

## SOME RESULTS ON DEGREE SET OF FUZZY GRAPHS

K. RADHA<sup>1\*</sup>, A. ROSEMINE<sup>2</sup>

<sup>1</sup>P.G. & Research Department of Mathematics,  
 Periyar E.V.R. College, Tiruchirappalli – 620023, Tamil Nadu, India.

<sup>2</sup>P.G. & Research Department of Mathematics,  
 Periyar E.V.R. College, Tiruchirappalli – 620023, Tamil Nadu, India,

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### ABSTRACT

*In this paper characterization of degree set in certain fuzzy graphs are given with counter examples.*

**Keywords:** degree of a vertex, degree sequence of a fuzzy graph, degree set of a fuzzy graph.

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### I. INTRODUCTION

The phenomena of uncertainty in real life situation were described in a mathematical framework by Zadeh in 1965. Fuzzy graph is the generalization of the ordinary graph. Therefore it is natural that though fuzzy graph inherits many properties similar to those of ordinary graph, it deviates at many places. A. Nagoor gani and K. Radha introduced incidence sequence of a fuzzy graph in [6]. K.Radha and A.Rosemine introduced degree sequence of fuzzy graph in [9] and introduced degree set of a fuzzy graph in [10]. In this paper we gave characterizations of degree sets for certain fuzzy graphs.

### II. PRELIMINARIES

A summary of basic definitions is given, which can be found in [1] - [10].

A fuzzy graph  $G$  is a pair of functions  $G: (\sigma, \mu)$  where  $\sigma$  is a fuzzy subset of a non empty set  $V$  and  $\mu$  is a symmetric fuzzy relation on  $\sigma$  (i.e.)  $\mu(xy) \leq \sigma(x) \wedge \sigma(y) \forall x, y \in V$ . The underlying crisp graph of  $G: (\sigma, \mu)$  is denote by  $G^*: (V, E)$  where  $E \subseteq V \times V$

In a fuzzy graph  $G: (\sigma, \mu)$  degree of vertex  $\mu \in V$  is  $d(u) = \sum_{u \neq v} \mu(uv)$ , The minimum degree of  $G$  is  $(G) = \wedge \{d_G(u) / u \in V\}$ , the maximum degree of  $G$  is  $\Delta(G) = \{d_G(u) / u \in V\}$ .

A sequence of real numbers  $(d_1, d_2, d_3, \dots, d_n)$  with  $d_1 \geq d_2 \geq d_3 \geq \dots \geq d_n$ , where  $d_i$  is equal to  $d(v_i)$ , is the degree sequence of a fuzzy graph. A sequence  $S = (d_1, d_2, d_3, \dots, d_n)$  of real numbers is said to be fuzzy graphic sequence if there exists a graph  $G$  whose vertices have degree  $d_i$  and  $G$  is called realization of  $S$ . A degree sequence of real numbers in which no two of its elements are equal is called perfect degree sequence. In crisp graph theory there is no perfect degree sequence. But fuzzy graphs may have perfect degree sequence. A degree sequence of real numbers in which exactly two of its elements are same is called quasi- perfect. A homomorphism of fuzzy graphs  $h: G \rightarrow G'$  is a map  $h: V \rightarrow V'$  such that  $\sigma(x) \leq \sigma'(h(x)) \forall x \in V$ ,  $\mu(xy) \leq \mu'(h(x)h(y)) \forall x, y \in V$ . A weak isomorphism of fuzzy graphs  $h: G \rightarrow G'$  is a map  $h: V \rightarrow V'$  which is a bijective homomorphism that satisfies  $\sigma(x) = \sigma'(h(x)) \forall x \in V$ ,  $\mu(xy) \leq \mu'(h(x)h(y)) \forall x, y \in S$ . A co-weak isomorphism of fuzzy graphs  $h: G \rightarrow G'$  is a map  $h: V \rightarrow V'$  which is a bijective homomorphism that satisfies  $\sigma(x) \leq \sigma'(h(x)) \forall x \in V$ ,  $\mu(xy) = \mu'(h(x)h(y)) \forall x, y \in S$ .

