International Journal of Mathematical Archive-8(7), 2017, 246-252
Available online through www.ijma.info ISSN 2229 – 5046

COMMUTATIVE THEOREM ON A NEAR-FIELD SPACES AND SUB NEAR-FIELD SPACES OVER A NEAR-FIELD

Dr N V NAGENDRAM*

**Professor of Mathematics,
Kakinada Institute of Technology & Science (K.I.T.S.),
Department of Humanities & Science (Mathematics),
Tirupathi (vill) Peddapuram (M), Divili 533 433,
East Godavari District. Andhra Pradesh. INDIA.**

(Received On: 04-07-17; Revised & Accepted On: 28-07-17)

ABSTRACT

The development of the general structure theory for near-field spaces and sub near-field spaces over a near-field, a great deal of work was done that showed under certain types of hypothesis, near-field spaces had to be commutative or almost commutative. For a good cross section of the kind of result that was obtained, one can look and in the bibliographies given in these.

Of these type of questions studied, one outstanding one remained open, It asked Suppose N is a near-field space in which, for any a, b $\in$ N, there are integers $m = m(a, b) \geq 1$, $n = n(a, b) \geq 1$ such that $a^m b^n = b^n a^m$. must the Commutator sub near-field space over a near-field N then be a nil sub near-field space? Equivalently, if N is as above and has no non zero nil sub near-field spaces, must N be commutative?

Keywords: *near-field spaces, sub near-field space, near-field space, semi simple near-field space.*

2000Mathematics Subject Classification: *43A10, 46B28, 46H25, 46H99, 46L10, 46M20, 51 M 10, 51 F 15, 03 B 30.*

INTRODUCTION

Some in depth study and generalization, main results cum progress on this was made. In a fairly recent articles Dr. N. V. Nagendram showed. Let N be a near-field space, M be a commutative sub near-field space over a near-field N and suppose that given s $\in$ N, $s^n \in$ M for some $n = n(s) \geq 1$. Then the commutative sub near-field space is nil sub near-field space over a near-field N. Dr. N. V. Nagendram's situation is a very special case of the question asked at the beginning, for, if $a^n \in$ M and $b^m \in$ M, then $a^n b^m = b^m a^n$, since M is commutative sub near-field space over a near-field N.

In a recent article, Dr. N. V. Nagendram introduced the concept of the Hypercenter of a near-field space over a near-field. The Hypercenter, S, of the near-field space over a near-field N is defined by $S = \{s \in N / sx^n = x^n s, n = n(x,s) \geq 1$, for all $x \in N\}$. Dr. N V Nagendram showed that if N has no non-zero nil sub near-field spaces, then S = Z, the center of N.

As pointed out, Dr. N.V. Nagendram's result followed from this theorem that identifies the center and Hypercenter. We cite the result here because we make much use of it in this paper.

The result we prove here settles the open question, mentioned at the outset, in the affirmative. We prove the

Theorem: Let N be a near-field space in which, given a, b $\in$ N, there exist integers $m = m(a, b) \geq 1$, $n = n(a, b) \geq 1$ such that $a^m b^n = b^n a^m$. Then, the Commutator sub near-field space N of a near-field is nil sub near-field space. In particular, if N has no non-zero sub near-field spaces, then N must be commutative near-field space over a near-field N.

*Corresponding Author: Dr. N. V. Nagendram Professor of Mathematics,
Kakinada Institute of Technology & Science, Tirupathi (v), Peddapuram(M),
Divili 533 433, East Godavari District, Andhra Pradesh. India. E-mail: nvn220463@yahoo.co.in.*

Proof: This theorem will be proved as a consequence of a series of lemmas, and reductions we shall make. Note that in the hypothesis of the theorem we may assume without any loss of generality that m = n, for, if $a^m b^n = b^n a^m$ then $a^{mn} b^{mn} = b^{mn} a^{mn}$.

In all that follows N will be a near-field space in which, for any pair a, b $\in$ N, there is an integer n = n(a, b) $\geq$ 1 such that $a^n b^n = b^n a^n$. we begin with a result that is known, we include it and its proof for the sake of completeness.

Lemma 1: If N is a division near-field space, then N is commutative.

Proof: we prove this lemma negation method. For that suppose the result is false. By result on the Hypercenter quoted earlier, there must be elements a and b in N such that commutes with na positive power of b. Let $C_m = \{x \in N \,/\, x\, b^{m!} = b^{m!}\, x\}$ and let $B = \bigcup_{m \geq 1} C_m$. Clearly, B is a subdivision near-field space of N and, since $a\, b^{m!} \neq b^{m!}\, a$ for all m $\geq$ 1, a$\notin$ B. Thus B $\neq$ N. However, given x $\in$ N, $x^m b^{m!} = b^{m!} x^m$ for some appropriate m $\geq$ 1, and so $x^m b^{m!} = b^{m!} x^m$. therefore, $x^m \in B\ \forall\ x \in N$. Hence N must be a near-field space. But N is not a division near-field space. With this contradiction, the lemma is proved.

