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SOLVING SUM OF FUZZY LINEAR PLUS LINEAR FRACTIONAL PROGRAMMING PROBLEM WITH BOUNDED DECISION VARIABLES

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ABSTRACT

In this paper we take $\alpha - cut$ on the objective function and $r - cut$ on the constraints for solving the problem in which objective function is sum of fuzzy linear and linear fractional function and constraint functions are in the form of linear inequalities with bounded decision variables.

Keywords: *Linear programming problem, Linear fractional programming problem, Triangular fuzzy number.*

1. INTRODUCTION

The problem of sum of Fuzzy linear and linear fractional arises when a sum of absolute and relative terms is to be maximized. A linear program can be regarded as a special case of a linear-fractional program in which the denominator is the constant function one. The proposed method is useful for large class of fractional programming models with bounded decision variables.

Isbell and Marlow [4] solved Linear Fractional programming problem using a sequence of linear programming problems. Zoints [13] solved sum of linear and linear fractional function. Tanaka *et al.* [12] proposed the general theory of fuzzy linear programming. Chadha [2] proposed method of Dual of the sum of a linear and linear fractional program. Li and Chen [5] solved fuzzy LFP using a fuzzy programming approach. Hirche [3] also solved problems with linear-plus-linear-fractional objective functions. Schaible [8] solved sum of linear and linear fractional function. Pramanik, Dey and Giri [6] solved Multi-objective linear plus linear fractional programming problem based on Taylor series approximation. Sharma and Kumar [9] solved problems based on linear plus linear fractional interval programming problem. Singh, Kumar and Singh [10, 11] gave Fuzzy method for multi-objective linear plus linear fractional programming problem and Fuzzy multi-objective linear plus linear fractional programming problem: approximation and goal programming approach. Sadia, Gupta, Qazi and Bari [7] suggested deriving the optimum solution of Multi-objective Linear plus Linear Fractional Programming Problem. Ammar and Muamer [1] solved Fuzzy Rough linear fractional programming problem by reducing it to the multi objective fuzzy linear fractional programming problem.

To formulate the problem that should be close to real world decision situations where the parameters of objective and constraints are not fixed, we have used fuzzy parameters and fuzzy decision variables. Firstly, the given problem with bounded decision variable is converted to the problem of non-negativity. Then, we obtain an (α, r) optimal value for the given problem where the objective and decision variables are triangular fuzzy numbers. To obtain the (α, r) optimal value, we take an α-cut on the triangular fuzzy objective function and r-cut on the triangular fuzzy constraints, hence we obtain the upper and lower bounds of the problem which can be solved numerically.

The given problem with bounded decision variable is converted to the problem of non-negativity and then converted to a linear programming problem which can be solved using Simplex or any other technique.

2. THE PROBLEM

In this section firstly we describe the general form of linear plus linear fractional programming problem with bounded decision variable:

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Let us consider the problem

Max $Z(x) = Ax + \alpha + \dfrac{Bx + \beta}{Cx + \gamma}$

s.t. $ax \le b$

$x \ge l, x \in X$ (2.1)

This primal problem with bounded decision variable can be converted to the problem of non-negativity with the help of the transformation

$w = x - l \ge 0$

As given below

Max $Z(w) = Aw + \alpha_1 + \dfrac{Bw + \beta_1}{Cw + \gamma_1}$

s.t. $w \in X_1$ (X_1 is a convex polyhedral set.)

where $X_1 = \{w : aw \le b_1, w \ge 0\}$ (2.2)

Now Let $y_1 = A^T w + \alpha_1$ and $y_2 = \dfrac{w}{C^T w + \gamma_1}$

The linear transformation of the given problem will be

Max $Z(y) = y_1 + py_2 + q$

s.t $ry_2 \le s$ (2.3)

where $p = (B^T - \dfrac{\beta_1}{\gamma_1} C^T)$, $q = \dfrac{\beta_1}{\gamma_1}$, $r = a + \dfrac{b_1}{\gamma_1} C^T$, $s = \dfrac{b_1}{\gamma_1}$

Problem (2.3) is a linear programming problem of fractional programming problem (2.2) and can be solved by traditional Simplex Method and hence from the solution of problem (2.2) we get the solution of given fractional programming problem (2.1).

