + d_G^N(x)$$

$$\Rightarrow d_G^P(w) \neq d_G^P(x) \quad (\text{or}) \quad d_G^N(w) \neq d_G^N(x)$$

$$\Rightarrow (d_G^P(w), d_G^N(w)) \neq (d_G^P(x), d_G^N(x)) \Rightarrow d_G(w) \neq d_G(x)$$

\Rightarrow there exists a vertex u which is adjacent to a vertices w and x have distinct degrees. Hence G is an irregular bipolar fuzzy graph.

Theorem 3.15: Let $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*(V, E)$ and B is a constant function. If G is strongly edge totally irregular bipolar fuzzy graph, then G is an irregular bipolar fuzzy graph.

Proof: Proof is similar to the above Theorem 3.14.

Remark 3.16: Converse of the above theorems 3.14 and 3.15 need not be true.

Theorem 3.17: Let $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*(V, E)$ and B is a constant function. Let G is strongly edge irregular bipolar fuzzy graph, then G is highly irregular bipolar fuzzy graph.

Proof: Proof is similar to the above Theorem 3.14.

Definition 3.17: Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*: (V, E)$. If every pair of edges have the same edge degree and each edge e_i have edge degree (c_i, k_i) with $c_i = |k_i|$, then G is called an equally strongly edge irregular bipolar fuzzy graph. Otherwise it is unequally strongly edge irregular bipolar fuzzy graph.

Result 3.18: An equally strongly edge irregular bipolar fuzzy graph is strongly edge irregular bipolar bipolar fuzzy graph.

Result 3.19: A strongly edge irregular bipolar fuzzy graph need not be an equally strongly edge irregular bipolar fuzzy graph.

Theorem 3.20: Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*: (V, E)$, a path on $2m$ ($m > 1$) vertices. If the membership value of the edges $e_1, e_2, e_3, \dots, e_{2m-1}$ are respectively $(c_1, k_1), (c_2, k_3), \dots, (c_n, k_{2m-1})$ such that $(c_1, k_1) < (c_2, k_3) < \dots < (c_{2m-1}, k_{2m-1})$, then G is both strongly edge irregular bipolar fuzzy graph and strongly edge totally irregular bipolar fuzzy graph.

Theorem 3.21: Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*: (V, E)$, a cycle on n ($n \geq 4$) vertices. If the membership value of the edges $e_1, e_2, e_3, \dots, e_n$ are respectively $(c_1, k_1), (c_2, k_3), \dots, (c_n, k_n)$ such that $(c_1, k_1) < (c_2, k_3) < \dots < (c_n, k_n)$, then G is both strongly edge irregular bipolar fuzzy graph and strongly edge totally irregular bipolar fuzzy graph.

Theorem 3.22: Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*: (V, E)$, a star $K_{1,n}$. If the membership values of no two edges are the same, then G is strongly edge irregular bipolar fuzzy graph and G is totally edge regular bipolar fuzzy graph.

Theorem 3.23: Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*: (V, E)$, a Barbell graph $B_{n,m}$. If the membership values of no two edges are the same, then G is strongly edge irregular bipolar fuzzy graph and G is not strongly edge totally irregular bipolar fuzzy graph.

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REFERENCES

1. S. Arumugam and S. Velammal, Edge domination in graphs, *Taiwanese Journal of Mathematics*, Volume 2, Number 2, June 1998, 173-179.
2. M.Akram and Wieslaw A.Dudek, Regular bipolar fuzzy graphs, *Neural Comput & Applic* (2012), 21 (Suppl 1) S197- S205.
3. A.Nagoor Gani and S.R.Latha, On Irregular Fuzzy graphs, *Applied Mathematical Sciences*, 6 (2012), 517-523.
4. A.Nagoor Gani and R.Radha, The degree of a vertex in some fuzzy graphs, *International Journal of Algorithms, Computing and Mathematics*, Volume 2, Number 3, August 2009, 107-116.
5. S.P.Nandhini and E.Nandhini, Strongly Irregular Fuzzy graphs, *International Journal of Mathematical Archive*, Vol5 (5), 2014, 110-114.
6. K. Radha and N. Kumaravel, On Edge Regular Bipolar Fuzzy Graphs, *Annals of Pure and Applied Mathematics*, Vol.10, No.2, 2015 129-139.
7. A.Rosenfeld, fuzzy graphs, In: L.A.Zadeh, K.S.Fu, M.Shimura, EDs., *Fuzzy sets and Their Applications*, Academic press (1975), 77-95.
8. N. R. Santhi Maheswari and C. Sekar, On Strongly Edge Irregular fuzzy graphs, *Kragujevac Journal of Mathematics*, Volume 40(1) (2016), Pages 125-135.
9. N. R. Santhi Maheswari and C. Sekar, On Neighbourly Edge Irregular Fuzzy Graphs, *International Journal of Mathematical Archive*- 6(10), 2015, 224-231.
10. N. R. Santhi Maheswari and C. Sekar, On Edge Irregular fuzzy graphs, *International Journal of Mathematics and Soft computing*, Vol.6, No.2, 2016, 131-143.

11. N. R. Santhi Maheswari and C. Sekar, On Neighbourly Edge Irregular bipolar Fuzzy Graphs, *Annals of Pure and Applied Mathematics*, Vol. 11, No. 1, 2016, 1-8. ISSN: 2279-087X(P), 2279-0888 (online) Published on 1 January 2016.
12. Sovan Samantha, Madhumangal Pal, Irregular Bipolar Fuzzy Graphs, *International Journal of Application of Fuzzy sets* Vol.2, (2012), 91-102.
13. L.A. Zadeh, Fuzzy Sets, *Information and control* 8, 1965, 338-353.

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