Lemma 2: If N is semi-simple near-field space then N is commutative near-field space.

Proof: To settle the semi-simple near-field space case, it is enough to handle the situation in which N is primitive near-field space. Suppose then N is primitive near-field space. If N is a division n ear-field space it must be commutative by lemma 1. If N is not a division near-field space by the density theorem of the near-field of 2 x 2 matrices, D_2 over a division near-field space is a homomorphic image of a sub near-field space of N. But, then, D_2 inherits the hypothesis $a^n b^n = b^n a^n$. This however, is patently false for a = e_{11} and b = $e_{11} + e_{12}$. Thus, N is a division near-field space and so is commutative near-field space.

To prove the theorem it is enough to prove that if N has no nonzero nil sub near-field spaces, then N must be commutative near-field space. We proceed by assuming this to be false. We now make a series of reductions, based on the falsity of the theorem, then will eventually lead us to a contradiction.

Since N has no nil sub near-field spaces, N is a sub direct product of prime near-field spaces of N_x, having no nonzero nil sub near-field spaces, in which there is a non nilpotent element c_α with the following property; given a nonzero sub near-field space V_x of N_x, then $a_\alpha^{t(V_\alpha)} \in Vx$ for some integer $t(V_\alpha) \geq 1$. Since each N_α inherits the hypothesis $a^n b^n = b^n a^n$, it is enough to prove each N_α is commutative.

Thus, we may assume, henceforth, that N is a prime near-field space, having no nonzero nil sub near-fields and containing a nilpotent element c such that $c^{t(V)} \in V$ for any nonzero sub near-field V of N, where t(V) $\geq$ 1. Here we assume that J(N) $\neq$ 0, where J(N) is the Jacobson radical sub near-field space of N.

Now Jacobson radical near-field space (N) is itself a prime near-field space, and, of course, $a^n b^n = b^n a^n$ for all a, b $\in$ J(N). Also since J(N) $\neq$ 0 is a sub near-field space of N, d = $c^i \in$ (N), some i, if U $\neq$ 0 is a sub near-field space of J(N), then U $\supset$ V= J(N) U J(N) $\neq$ 0 is a sub near-field space of N. Hence, $c^k \in$ V for some k, where $c^{ki} = d^i$ is in V and so, is in U. In short, (N) has all the properties of N. If J(N) is commutative near-field space, then N is commutative near-field space. Thus, from now on, we may assume without loss of generality that N = J(N), that is, N is its own radical sub near-field space and near-field space.

Since N = J(N), given x $\in$ N $\exists$ x' $\in$ N $\ni$ x +x' + xx' = x + x' +x'x = 0. The mapping $\varphi : N \to N$ given by $\varphi(y) = (1 + x)\, y\, (1+ x') = y + xy + y'x + yx' + xyx'$ is an automorphism of N. we write it formally as $\varphi(y) = (1 + x)\, y\, (1+ x)^{-1}$. This completes the proof of the lemma.

Lemma 3: If N = J(N) has no zero-divisors, then N must be commutative near-field space.

Proof: Let Z be the center of near-field space N and suppose that x $\notin$ Z. Since the Hypercenter of N coincides with Z in our situation, x is not in the Hypercenter of N. Thus, there is an element a $\in$ N such that $xa^m \neq a^m x$ for all m > 0. By our basic hypothesis on N, we can find an integer n such that both $[(1 + x)a(1 + x)^{-1}]^n$ and $[(1 + ax)a(1 + ax)^{-1}]^n$ commute with a^n. Thus, both $a_1 = [(1 + x)a^n(1 + x)^{-1}]$ and $a_2 = [(1 + ax)a^n (1 + xa)^{-1}]$ commute with a^n.

Now $(1+x)a^n = a_1(1 + x)$, $(1 + ax)a^n = a_2(1 + ax)$ (1)

Multiply the first equation from the left by a and subtract the second equation from this, we get
$a^n(a – 1) = aa_1 – a_2 + (aa_1 – a_2a)x$ (2)

Since the left side of (2) commutes with a^n and a, a_1 and a_2 commute with a^n, we get from (2), on commuting it with a^n, that

$(aa_1 - a_2a)(xa^n - a^nx) = 0$ (3)

Since N has no zero divisors and since $xa^n \neq a^nx$, we must have, from (3), that $aa_1 = a_2a$.

Since $aa_1 = a_2a$, (2) reduces to $a^n(a - 1) = aa_1 - a_1 = a_2a - a_2 = a_2(a - 1)$ and since a is the radical, a – 1 is formally invertible, hence, $a^n = a_2$. But then $aa_1 = a_2a = a^na = a^{n+1}$, which gives us $a_1 = a^n$. Using $a_1 = a^n$ we get from (1) the contradiction that $xa^n = a^nx$. This completes the proof of the lemma.