Now, we consider the following problem, where the cost of objective function, coefficients and decision variables are fuzzy triangular numbers:

$$Max\,\tilde{z} = \tilde{A}_j \tilde{x}_j + \tilde{\alpha} + \frac{\tilde{B}_j \tilde{x}_j + \tilde{\beta}}{\tilde{C}_j \tilde{x}_j + \tilde{\gamma}}$$

s.t. $\tilde{a}_{ij} \tilde{x}_j \le \tilde{b}_i \quad i = 1,2,...,m$ and $j = 1,2,...,n \; \tilde{x}_j \ge l, \tilde{x}_j \in X$

Here $\tilde{A}_j, \tilde{B}_j, \tilde{C}_j, \tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}, \tilde{a}_{ij}, \tilde{b}_i, \tilde{x}_j$ are fuzzy triangular numbers. This primal problem with bounded decision variable can be converted to the problem of non-negativity with the help of the transformation

$\tilde{w}_j = \tilde{x}_j - l \ge 0$ as given below:

Max $Z(w) = \tilde{A}_j \tilde{w}_j + \tilde{\alpha}_1 + \dfrac{\tilde{B}_j \tilde{w}_j + \tilde{\beta}_1}{\tilde{C}_j \tilde{w}_j + \tilde{\gamma}_1}$

s.t. $w_j \in X_1$

where $X_1 = \{w_j : \tilde{a}_{ij} \tilde{w}_j \le \tilde{d}_j, w_j \ge 0\}$ (2.4)

X_1 is a convex polyhedral set.

If *x* be the feasible solution of fractional programming problem (2.1), then there exist a feasible solution *w* ($x = w + l$) of fractional programming problem (2.4) because *w* satisfies both constraints and non negative constraints.

THEOREMS

Theorem 3.1: If x^* be the optimal solution of fractional programming problem (2.1) then there exist an optimal solution w^* ($x^* = w^* + l$) of fractional programming problem (2.4).

Theorem 3.2: If w^* be the optimal solution of fractional programming problem (2.4) then there exist $x^* = w^* + l$ which satisfy the fractional programming problem (2.1) and the extreme values of the two objective functions are equal.

4. ALGORITHM

The fuzzy triangular numbers are:

$$\tilde{A}_j = \left(A_j^{(1)}, A_j^{(2)}, A_j^{(3)}\right), \tilde{B}_j = \left(B_j^{(1)}, B_j^{(2)}, B_j^{(3)}\right),\ \tilde{C}_j = \left(C_j^{(1)}, C_j^{(2)}, C_j^{(3)}\right),\ \tilde{\alpha}_1 = \left(\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{3}\right),$$

$$\tilde{\beta}_1 = \left(\beta_1^{(1)}, \beta_1^{(2)}, \beta_1^{(3)}\right),\ \tilde{\gamma}_1 = \left(\gamma_1^{(1)}, \gamma_1^{(2)}, \gamma_1^{(3)}\right),\ \tilde{a}_{ij} = \left(a_{ij}^{(1)}, a_{ij}^{(2)}, a_{ij}^{(3)}\right),\ \tilde{w}_j = \left(w_j^{(1)}, w_j^{(2)}, w_j^{(3)}\right)$$

The above problem can be written as:

Max $\tilde{Z}$ =

$$\left(A_j^{(1)}, A_j^{(2)}, A_j^{(3)}\right)\ \left(w_j^{(1)}, w_j^{(2)}, w_j^{(3)}\right) + \left(\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{3}\right) + \frac{\left(B_j^{(1)}, B_j^{(2)}, B_j^{(3)}\right)\left(w_j^{(1)}, w_j^{(2)}, w_j^{(3)}\right) + \left(\beta_1^{(1)}, \beta_1^{(2)}, \beta_1^{(3)}\right)}{\left(C_j^{(1)}, C_j^{(2)}, C_j^{(3)}\right)\left(w_j^{(1)}, w_j^{(2)}, w_j^{(3)}\right) + \left(\gamma_1^{(1)}, \gamma_1^{(2)}, \gamma_1^{(3)}\right)}$$

s.t. $\left(a_{ij}^{(1)}, a_{ij}^{(2)}, a_{ij}^{(3)}\right)\ \left(w_j^{(1)}, w_j^{(2)}, w_j^{(3)}\right) \le \left(d_j^{(1)}, d_j^{(2)}, d_j^{(3)}\right)$