**Corresponding Author: K. Radha<sup>1\*</sup>,**

**<sup>1</sup>P.G. & Research Department of Mathematics,  
 Periyar E.V.R. College, Tiruchirappalli – 620023, Tamil Nadu, India.**

An isomorphism  $h: G \rightarrow G'$  is a map  $h: V \rightarrow V'$  which is a bijective that satisfies  $\sigma(x) = \sigma'(h(x)) \forall x \in V$ ,  $\mu(xy) = \mu'(h(x)h(y)) \forall x, y \in S$ . The union of two fuzzy graphs  $G_1: (\sigma_1, \mu_1)$  and  $G_2: (\sigma_2, \mu_2)$  is  $G_1$  defined to be a fuzzy graph  $G = G_1 \cup G_2: (\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$  where  $V = V_1 \cup V_2$  and  $E = E_1 \cup E_2$  with

$$\sigma_1 \cup \sigma_2(u) = \begin{cases} \sigma_1(u) & \text{if } u \in V_1 - V_2 \\ \sigma_2(u) & \text{if } u \in V_2 - V_1 \\ \sigma_1(u) \vee \sigma_2(u) & \text{if } u \in V_1 \cap V_2 \end{cases}$$

$$\text{and } \mu_1 \cup \mu_2(e) = \begin{cases} \mu_1(e) & \text{if } e \in E_1 - E_2 \\ \mu_2(e) & \text{if } e \in E_2 - E_1 \\ \mu_1(e) \vee \mu_2(e) & \text{if } e \in E_1 \cap E_2 \end{cases}$$

The join of two fuzzy graphs  $G_1: (\sigma_1, \mu_1)$  and  $G_2: (\sigma_2, \mu_2)$  is  $G_1$  defined to be a fuzzy graph  $G_1 + G_2: (\sigma_1 + \sigma_2, \mu_1 + \mu_2)$  where  $V = V_1 \cup V_2$  and  $E = E_1 \cup E_2 \cup E'$  where  $E'$  is the set of all edges joining the vertices of  $V_1$  with vertices of  $V_2$  such that

$$\text{and } (\sigma_1 + \sigma_2)(u) = (\sigma_1 \cup \sigma_2)(u) \text{ for all } u \in V_1 \cup V_2$$

$$\text{and } (\mu_1 + \mu_2)(u) = \begin{cases} (\mu_1 \cup \mu_2)(uv) & \text{if } uv \in E_1 \cup E_2 \\ \sigma_1(u) \wedge \sigma_2(u) & \text{if } uv \in E' \end{cases}$$

The set of distinct positive real numbers occurring in a degree sequence of a fuzzy graph is called its degree set.

A set of positive real numbers is called a degree set if it is the degree set of some fuzzy graph. The fuzzy graph is said to realize the degree set.

### 3. CHARACTERIZATIONS

**Theorem 3.1:** Let  $d$  be any positive real number, then  $\{d\}$  is a degree set of a fuzzy graph on a cycle if and only if  $0 < d \leq 2$ .

**Proof:** Let  $C_n$  be a cycle  $v_1 v_2 v_3 v_4 \dots v_n v_1$  on  $n$  vertices where  $n$  is any positive integer.

Suppose that  $\{d\}$  is the degree set of a fuzzy graph  $(\sigma, \mu)$  on  $C_n$ .

$$\text{Then } d(v_i) = \mu(v_{i-1} v_i) + \mu(v_i v_{i+1}) \quad \forall i=1, 2, \dots, n.$$

$$\leq 1 + 1$$

$$\leq 2.$$

$$\text{Since } \mu(v_i v_{i+1}) > 0, d(v_i) > 0, \forall i=1, 2, \dots, n.$$

$$\text{Hence } 0 < d \leq 2.$$

Conversely, assume that  $0 < d \leq 2$ . Then  $0 < d/2 \leq 1$ . Assign  $\mu(v_i v_{i+1}) = d/2 \quad \forall i=1, 2, \dots, n$  where  $v_{n+1} = v_n$ . Assign any value satisfying the condition of fuzzy graph as  $\sigma(v_i)$  for all  $i$ . Then

$$d(v_i) = \mu(v_{i-1} v_i) + \mu(v_i v_{i+1}) \quad \forall i=1, 2, \dots, n.$$

$$= \frac{d}{2} + \frac{d}{2}$$

$$= d, \quad \forall i=1, 2, \dots, n.$$

Thus  $\{d\}$  is a degree set of a fuzzy graph  $(\sigma, \mu)$  on  $C_n$ .

**Theorem 3.2:** Let  $d$  be any positive real number, then  $\{d\}$  is a degree set of a fuzzy graph on a complete graph  $K_n$  if and only if  $0 < d \leq n-1$ .