Note: Thus we may assume that N has zero-divisor sub near-field spaces. But a prime near-field space that has nontrivial zero-devisors must have nonzero nil potent elements. Thus, we have an element $a \neq 0$ in N such that $a^2 = 0$.

Lemma 4: If $a^2= 0$, $a \neq 0$ then aNa is a nil right sub near-field space of N.

Proof: By our basic hypothesis on N, there exists an integer $n\geq 1$ such that $[(1 + a)(ax)^n(1 + a)^{-1}] = [(1 + a)(ax)(1 + a)^{-1}]^n$ and $(ax)^n$ commute. Since $a^2 = 0$, $(1 + a)^{-1} = 1 - a$ thus $[(1+a)(ax)^n(1 - a)(ax)^n] = (ax)^n [(1 - a)(ax)^n(1 - a)]$. Using $a^2 = 0$ this reduces to $(ax)^{2n} = (ax)^{2n}(1 - a)$, hence $(ax)^{2n}a = 0$, and so $(ax)^{2n+1} = 0$. Thus indeed, aN is nil sub near-field space of N. A near-field space has a nonzero nil right sub near-field space, it have a nonzero nil two sided sub near-field space. This completes the proof of the lemma.

Lemma 5: Every zero-divisor near-field space in N is nilpotent sub near-field space.

Proof: First, we recall exactly what hypothesis N carries, in addition to the basic one that $a^nb^n = b^na^n$. we have that N is prime near-field space, N = J(N), and that there is an element $c \in N$, which is not nilpotent, such that $c^{t(V)} \in V$ for any sub near-field space $V \neq 0$ of N so N has no nil sub near-field spaces. In addition, N has zero-divisors.

Suppose that ab = 0 where $a \neq 0$, $b \neq 0$. Let $\lambda = \{x \in N / xb^m = 0$ for some integer m$\}$, and let $\rho = \{ x \in N / b^mx = 0$ for some integer m$\}$. Clearly, ρ is a right sub near-field space and λ is a left sub near-field space, of N. we claim that $\rho = \lambda$. For if $r \in \lambda$, then $rb^m = 0$ for some m. If r' is the quasi-inverse of r, that is, if r + r' + r'r = 0, then $r'b^m = 0$. Now for some integer n, $(1 + r)b^{mn}(1 + r')b^{mn} = b^{mn}(1 + r)b^{mn}(1 + r')$.

Using $rb^m = r'b^m = 0$, we get from this last relation that that $b^{2mn} = b^{2mn} (1+r')$, hence, $b^{2mn}r' = 0$. But then, $b^{2mn}r = 0$ and so $r \in \rho$. Hence, $\lambda \subset \rho$. Similarly, $\rho \subset \lambda$; hence, $\rho = \lambda$ is a two sided sub near-field space of N.

Since ab = 0, $a \neq 0$, we have that $\rho = \lambda \neq 0$. Thus $c^k \in \lambda$ for some k. Hence, $c^kb^t = 0$ for some t. Let $V = \{x \in N / c^mx = 0$ for some m$\}$. As we did for λ and ρ above, we have that V is a sub near-field space of N. If $V \neq 0$ we would have that $c^r \in V$ for some r, giving us the contradiction $0 = c^mc^r = c^{m+r}$, since c is not nilpotent. Thus, V = 0. But $b^t \in V$. Hence, bt = 0. In other words, we have shown that every zero-divisor in N is nilpotent. This completes the proof of the lemma.

Lemma 6: If N is not commutative near-field space then N must be torsion-free near-field space.

Proof: If N is not commutative it must have an element $a \neq 0$ such that $a^2 = 0$. If $x \in N$, there is an integer $n \ni (1 + a)(x)^n(1 + a)^{-1}x^n = x^n(1 + a)x^n(1 + a)^{-1}$ since $(1+a)^{-1} = 1 - a$, we get from this relation that

$ax^{2n} - 2x^nax^n + x^{2n}a = ax^nax^n - x^nax^na$ (4)

If char $N \neq 2$, we can find an integer n so that both (4) holds and $(1 - a)(x)^n(1 - a)^{-1} x)^n = x^n (1 - a)x^n(1 - a)^{-1}$. This gives us , as above, that

$ax^{2n} - 2x^nax^n + x^{2n}a = x^nax^na - ax^nax^n$ (4)

Adding (4), (5) and using that char $N \neq 2$ gives us that

$ax^{2n} - 2x^nax^n + x^{2n}a = 0$, that is, that $(ax^n - x^na)a^n = x^n(ax^n - x^na)$. If N is not torsion free sub near-field space, then char $N = \rho \neq 0$ and from $(ax^n - x^na) x^n = x^n(ax^n - x^na)$ we get that $ax^{pn} - x^{pn}a = px^{n(p-1)}(ax^n - x^na) = 0$. This says that a commutes with some power of every element; hence, a must be in the Hypercenter of N. Since N has no nil sub near-field spaces, its Hypercenter is its center. Thus, $a \in Z$, the center of N. But the center of a prime near-field space has no nilpotent elements, hence this is not possible. To show that N is torsion free sub near – field space, therefore, we must merely rule out the possibility that char N = 2.