$\left(w_j^{(1)}, w_j^{(2)}, w_j^{(3)}\right) \ge 0 \quad i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$

Step-3: To determine the (α, r) optimal value, we take $\alpha - cut$ of the objective function and $r - cut$ to the constraints. Then the problem becomes:

$$\text{Max } \tilde{Z} = \left(A_j^{(2)} - A_j^{(1)}(1-\alpha), A_j^{(2)} + A_j^{(3)}(1-\alpha)\right)\left(w_j^{(2)} - w_j^{(1)}(1-\alpha), w_j^{(2)} + w_j^{(3)}(1-\alpha)\right)$$
$$+\left(\alpha_1^{(2)} - \alpha_1^{(1)}(1-\alpha), \alpha_1^{(2)} + \alpha_1^{3}(1-\alpha)\right)$$
$$+\left(\frac{\left(B_j^{(2)} - B_j^{(1)}(1-\alpha), B_j^{(2)} + B_j^{(3)}(1-\alpha)\right)\left(w_j^{(2)} - w_j^{(1)}(1-\alpha), w_j^{(2)} + w_j^{(3)}(1-\alpha)\right) + \left(\beta_1^{(2)} - \beta_1^{(1)}(1-\alpha), \beta_1^{(2)} + \beta_1^{(3)}(1-\alpha)\right)}{\left(C_j^{(2)} - C_j^{(1)}(1-\alpha), C_j^{(2)} + C_j^{(3)}(1-\alpha)\right)\left(w_j^{(2)} - w_j^{(1)}(1-\alpha), w_j^{(2)} + w_j^{(3)}(1-\alpha)\right) + \left(\gamma_1^{(2)} - \gamma_1^{(1)}(1-\alpha), \gamma_1^{(2)} + \gamma_1^{(3)}(1-\alpha)\right)}\right)$$

s.t. $\left(a_{ij}^{(2)} - a_{ij}^{(1)}(1-r), a_{ij}^{(2)} + a_{ij}^{(3)}(1-r)\right)\left(w_j^{(2)} - w_j^{(1)}(1-r), w_j^{(2)} + w_j^{(3)}(1-r)\right)$

$$\le \left(d_j^{(2)} - d_j^{(1)}(1-r), d_j^{(2)} + d_j^{(3)}(1-r)\right)$$

$w_j^{(2)} - w_j^{(1)} \ge 0,\ w_j^{(3)} - w_j^{(2)} \ge 0 \quad i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$

The above problem is equivalent to

$$\text{Max } \tilde{Z} = (A_j^{(2)} - A_j^{(1)}(1-\alpha))(w_j^{(2)} - w_j^{(1)}(1-\alpha)) + (\alpha_1^{(2)} - \alpha_1^{(1)}(1-\alpha)),$$
$$(A_j^{(2)} + A_j^{(3)}(1-\alpha))(w_j^{(2)} + w_j^{(3)}(1-\alpha)) + (\alpha_1^{(2)} + \alpha_1^{3}(1-\alpha))$$
$$+\frac{(B_j^{(2)} - B_j^{(1)}(1-\alpha))(w_j^{(2)} - w_j^{(1)}(1-\alpha)) + (\beta_1^{(2)} - \beta_1^{(1)}(1-\alpha))}{(C_j^{(2)} - C_j^{(1)}(1-\alpha))(w_j^{(2)} - w_j^{(1)}(1-\alpha)) + (\gamma_1^{(2)} - \gamma_1^{(1)}(1-\alpha))},$$
$$\frac{(B_j^{(2)} + B_j^{(3)}(1-\alpha))(w_j^{(2)} + w_j^{(3)}(1-\alpha)) + (\beta_1^{(2)} + \beta_1^{(3)}(1-\alpha))}{(C_j^{(2)} + C_j^{(3)}(1-\alpha))(w_j^{(2)} + w_j^{(3)}(1-\alpha)) + (\gamma_1^{(2)} + \gamma_1^{(3)}(1-\alpha))}$$

s.t.