**Proof:** Let  $K_n$  be a complete graph on  $n$  vertices say,  $v_1, v_2, v_3, v_4, \dots, v_n$ . Suppose that  $\{d\}$  is a degree set of a fuzzy graph on a complete graph  $K_n$ .

$$\text{Then } d(v_i) = \sum_{j=1, j \neq i}^n \mu(v_i v_j) \quad \forall i=1, 2, \dots, n$$

$$\leq \sum_{j=1, j \neq i}^n 1 \quad \forall i=1, 2, \dots, n$$

$$\leq n-1$$

$$\text{Since } \mu(v_i v_{i+1}) > 0 \quad \forall i=1, 2, \dots, n.$$

$$\text{Hence } 0 < d \leq n-1.$$

Conversely assume that  $0 < d \leq n-1$ . Then  $0 < d/(n-1) \leq 1$ . Assign  $\mu(v_i v_j) = d/(n-1) \forall i \neq j$

Assign any value in  $[\frac{d}{n-1}, 1]$  as  $\sigma(v_i)$  for all  $i$ .

$$\begin{aligned} \text{Then } d(v_i) &= \sum_{j=1, j \neq i}^n \mu(v_i v_j) \quad \forall i=1, 2, \dots, n \\ &= \sum_{j=1, j \neq i}^n \frac{d}{n-1} \quad \forall i=1, 2, \dots, n \\ &= (n-1) \frac{d}{n-1} \quad \forall i=1, 2, \dots, n \\ &= d. \quad \forall i=1, 2, \dots, n \end{aligned}$$

Thus  $\{d\}$  is a degree set of a fuzzy graph  $(\sigma, \mu)$  on  $K_n$ .

**Theorem 3.3:** Let  $d$  be any positive real number, then  $\{d\}$  is a degree set of a fuzzy graph on a path  $P_n$  if and only if  $0 < d \leq 1$  and  $n=2$ .

**Proof:** Let  $P_n$  be a path on  $n$  vertices say,  $v_1, v_2, v_3, v_4, \dots, v_n$ . Suppose that  $\{d\}$  is a degree set of a fuzzy graph  $(\sigma, \mu)$  on a path  $P_n$ .

Then  $d(v_i) = d \quad \forall i=1, 2, \dots, n, n \geq 2$ . Therefore  $d = d(v_1) = \mu(v_1 v_2) \Rightarrow 0 < d \leq 1$ .

If possible assume that  $n \geq 2$ . then  $d(v_2) = \mu(v_1 v_2) + \mu(v_2 v_3)$   
 $= d + \mu(v_2 v_3)$   
 $> d$  which is a contradiction.

Hence  $n = 2$ .

Conversely, assume that  $0 < d \leq 1$  and  $n = 2$ .

A path  $P_2$  on two vertices is same as the complete graph  $K_2$  on two vertices. Therefore by theorem 3.2,  $\{d\}$  is a degree set of a fuzzy graph  $(\sigma, \mu)$  on a path  $P_2$ .

**Theorem 3.4:** Let  $d$  be any positive real number, then  $\{d\}$  is a degree set of a fuzzy graph on a star  $K_{1,n}$  if and only if  $0 < d \leq 1$  and  $n=1$ .

**Proof:** Let  $K_{1,n}$  be star  $v_0, v_1, v_2, v_3, \dots, v_n$  with  $n+1$  vertices and  $v_0$  as its center. Let  $\{d\}$  be a degree set of a fuzzy graph  $(\sigma, \mu)$  on a star  $K_{1,n}$ . Then  $d = d(v_i) = \mu(v_i v_0), \quad \forall i=1, 2, \dots, n$   
 $\Rightarrow 0 < d \leq 1$ .

$$\begin{aligned} \text{Also } d(v_0) &= \sum_{i=1}^n \mu(v_i v_0) \\ &= \sum_{i=1}^n d \\ &= nd. \end{aligned}$$

Since  $\{d\}$  is a degree set of a fuzzy graph on a star  $K_{1,n}$ ,  $d(v_0) = d$ .

Therefore  $nd = d \Leftrightarrow n=1$ . Hence the proof.

**Theorem 3.5:** If  $\{d_1, d_2\}$  is the degree set of a fuzzy graph on a cycle then  $0 < d_i \leq 2$  for  $i=1, 2$ .