If char N = 2, then (4) reduces to $ax^{2n} + x^{2n}a = (ax^n)^2 + (x^na)^2$. Let $y = x^n$. Hence, $ay^2 + y^2a = (ay)^2 + (ya)^2$, whence, multiplying by a, $ay^2a = ayaya$.

Now, the relation $ay^2 + y^2a = (ay)^2 + (ya)^2$ comes from the fact that $(1+a)y(1+a)^{-1}$ commutes with y. But $(1+ a)y'(1+a)^{-1}$ then also commutes with y; hence, $ay^{2r} + y^{2r}a = (ay^r)^2 + (y^ra)^2$ for all r. Thus, $ay^{2r}a = ay^ray^ra$.

aN is nil sub near-field space, hence, $(ay)^{2m} = 0$ for some m. But then $ay^{2m}a = ay^{2m-1}a = ... = (ay)^{2m}a = 0$. Since $ay^{2(m+1)} + y^{2(m+1)}a = 0$, we get that $ay^{2m+1} = y^{2m+1}a$. Recalling that $y = x^n$, we have that a commutes with a power of x for every $x\in N$. Thus again, a is in the Hypercenter of N, hence, in the center of N. This can not be, since a is nilpotent. Thus, char N = 2 is not possible. In short, the only way out is that N is torsion free sub-near-field space.

Note: N is torsion-free sub near-field space, we also have that if $a^2 = 0$ and $x \in N$, then $(ax^n - x^na)x^n = x^n(ax^n - x^na)$ for some integer $n \geq 1$.

Lemma 7: If $a \neq 0$, $a^2 = 0$ and if $x \in N$, then for some integer $n \geq 1$, $(ax^n – x^na)x^n = x^n(ax^n – x^na)$.

Lemma 8: Let N be a 2- torsion free near-field space and let a, b$\in$N. Suppose that $a^n = 0$ and that $(ab – ba)b = b(ab – ba)$. Then, for any $n \geq 3$,

$$(a+b)^n = b^n + nb^{n-1}a + \frac{n(n-1)}{2}b^{n-2}(ab-ba) + \frac{n(n-1)(n-2)}{6}b^{n-3}aba.$$ Also $abab^i = b^iaba$ for any $i \geq 1$.

Proof: Since $(ab – ba)b = b(ab – ba)$, $ab^2 + b^2a = 2bab$. Multiplying from the left by *a* yields $ab^2a = 2abab$ and multiplying from the right by *a* yields $aba^2 = 2baba$. Since N is 2-torsion free sub near-field space, we get that abab = baba. Because aba commutes with b it commutes with all b^i. The proof is a straightforward method of induction. We can now finish the proof of the theorem.

Proof of the theorem: Assuming that the theorem was false, we have reduced down to the following situation. N is prime near-field space, torsion free without nil sub near-field spaces, has zero divisors, and all its zero divisors are nilpotent. Furthermore, if $a \neq 0$, $a^2 = 0$, then for any $x \in N$, there is an integer *n* such that $(ax^n - x^na)x^n = x^n(ax^n - x^na)$. we show that there lead us to a contradiction.

We claim that, given $x \in N$ and $a^2 = 0$, then $ax^r = x^ra$ for some $r \geq 1$, depending on x and a. If x is a zero divisor, then it is certainly correct, for x must be nilpotent, hence, $x^r = 0$ for some r. Thus, we may assume that x is regular that is non zero divisor. Then for some $n \geq 1$, $(ax^n - x^na)x^n = x^n(ax^n - x^na)$. Let $b = x^n$. by our basic assumption on N, there is an integer $m \geq 1$ such that $(a+b)^n b^m = b^m(a + b)^m$. Clearly, we can pick $m \geq 3$ then

$$(a+b)^m = b^m + nb^{m-1}a + \frac{m(m-1)}{2}b^{m-2}(ab-ba) + \frac{m(m-1)(m-2)}{6}b^{m-3}aba..$$

On the right-hand side, b^m, $b^{m-2}(ab - ba)$, and $b^{m-3}aba$ all commute with b, hence with b^m, since the left hand-side commutes with b^m, we end up with $mb^{m-1}ab^m = b^m(mb^{m-1}a)$. This gives us that $mb^{m-1}(ab^m - b^ma) = 0$, since N is torsion free near-field space, we have that $b^{m-1}(ab^m - b^ma)$. But $b = x^n$ and since x is regular, b, and so b^{m-1}, must be regular. The upshot of this is that $ab^m = b^ma$, which is to say that $ax^{mn} = x^{mn}a$.