$$\left((a_{ij}^{(2)}-a_{ij}^{(1)}(1-r))(w_j^{(2)}-w_j^{(1)}(1-r))\right)\le\left(d_j^{(2)}-d_j^{(1)}(1-r)\right)$$

$$\left((a_{ij}^{(2)}+a_{ij}^{(3)}(1-r))(w_j^{(2)}+w_j^{(3)}(1-r))\right)\le\left(d_j^{(2)}+d_j^{(3)}(1-r)\right)$$

$$w_j^{(2)}-w_j^{(1)}\ge 0,\;\; w_j^{(3)}-w_j^{(2)}\ge 0 \quad i=1,2,\dots,m \text{ and } j=1,2,\dots,n$$

Therefore,

Lower bound $Z^L = (A_j^{(2)}-A_j^{(1)}(1-\alpha))(w_j^{(2)}-w_j^{(1)}(1-\alpha))+(\alpha_1^{(2)}-\alpha_1^{(1)}(1-\alpha))$

$$+\frac{(B_j^{(2)}-B_j^{(1)}(1-\alpha))(w_j^{(2)}-w_j^{(1)}(1-\alpha))+(\beta_1^{(2)}-\beta_1^{(1)}(1-\alpha))}{(C_j^{(2)}-C_j^{(1)}(1-\alpha))(w_j^{(2)}-w_j^{(1)}(1-\alpha))+(\gamma_1^{(2)}-\gamma_1^{(1)}(1-\alpha))}$$

s.t.

$$\left((a_{ij}^{(2)}-a_{ij}^{(1)}(1-r))(w_j^{(2)}-w_j^{(1)}(1-r))\right)\le\left(d_j^{(2)}-d_j^{(1)}(1-r)\right)$$

$$\left((a_{ij}^{(2)}+a_{ij}^{(3)}(1-r))(w_j^{(2)}+w_j^{(3)}(1-r))\right)\le\left(d_j^{(2)}+d_j^{(3)}(1-r)\right)$$

$$w_j^{(2)}-w_j^{(1)}\ge 0,\;\; w_j^{(3)}-w_j^{(2)}\ge 0$$

And Upper Bound $Z^U =$

$$(A_j^{(2)}+A_j^{(3)}(1-\alpha))(w_j^{(2)}+w_j^{(3)}(1-\alpha))+(\alpha_1^{(2)}+\alpha_1^{3}(1-\alpha))+$$

$$\frac{(B_j^{(2)}+B_j^{(3)}(1-\alpha))(w_j^{(2)}+w_j^{(3)}(1-\alpha))+(\beta_1^{(2)}+\beta_1^{(3)}(1-\alpha))}{(C_j^{(2)}+C_j^{(3)}(1-\alpha))(w_j^{(2)}+w_j^{(3)}(1-\alpha))+(\gamma_1^{(2)}+\gamma_1^{(3)}(1-\alpha))}$$

s.t.

$$\left((a_{ij}^{(2)}-a_{ij}^{(1)}(1-r))(w_j^{(2)}-w_j^{(1)}(1-r))\right)\le\left(d_j^{(2)}-d_j^{(1)}(1-r)\right)$$

$$\left((a_{ij}^{(2)}-a_{ij}^{(3)}(1-r))(w_j^{(2)}-w_j^{(3)}(1-r))\right)\le\left(d_j^{(2)}-d_j^{(3)}(1-r)\right)$$

$$w_j^{(2)}-w_j^{(1)}\ge 0,\;\; w_j^{(3)}-w_j^{(2)}\ge 0$$

For specific values of α and r the above two problems can be solved as discussed in Section 2.

5. CONCLUSION

In the present paper (α, r) optimal value has computed for a sum of fuzzy linear plus linear fractional programming problem having bounded decision variables, which has converted to the problem of non-negativity. With certain assumptions, linear programming problem has been obtained, which can be solved using Simplex or any other technique.

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