**Proof:** Let  $\{d_1, d_2\}$  be the degree set of a fuzzy graph on a cycle  $v_1 v_2 v_3 v_4 \dots v_n v_1$

Then  $d_1 = d(v_i)$  for some  $i$

$d_2 = d(v_j)$  for some  $j$

Therefore  $d_1 = \mu(v_{i-1} v_i) + \mu(v_i v_{i+1}) \leq 2$  and  $d_2 = \mu(v_{j-1} v_j) + \mu(v_j v_{j+1}) \leq 2$ .

Also  $\mu(v_i v_{i+1}) > 0 \quad \forall i=1, 2, \dots, n$ . Hence  $0 < d_i \leq 2$  for  $i=1, 2$ .

**Theorem 3.6:** If  $\{d_1, d_2, d_3, \dots, d_n\}$  is the degree set of a fuzzy graph on a cycle then  $0 < d_i \leq 2$  for  $i=1, 2, \dots, n$ .

**Proof:** The proof is similar to the proof of the theorem 3.5, since each  $d_i$  is the sum of the two edges incident on it.

**Theorem 3.7:** If there exists a  $t \in (0,1]$  such that  $0 < d_i - t \leq 1$ ;  $i=1, 2$  then  $\{d_1, d_2\}$  is the degree set of a fuzzy graph on an even cycle.

**Proof:** Consider an even cycle  $C: v_1 v_2 v_3 v_4 \dots v_{2n} v_1$

Assign  $\mu(v_1 v_2) = \mu(v_3 v_4) = \mu(v_5 v_6) = \dots = \mu(v_{2n-1} v_{2n}) = t$  and  $\mu(v_2 v_3) = d_1 - t$ ;  $\mu(v_{2n} v_1) = d_2 - t$ . For all the remaining edges assign either  $d_1 - t$  or  $d_2 - t$  as their membership values.

$$\begin{aligned} \text{Then, } d(v_1) &= \mu(v_1 v_2) + \mu(v_{2n} v_1) \\ &= t + d_2 - t \\ &= d_2. \end{aligned}$$

$$\begin{aligned} d(v_2) &= \mu(v_1 v_2) + \mu(v_2 v_3) \\ &= t + d_1 - t \\ &= d_1. \end{aligned}$$

Similarly all the remaining vertices are of either degree  $d_1$  or  $d_2$ .

Assign any value satisfying the condition of fuzzy graph as the membership values of the vertices. Thus  $\{d_1, d_2\}$  is the degree set of a fuzzy graph on an even cycle.

**Example 3.8:** Let us illustrate the procedure described in the above theorem by the following example.

Let  $d_1 = 1.9$ ,  $d_2 = 1.5$ , then  $t \geq 0.9$ , take  $t = 0.9$ . Consider a fuzzy graph  $G$  on cycle with six vertices  $v_1 v_2 v_3 v_4 v_5 v_6$  and assign  $\mu(v_1 v_2) = \mu(v_3 v_4) = \mu(v_5 v_6) = 0.9$ ,  $\mu(v_2 v_3) = d_1 - t = 1.9 - 0.9 = 1$  and  $\mu(v_4 v_5) = \mu(v_6 v_1) = d_2 - t = 1.5 - 0.9 = 0.6$ , then  $d(v_1) = d(v_4) = d(v_5) = d(v_6) = 1.5$ ,  $d(v_2) = d(v_3) = 1.9$  (fig.1). Hence the following fuzzy graph is of degree set  $\{1.9, 1.5\}$

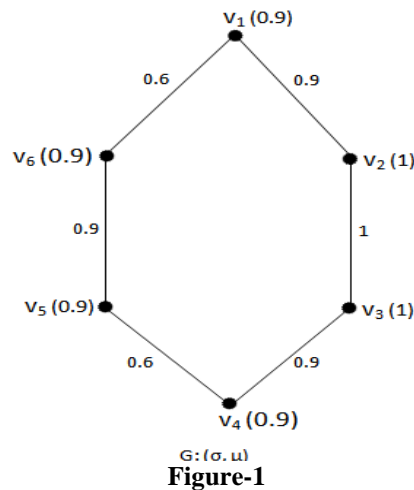


Figure-1

**Theorem 3.9:** If there exists a  $t \in (0,1]$  such that  $0 < d_i - t \leq 1$ ;  $i=1, 2$  and  $2t$  is either equal to  $d_1$  or  $d_2$  then  $\{d_1, d_2\}$  is the degree set of a fuzzy graph on an odd cycle.