Thus, *a* commutes with some power of every element in N and so *a* is in the Hypercenter of N. Since N has no nil sub near-field spaces, the Hypercenter of N is merely the center of N. Hence, the element $a \neq 0$, which is nilpotent, is in the center of the prime near-field space N. This is a contradiction ⊗. This completes the proof of the theorem.

ACKNOWLEDGEMENT

The author being a Professor is indebted to the referee for his various valuable comments leading to the improvement of the advanced research article. This work was supported by the chairman Sri B Srinivasa Rao, Kakinada Institute of Technology & Science (K.I.T.S.), R&D education Department S&H(Mathematics), Divili 533 433. Andhra Pradesh INDIA.

REFERENCES

1. A I Lightman , Rings that are radical over a commutative subring, math, Sbormik (N.S.) 83,(1970) 513-523.
2. Clay, J R , Near-rings-Genses and applications, Oxford University Press, Oxford, 1992.
3. Graninger G, Left modules for left near—rings, Doctoral theis, Univ. Of Arizona, 1988.
4. Hungerford T W, Algebra, springer-verlag, New Yark, 1974.
5. Mahmood S J and Mansouri M F, Tensor products of near—ring Modules, Kulwer academic Publishers, Netherland, 1997.
6. Mathna N M Q, Near-rings and their modules, Master's Thesis, King Saud Univ. KSA, 1990.
7. I N Herstein, two remarks on the commutativity of rings, contd. Math 7, 1955, 411-412.