**Proof:** Consider an odd cycle  $C: v_1 v_2 v_3 v_4 \dots v_{2n+1} v_1$

Assign  $\mu(v_1 v_2) = \mu(v_3 v_4) = \mu(v_5 v_6) = \dots = \mu(v_{2n+1} v_1) = t$  and  $\mu(v_2 v_3) = d_2 - t$ ;  $\mu(v_{2n} v_{2n+1}) = d_1 - t$ . For all the remaining edges assign either  $d_1 - t$  or  $d_2 - t$  as their membership values. Then,

$$\begin{aligned} d(v_1) &= \mu(v_1 v_2) + \mu(v_{2n+1} v_1) \\ &= t + t \\ &= 2t. \end{aligned}$$

$$\begin{aligned} d(v_2) &= \mu(v_1 v_2) + \mu(v_2 v_3) \\ &= t + d_2 - t \\ &= d_2. \end{aligned}$$

$$\begin{aligned} d(v_{2n+1}) &= \mu(v_{2n} v_{2n+1}) + \mu(v_{2n+1} v_1) \\ &= d_1 - t + t \\ &= d_1. \end{aligned}$$

Similarly all the remaining vertices are of either degree  $d_1$  or  $d_2$ .

Assign any value satisfying the condition of fuzzy graph as the membership values of the vertices. Thus  $\{d_1, d_2\}$  is the degree set of a fuzzy graph on an odd cycle.

**Example 3.10:** Let us illustrate the procedure described in the above theorem by the following example.

Let  $d_1 = 2$ ,  $d_2 = 1.5$ , then  $t \geq 1$ , take  $t = 1$ .

Consider a fuzzy graph  $G$  on cycle with five vertices  $v_1 v_2 v_3 v_4 v_5$  and

Assign  $\mu(v_1 v_2) = \mu(v_3 v_4) = \mu(v_5 v_1) = 1$ ,  $\mu(v_2 v_3) = d_1 - t = 2 - 1 = 1$  and  $\mu(v_4 v_5) = d_2 - t = 1.5 - 1 = 0.5$ , then  $d(v_1) = 2(t) = d_1 = 2$ ,  $d(v_4) = d(v_5) = 1.5$ ,  $d(v_2) = d(v_3) = 2$ .

Hence the following fuzzy graph is of degree set  $\{2, 1.5\}$

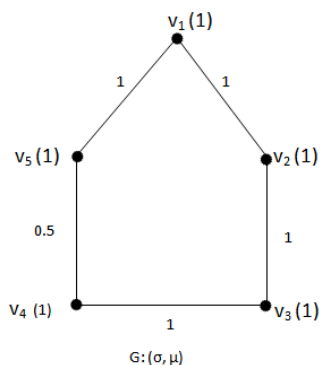


Figure-2

**Theorem 3.11:** If  $d_1$  and  $d_2$  are two positive real numbers, then  $\{d_1, d_2\}$  is a degree set of a fuzzy graph on a star  $K_{1,n}$  if and only if  $0 < d_2 \leq 1$  and  $d_1 = nd_2$ .

**Proof:** Let  $K_{1,n}$  be star  $v_0, v_1, v_2, v_3, \dots, v_n$  with  $n+1$  vertices and  $v_0$  as its center.

Let  $\{d_1, d_2\}$  is a degree set of a fuzzy graph  $(\sigma, \mu)$  on a path  $K_{1,n}$ .

Then  $d(v_i) = \mu(v_i v_0)$ ,  $\forall i=1, 2, \dots, n$ ,  $d(v_0) = \sum_{i=1}^n \mu(v_i v_0)$  and  $d_1 < d_2$

We must have  $d_2 = d(v_i) = \sum_{i=1}^n \mu(v_i v_0)$  and  $d_1 = d(v_0)$ ,

Therefore  $0 < d_2 \leq 1$  and  $d(v_0) = \sum_{i=1}^n \mu(v_i v_0)$   
 $= \sum_{i=1}^n d_2$   
 $= nd_2$ .

Conversely assume that  $0 < d_1 \leq 1$  and  $d_1 = nd_2$ . Assign  $\mu(v_i v_0) = d_2 \forall i=1, 2, \dots, n$ . Assign any value satisfying the condition of fuzzy graph as the membership values of the vertices. Then  $d(v_i) = \mu(v_i v_0) = d_2$  and

$$\begin{aligned} d(v_0) &= \sum_{i=1}^n \mu(v_i v_0) \\ &= \sum_{i=1}^n d_2 \\ &= nd_2 = d_1. \end{aligned}$$

Hence  $\{d_1, d_2\}$  is a degree set of a fuzzy graph  $(\sigma, \mu)$  on a star  $K_{1,n}$ .

**Example 3.12:** Let us illustrate the procedure described in the above theorem by the following example.

Consider the set  $\{1.5, 0.5\}$ . Let  $d_1 = 1.5$ ,  $d_2 = 0.5$ .

Consider a fuzzy graph  $G$  on a star  $K_{1,3}$  with vertices  $v_0 v_1 v_2 v_3$

Assign  $\mu(v_0 v_i) = 0.5$  for  $i = 1, 2, 3, 4$ .

Then  $d(v_0) = 1.5 = d_1$ ,  $d(v_1) = d(v_2) = d(v_3) = d(v_4) = 0.5$ .

Hence  $\{1.5, 0.5\}$  is the degree set of the fuzzy graph  $G:(\sigma, \mu)$  on the cycle in figure 3

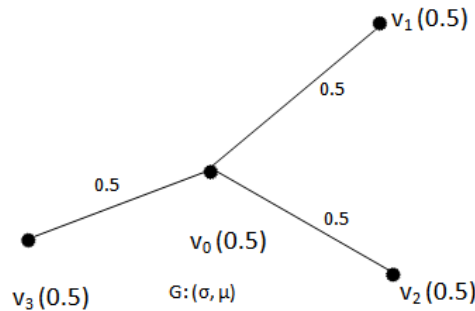


Figure-3

**Theorem 3.13:** If  $d_1$  and  $d_2$  are two real numbers such that  $0 < d_2 \leq 3$  and  $n d_2 = 3 d_1$  then  $\{d_1, d_2\}$  is the degree set of a fuzzy graph on a wheel  $W_n$ .

**Proof:** Let  $W_n$  be wheel on  $v_0, v_1, v_2, v_3, \dots, v_n$  with  $n+1$  vertices and  $v_0$  as its center.

Since  $0 < d_2 \leq 3$ ,  $0 < \frac{d_2}{3} \leq 1$ . Assign  $\mu(v_i v_{i+1}) = \frac{d_2}{3} \forall i=1,2,\dots,n$  where  $v_{n+1} = v_1$  and  $\mu(v_0 v_i) = \frac{d_2}{3} \forall i=1,2,\dots,n$ . Assign any value satisfying the condition of fuzzy graph as the membership values of the vertices.

$$\begin{aligned} \text{Then for } i=1,2,\dots,n \quad d(v_i) &= \mu(v_i v_{i+1}) + \mu(v_i v_0) + \mu(v_{i-1} v_i) \\ &= \frac{d_2}{3} + \frac{d_2}{3} + \frac{d_2}{3} \\ &= d_2. \end{aligned}$$

$$\begin{aligned} \text{and} \quad d(v_0) &= \sum_{i=1}^n \mu(v_i v_0) \\ &= \sum_{i=1}^n \frac{d_2}{3} \\ &= n \frac{d_2}{3} \\ &= d_1. \end{aligned}$$

Hence  $\{d_1, d_2\}$  is a degree set of a fuzzy graph  $(\sigma, \mu)$  on a star  $W_n$ .

**Remark 3.14:** For any non negative real number  $d$ ,  $\{d\}$  cannot be a degree set of a fuzzy graph on a bistar.

**Theorem 3.15:** Let  $d_1$  and  $d_2$  be any two nonnegative real number such that  $d_1 < d_2$ . Then  $\{d_1, d_2\}$  is a degree set of a fuzzy graph on a bistar with  $2m+2$  vertices  $m \geq 1$  if and only if  $0 < d_2 \leq 1$  and  $0 < d_1 - m d_2 \leq 1$ .

**Proof:** Let  $G$  be a bistar on  $n$  vertices  $v_0, v_1, v_2, v_3, \dots, v_m, u_0, u_1, u_2, u_3, \dots, u_m$  which is obtained by joining the apex vertices  $v_0$  and  $u_0$  of the two stars with the vertex sets  $(v_0, v_1, v_2, v_3, \dots, v_m)$  and  $(u_0, u_1, u_2, u_3, \dots, u_m)$ .