8. I N Herstein Topiccs in ring theory Chicago lectures in Mathematics, University of Chicago press 11 1969.
9. I kaplansky A theorem on division rings contd., J Math , N.S. 83, 1970, 513 – 523.
10. N V Nagendram,T V Pradeep Kumar and Y V Reddy On “Semi Noetherian Regular Matrix δ–Near Rings and their extensions”, International Journal of Advances in Algebra (IJAA), Jordan, ISSN 0973 - 6964, Vol.4, No.1, (2011), pp.51-55.
11. N V Nagendram,T V Pradeep Kumar and Y V Reddy “A Note on Bounded Matrices over a Noetherian Regular Delta Near Rings”,n(BMNR-delta-NR) published in International Journal of Contemporary Mathematics, Vol.2, No.1, June 2011, Copyright@MindReaderPublications, ISSNNo:0973-6298, pp.13-19.
12. N V Nagendram,T V Pradeep Kumar and Y V Reddy “A Note on Boolean Regular Near-Rings and Boolean Regular δ–Near Rings”, (BR-delta-NR) published in International Journal of Contemporary Mathematics,IJCM Int. J. of Contemporary Mathematics ,Vol. 2, No. 1, June 2011,Copyright @ Mind Reader Publications, ISSN No: 0973-6298, pp. 29 - 34.
13. N V Nagendram,T V Pradeep Kumar and Y V Reddy “on p-Regular δ–Near-Rings and their extensions”, (PR-delta-NR) accepted and to be published in int. J. Contemporary Mathematics (IJCM),0973-6298,vol.1, no.2, pp.81-85,June 2011.
14. N V Nagendram, T V Pradeep Kumar and Y V Reddy “On Strongly Semi –Prime over Noetherian Regular δ–Near Rings and their extensions”,(SSPNR-delta-NR) published in International Journal of Contemporary Mathematics, Vol.2, No.1, June 2011, , pp.83-90.
15. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Structure Theory and Planar of Noetherian Regular δ-Near–Rings (STPLNR-delta-NR)”, International Journal of Contemporary Mathematics, IJCM ,published by IJSMA, pp.79-83, Dec, 2011.
16. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Matrix’s Maps over Planar of Noetherian Regular δ-Near–Rings (MMPLNR-delta-NR)”, International Journal of Contemporary Mathematics ,IJCM, published by IJSMA, pp.203-211, Dec, 2011.
17. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On IFP Ideals on Noetherian Regular-δ- Near Rings(IFPINR-delta-NR)”, Int. J. of Contemporary Mathematics, Copyright @ Mind Reader Publications, ISSN No: 0973-6298, Vol. 2, No. 1, pp.53-58, June 2011.
18. N V Nagendram, B Ramesh paper "A Note on Asymptotic value of the Maximal size of a Graph with rainbow connection number 2*(AVM-SGR-CN2*)" published in an International Journal of Advances in Algebra(IJAA) Jordan @ Research India Publications, Rohini, New Delhi, ISSN 0973-6964 Volume 5, Number 2 (2012), pp. 103-112.
19. N V Nagendram research paper on "Near Left Almost Near-Fields (N-LA-NF)" communicated to for 2nd intenational conference by International Journal of Mathematical Sciences and Applications, IJMSA@ mindreader publications, New Delhi on 23-04-2012 also for publication.
20. N V Nagendram, T Radha Rani, Dr T V Pradeep Kumar and Dr Y V Reddy “A Generalized Near Fields and (m, n) Bi-Ideals over Noetherian regular Delta-near rings (GNF-(m, n) BI-NR-delta-NR)", published in an International Journal of Theoretical Mathematics and Applications (TMA),Greece,Athens,dated 08-04-2012.
21. N V Nagendram,Smt.T.Radha Rani, Dr T V Pradeep Kumar and Dr Y V Reddy "Applications of Linear Programming on optimization of cool freezers (ALP-on-OCF)" Published in International Journal of Pure and Applied Mathematics, IJPAM-2012-17-670 ISSN-1314-0744 Vol-75 No-3(2011).
22. N V Nagendram "A Note on Algebra to spatial objects and Data Models(ASO-DM)" Published in international Journal American Journal of Mathematics and Mathematical Sciences, AJMMS,USA, Copyright @ Mind Reader Publications, Rohini , New Delhi, ISSN. 2250-3102, Vol.1, No.2 (Dec. 2012),pp. 233 – 247.
23. N V Nagendram, Ch Padma, Dr T V Pradeep Kumar and Dr Y V Reddy "A Note on Pi-Regularity and Pi-S-Unitality over Noetherian Regular Delta Near Rings (PI-R-PI-S-U-NR-Delta-NR)" Published in International Electronic Journal of Pure and Applied Mathematics, IeJPAM-2012-17-669 ISSN-1314-0744 Vol-75, No-4(2011).
24. N V Nagendram, Ch Padma, Dr T V Pradeep Kumar and Dr Y V Reddy "Ideal Comparability over Noetherian Regular Delta Near Rings(IC-NR-Delta-NR)" Published in International Journal of Advances in Algebra(IJAA,Jordan),ISSN 0973-6964 Vol:5,NO:1(2012), pp.43-53@ Research India publications, Rohini, New Delhi.
25. N. V. Nagendram, S. Venu Madava Sarma and T. V. Pradeep Kumar, “A Note On Sufficient Condition Of Hamiltonian Path To Complete Graphs (SC-HPCG)”, IJMA-2(11), 2011, pp.1-6.
26. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Noetherian Regular Delta Near Rings and their Extensions(NR-delta-NR)”,IJCMS,Bulgaria,IJCMS-5-8-2011,Vol.6,2011,No.6,255-262.
27. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Semi Noehterian Regular Matrix Delta Near Rings and their Extensions (SNRM-delta-NR)”,Jordan,@ResearchIndiaPublications,AdvancesinAlgebraISSN 0973-6964 Volume 4, Number 1 (2011), pp.51-55© Research India Publicationspp.51-55.
28. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Boolean Noetherian Regular Delta Near Ring (BNR-delta-NR)s”, International Journal of Contemporary Mathematics,IJCM Int. J. of Contemporary Mathematics ,Vol. 2, No. 1-2,Jan-Dec 2011 , Mind Reader Publications,ISSN No: 0973-6298,pp. 23-27.
29. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Bounded Matrix over a Noetherian Regular Delta Near Rings(BMNR-delta-NR)”, Int. J. of Contemporary Mathematics,Vol. 2, No. 1-2, Jan-Dec 2011 ,Copyright @ Mind Reader Publications, ISSN No: 0973-6298, pp.11-16.

30. N V Nagendram,Dr T V Pradeep Kumar and Dr Y V Reddy “On Strongly Semi Prime over Noetherian Regular Delta Near Rings and their Extensions(SSPNR-delta-NR)”, Int. J. of Contemporary Mathematics,Vol. 2, No. 1, Jan-Dec 2011 ,Copyright @ Mind Reader Publications ,ISSN No: 0973-6298,pp.69-74.
31. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On IFP Ideals on Noetherian Regular Delta Near Rings(IFPINR-delta-NR)”, Int. J. of Contemporary Mathematics,Vol. 2, No. 1-2, Jan-Dec 2011 ,Copyright @ Mind Reader Publications, ISSN No: 0973-6298, pp.43-46.
32. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Structure Thoery and Planar of Noetherian Regular delta-Near–Rings (STPLNR-delta-NR)”, International Journal of Contemporary Mathematics, IJCM ,accepted for Ist international conference conducted by IJSMA, New Delhi December 18,2011, pp:79-83,Copyright @ Mind Reader Publications and to be published in the month of Jan 2011.
33. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “On Matrix’s Maps over Planar of Noetherian Regular delta-Near–Rings (MMPLNR-delta-NR)”, International Journal of Contemporary Mathematics ,IJCM, accepted for Ist international conference conducted by IJSMA, New Delhi December 18, 2011, pp:203-211, Copyright @ Mind Reader Publications and to be published in the month of Jan 2011.
34. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “Some Fundamental Results on P- Regular delta-Near–Rings and their extensions (PNR-delta-NR)”,International Journal of Contemporary Mathematics ,IJCM,Jan-December’2011,Copyright@MindReader Publications, ISSN:0973-6298, vol.2, No.1-2,PP.81-85.
35. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy “A Generalized ideal based-zero divisor graphs of Noetherian regular Delta-near rings (GIBDNR- d-NR)", International Journal of Theoretical Mathematics and Applications (TMA)accepted and published by TMA, Greece, Athens,ISSN:1792- 9687 (print),vol.1, no.1, 2011, 59-71, 1792-9709 (online), International Scientific Press, 2011.
36. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "Inversive Localization of Noetherian regular Delta-near rings (ILNR- Delta-NR)" , International Journal of Pure And Applied Mathematics published by IJPAM-2012-17-668, ISSN.1314-0744 vol-75 No-3, SOFIA, Bulgaria.
37. N VNagendram1, N Chandra Sekhara Rao2 "Optical Near field Mapping of Plasmonic Nano Prisms over Noetherian Regular Delta Near Fields (ONFMPN-NR-Delta-NR)" accepted for 2nd international Conference by International Journal of Mathematical Sciences and Applications, IJMSA @ mind reader publications, New Delhi going to conduct on 15 – 16 th December 2012 also for publication.
38. N V Nagendram, K V S K Murthy (Yoga), "A Note on Present Trends on Yoga Apart From Medicine Usage and Its Applications (PTYAFMUIA)" Pubished by the International Association of Journal of Yoga Therapy, IAYT 18 th August, 2012.
39. N VNagendram, B Ramesh, Ch Padma, T Radha Rani and S V M Sarma research article "A Note on Finite Pseudo Artinian Regular Delta Near Fields (FP AR-Delta-NF)" communicated to International Journal of Advances in Algebra, IJAA, Jordan on 22 nd August 2012.
40. N V Nagendram “Amenability for dual concrete complete near-field spaces over a regular delta near-rings (ADC-NFS-R-δ-NR)" accepted for 3nd international Conference by International Journal of Mathematical Sciences and Applications, IJMSA @ mind reader publications, New Delhi going to conduct on 15 – 16 th December 2014 also for publication.
41. N V Nagendram “Characterization of near-field spaces over Baer-ideals" accepted for 4th international Conference by International Journal Conference of Mathematical Sciences and Applications, IJCMSA @ mind reader publications, New Delhi going to conduct on 19–20 th December 2015 at Asian Institute of Technology AIT, Klaung Lange 12120, Bangkok, Thailand.
42. N V Nagendram,, S V M Sarma Dr T V Pradeep Kumar “ A note on sufficient condition of Hamiltonian path to Complete Graphs” published in International Journal of Mathematical archive IJMA, ISSN 2229-5046, Vol.2, No..2, Pg. 2113 – 2118, 2011.
43. N V Nagendram, S V M Sarma, Dr T V Pradeep Kumar "A note on Relations between Barnette and Sparse Graphs" publishd in an International Journal of Mathematical Archive (IJMA), An International Peer Review Journal for Mathematical, Science & Computing Professionals, 2(12), 2011, pg no.2538-2542, ISSN: 2229 – 5046.
44. N V Nagendram "On Semi Modules over Artinian Regular Delta Near Rings(S Modules-AR-Delta-NR) Accepted and published in an International Journal of Mathematical Archive (IJMA)", An International Peer Review Journal for Mathematical, Science & Computing Professionals ISSN 2229-5046, IJMA-3-474, 2012.
45. N V Nagendram "A note on Generating Near-field efficiently Theorem from Algebraic K - Theory" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.3, No.10, Pg. 1 – 8, 2012.
46. N V Nagendram and B Ramesh on "Polynomials over Euclidean Domain in Noetherian Regular Delta Near Ring Some Problems related to Near Fields of Mappings(PED-NR-Delta-NR & SPR-NF)" Accepted and published in an International Journal of Mathematical Archive (IJMA), An International Peer Review Journal for Mathematical, Science & Computing Professionals ISSN 2229-5046,vol.3,no.8,pp no. 2998-3002, 2012.
47. N V Nagendram "Semi Simple near-fields Generating efficiently Theorem from Algebraic K-Theory" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.3, No.12, Pg. 1 – 7, 2012.
48. N V Nagendram "---------------------------" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.3, No.10, Pg. 3612 – 3619, 2012.