Suppose that  $\{d_1, d_2\}$  is a degree set of a fuzzy graph  $G:(\sigma, \mu)$  on a bistar graph.

For  $i=1,2,3,\dots,m$ ,  $d(u_i)$  and  $d(v_i)$  are the membership value of the edge  $u_i u_0, v_i v_0$  respectively.

$$\text{Also, } d(v_0) = \sum_{i=1}^m \mu(v_i v_0) + \mu(u_0 v_0)$$

$$d(u_0) = \sum_{i=1}^m \mu(v_i u_0) + \mu(u_0 v_0).$$

Since  $d_1 < d_2$ ,  $d(v_0) = d(u_0) = d_1$  and  $d(v_i) = d(u_i) = d_2$  for  $i=1,2,\dots,m$

$$\text{Therefore } 0 < d_2 \leq 1 \text{ and } d_2 = \sum_{i=1}^m d_1 + \mu(u_0 v_0) = m d_1 + \mu(u_0 v_0)$$

Hence  $0 < d_1 - m d_2 \leq 1$ .

Conversely  $\{d_1, d_2\}$  satisfy the given hypothesis. Assign  $\mu(v_0 v_i) = \mu(u_0 u_i) = d_2$ .

$\mu(v_0 u_0) = d_1 - m d_2$ . Assign any value satisfying the condition of fuzzy graph as the membership values of the vertices.

Then  $d(v_i) = \mu(v_i v_0) = d_2$ ,  $d(u_i) = \mu(u_i u_0) = d_2$  for  $i=1,2,3,\dots,m$  and

$$\begin{aligned} d(v_0) &= \sum_{i=1}^m \mu(v_i v_0) + \mu(u_0 v_0) \\ &= \sum_{i=1}^m m d_2 + d_1 - m d_2 \\ &= d_1. \end{aligned}$$

Hence  $\{d_1, d_2\}$  is an edge degree set of a fuzzy graph  $(\sigma, \mu)$  on a bistar with  $2m+2$  vertices.

The above theorem 3.15 can also be stated as follows:

**Theorem 3.16:**  $\{d_1, d_2\}$  is a degree set of a fuzzy graph on a bistar if and only if  $0 < d_2 \leq 1$  and  $\left(\frac{n-2}{2}\right) d_2 < d_1 \leq \left(\frac{n-2}{2}\right) d_2 + 1$  where  $n$  is an even integer greater than or equal to 4.

**Proof:** Let  $m = \left(\frac{n-2}{2}\right)$  then  $\left(\frac{n-2}{2}\right) d_2 < d_1 \leq \left(\frac{n-2}{2}\right) d_2 + 1 = m d_2 < d_1 \leq m d_2 + 1$   
 $= 0 < d_1 - m d_2 \leq 1$ .

Hence the proof is similar to the proof of the theorem 3.13.

**Example 3.17:** Let us illustrate the procedure described in the above theorem by the following example.

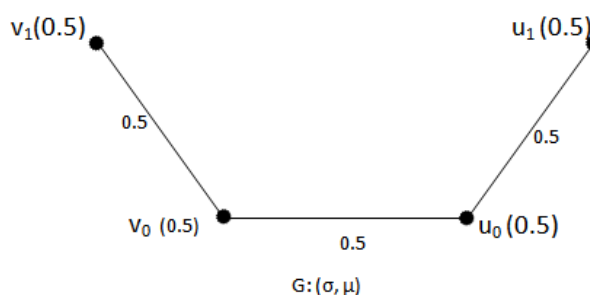
Let  $d_1 = 1$ ,  $d_2 = 0.5$ . Then the value  $d_1 - m d_2 = 1 - 0.5 = 0.5$ , since  $0 < 0.5 \leq 1$ .

Therefore  $m = 1$ ,

Consider a fuzzy graph  $G$  on a bistar with vertices  $v_1 v_0 u_0 u_1$ , assign  $\mu(v_1 v_0) = \mu(u_1 u_0) = 0.5$  then  $d(v_1) = d(u_1) = 0.5 = d_2$ .

Also assign  $\mu(v_0 u_0) = 0.5 \Rightarrow d(v_0) = d(u_0) = 1 = d_1$ .

Hence  $\{1, 0.5\}$  is the degree set of the fuzzy graph in fig.4.



**Figure-4**

## IV. CONCLUSION

In fuzzy graph theory degree of an edge is a parameter of a graph. In this paper we made a study about the edge degree set of a fuzzy graph.

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