49. N V Nagendram, E Sudeeshna Susila, "Applications of linear infinite dimensional system in a Hilbert space and its characterizations in engg. Maths (AL FD S HS & EM)", IJMA, ISSN.2229-5046, Vol.4, No.7, Pg. 1 – 11 (19 – 29), 2013.
50. N VNagendram, Dr T V Pradeep Kumar, "Compactness in fuzzy near-field spaces (CN-F-NS)", IJMA, ISSN. 2229 – 5046, Vol.4, No.10, Pg. 1 – 12, 2013.
51. N V Nagendram,Dr T V Pradeep Kumar and Dr Y Venkateswara Reddy, " Fuzzy Bi-Γ ideals in Γ semi near – field spaces (F Bi-Gamma I G)" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.4, No.11, Pg. 1 – 11, 2013.
52. N V Nagendram," EIFP Near-fields extension of near-rings and regular delta near-rings (EIFP-NF-E-NR)" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229 - 5046, Vol.4, No.8, Pg. 1 –11, 2013.
53. N V Nagendram, E Sudeeshna Susila, "Generalization of ($\in,\in$Vqk) fuzzy sub near-fields and ideals of near-fields(GF-NF-IO-NF)", IJMA, ISSN.2229-5046, Vol.4, No.7,Pg. 1 – 11, 2013.
54. N V Nagendram, Dr T V Pradeep Kumar," A note on Levitzki radical of near-fields(LR-NF)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.4, No.4, Pg.288 – 295, 2013.
55. N V Nagendram, "Amalgamated duplications of some special near-fields(AD-SP-N-F)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.4, No.2, Pg.1 – 7, 2013.
56. N V Nagendram," Infinite sub near-fields of infinite near-fields and near-left almost near-fields (IS-NF-INF-NL-A-NF)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.4, No.2, Pg. 90 – 99, 2013.
57. N V Nagendram "Tensor product of a near-field space and sub near-field space over a near-field" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.8, No.6, Pg. 8 – 14, 2017.
58. N V Nagendram, E Sudeeshna Susila, Dr T V Pradeep Kumar "Some problems and applications of ordinary differential equations to Hilbert Spaces in Engg mathematics (SP-O-DE-HS-EM)", IJMA, ISSN.2229-5046, Vol.4, No.4, Pg. 118 – 125, 2013.
59. N V Nagendram,Dr T V Pradeep Kumar and D Venkateswarlu, " Completeness of near-field spaces over near-fields (VNFS-O-NF)" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.5, No.2, Pg. 65 – 74, 2014
60. Dr N V Nagendram "A note on Divided near-field spaces and ϕ-pseudo – valuation near-field spaces over regular δ-near-rings (DNF-ϕ-PVNFS-O-δ-NR)" published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.6, No.4, Pg. 31 – 38, 2015.
61. Dr. N V Nagendram "A Note on B_1-Near-fields over R-delta-NR (B_1-NFS-R-δ-NR)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.6, No.8, Pg. 144 – 151, 2015.
62. Dr. N V Nagendram " A Note on TL-ideal of Near-fields over R-delta-NR(TL-I-NFS-R-δ-NR)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.6, No.8, Pg. 51 – 65, 2015.
63. Dr. N V Nagendram "A Note on structure of periodic Near-fields and near-field spaces (ANS-P-NF-NFS)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.7, No.4, Pg. 1 – 7, 2016.
64. Dr. N V Nagendram "Certain Near-field spaces are Near-fields(C-NFS-NF)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.7, No.4, Pg. 1 – 7, 2016.
65. Dr. N V Nagendram "Sum of Annihilators Near-field spaces over Near-rings is Annihilator Near-field space (SA-NFS-O-A-NFS)", Published by International Journal of Mathematical Archive, IJMA, ISSN. 2229-5046, Vol.7, No.1, Pg. 125 – 136, 2016.
66. Dr. N V Nagendram "A note on commutativity of periodic near-field spaces", Published by IJMA, ISSN. 2229 - 5046, Vol.7, No. 6, Pg. 27 – 33, 2016.
67. Dr N V Nagendram "Densely Co-Hopfian sub near-field spaces over a near-field, IMA, ISSN No.2229-5046, 2016, Vol.7, No.10, Pg 1-12.
68. Dr N V Nagendram, "Closed(or open) sub near-field spaces of commutative near-field space over a near-field", 2016, Vol.7, No,9,ISSN NO.2229 – 5046, Pg No.57 – 72.
69. Dr N V Nagendram, "Homomorphism of near-field spaces over a near-field "IJMA Jan 2017, Vol.8, No, 2, ISSN NO.2229 – 5046, Pg No. 141 – 146.
70. Dr N V Nagendram, "Sigma – toe derivations of near-field spaces over a near-field "IJMA Jan 2017, Vol.8, No, 4, ISSN NO.2229 – 5046, Pg No. 1 – 8.
71. Dr N V Nagendram, "On the hyper center of near-field spaces over a near-field "IJMA Feb 2017, Vol.8, No, 2, ISSN NO. 2229 – 5046, Pg No. 113 – 119.

Source of support: Nil, Conflict of interest: None Declared.