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CONTRA REGULAR MILDLY GENERALIZED CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is introduce and investigate new class of mappings called Contra Regular Mildly Generalized Continuous (briefly contra RMG-continuous) maps, we get several characterizations and some of their properties. Also we investigate its relationship with other type of maps.

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1. INTRODUCTION

In 1996, Dontchev[4] introduced the notation of contra-continuity. J. Dontchev and T. Noiri[5] introduced and investigated contra semi-continuous functions and RC-continuous functions between topological spaces. The purpose of this paper is to introduce a new class of functions namely contra RMG-continuous functions in topological spaces.

2. PRELIMINARIES

Throughout this paper (X, τ), (Y, σ) and (Z, η) or simply X, Y and Z will always denote topological spaces on which no separation axioms are assumed unless explicitly stated. Int(A), Cl(A), RMG-cl(A), and RMG-int(A) denote the interior of A, closure of A, RMG-closure of A and RMG-interior of A respectively. $X-A$ or A^c denotes the complement of A in X. We recall the following definitions and results.

Definition 2.1 A subset A of a topological space X is called

i) Regular open [19], if $A = \mathrm{int}(\mathrm{cl}(A))$and regular closed if$\mathrm{cl}(\mathrm{int}(A)) = A$.
ii) Pre-open [10], if $A \subseteq \mathrm{int}(\mathrm{cl}(A))$ and pre-closed if $\mathrm{cl}(\mathrm{int}(A)) \subseteq A$.
iii) Semi open [12], if $A \subseteq \mathrm{cl}(\mathrm{int}(A))$ and semi-closed if $\mathrm{int}(\mathrm{cl}(A) \subseteq A$.
iv) α-open [15], if $A\subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))$ and α-closed if $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(A)) \subseteq A$.
v) Semi pre open [1], if $A \subseteq \mathrm{cl}(\mathrm{int}(\mathrm{cl}(A)))$ and semi pre closed if $\mathrm{int}(\mathrm{cl}(\mathrm{int}(A))) \subseteq A$.
vi) π-open [6], if A is a finite union of regular open sets. The complement of π-open set is called the π-closed set.
vii) A subset A of X is called δ-closed [20] if $A = \mathrm{cl}_\delta(A)$, where $\mathrm{cl}_\delta(A) = \{x \in X : \mathrm{int}(\mathrm{cl}(U)) \cap A \neq \emptyset, U\in A\}$.

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Definition 2.2: A subset of a topological space (X, τ) is called

1. Generalized closed (briefly g-closed) [13] if cl(A) ⊆U whenever A⊆U and U is open in X.
2. Generalized α-closed (briefly gα-closed) [11] if α-cl(A) ⊆U whenever A⊆U and U is α-open in X.
3. Weakly generalized closed (briefly wg-closed) [14] if cl(int(A)) ⊆U whenever A⊆U and U is open in X.
4. Strongly generalized closed (briefly g*-closed) [18] if cl(A) ⊆U whenever A⊆U and U is g-open in X.
5. Weakly closed (briefly w-closed) [18] if cl(A) ⊆U whenever A⊆U and U is semi-open in X.
6. Mildly generalized closed (briefly mildly g-closed) [17] if cl(int(A)) ⊆U whenever A⊆U and U is g-open in X.
7. Regular weakly generalized closed (briefly rwg-closed) [14] if cl(int(A)) ⊆U whenever A⊆U and U is regular open in X.
8. Regular weakly closed (briefly rw-closed)[21] if cl(A)⊆U whenever A⊆U and U is regular semi open in X.
9. Regular generalized closed (briefly rg-closed) [16] if cl(A)⊆U whenever A⊆U and U is regular open set in X.
10. π-generalized closed (briefly πg-closed)[6] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.
11. Regular Mildly Generalized closed (briefly RMG-closed)[22] if cl(int(A)) ⊆ U whenever A ⊆ U and U is rg-open in X.

    The complements of the above mentioned closed sets are their respective open sets.

Definition 2.3: A function f: (X, τ)→(Y, σ) is called

i) RMG-continuous [24] if $f^{-1}(V)$ is RMG-closed set of (X, τ) for every closed set V of (Y, σ).
ii) RMG-Irresolute [24] if $f^{-1}(V)$ is RMG-closed set of (X, τ) for every RMG closed set V of (Y,σ).
iii) Strongly RMG-continuous [24] if $f^{-1}(V)$ is open set in (X, τ) for every RMG-open set V in (Y,σ).
iv) Perfectly RMG-continuous [24] if $f^{-1}(V)$ is clopen set in (X, τ) for every RMG-open set V in (Y,σ).

Definition 2.4: A function f: (X, τ) → (Y, σ) is called

i) contra-continuous [4] if $f^{-1}(V)$ is closed in (X, τ) for every open set V in (Y, σ).
ii) contra pre-continuous [8] if $f^{-1}(V)$ is pre-closed in (X, τ) for every open set V in (Y, σ).
iii) contra gc-continuous [3] if $f^{-1}(V)$ is gc-closed in (X, τ) for every open set V in (Y, σ).
iv) contra semi-continuous [5] if $f^{-1}(V)$ is semi-closed in (X, τ) for every open set V in (Y, σ).
v) contra semi pre-continuous [2] if $f^{-1}(V)$ is semi pre-closed in (X, τ) for every open set V in (Y, σ).
vi) contra πg-continuous [7] if $f^{-1}(V)$ is πg-closed in (X, τ) for every open set V in (Y, σ).

Definition 2.5: A function f: (X, τ)→(Y, σ) is called

i) contra w-continuous if $f^{-1}(V)$ is w-closed in (X, τ) for every open set V in (Y, σ).
ii) contra gα-continuous if $f^{-1}(V)$ is gα-closed in (X, τ) for every open set V in (Y, σ).
iii) contra rg-continuous if $f^{-1}(V)$ is rg-closed in (X, τ) for every open set V in (Y, σ).
iv) contra rw-continuous if $f^{-1}(V)$ is rw-closed in (X, τ) for every open set V in (Y, σ).
v) contra wg-continuous if $f^{-1}(V)$ is wg-closed in (X, τ) for every open set V in (Y, σ).
vi) contra rwg-continuous if $f^{-1}(V)$ is rwg-closed in (X, τ) for every open set V in (Y, σ).
vii) contra g*-continuous if $f^{-1}(V)$ is g^* closed in (X, τ) for every open set V in (Y, σ).
viii) viii)contra mildly g-continuous if $f^{-1}(V)$ is mildly g-closed in (X, τ) for every open set V in (Y, σ).

Definition 2.6: A function f: (X, τ)→(Y, σ) is called

i) strongly continuous[21] if $f^{-1}(V)$ is both open and closed set in (X, τ) for each set V of (Y, σ).
ii) strongly-w-continuous [21] if $f^{-1}(V)$ is open in (X, τ) for every w-open set V in (Y, σ).
iii) perfectly continuous[18] if $f^{-1}(V)$ is clopen in (X, τ) for every open set V in (Y, σ).

Lemma 2.7:

i) Every closed set is RMG-closed.[22]
ii) Every pre-closed (respectively w-closed, gα-closed) set is RMG-closed set in X.[22]
iii) Every RMG-closed is Mildly-g-closed set (respectively wg-closed, wπg-closed, rwg-closed) sets in X.[22]

Lemma 2.8: [23] A is RMG-open iff U⊂ int(cl(A)), whenever U is RMG-closed and U⊂A.

Lemma 2.9: [24] A space (X, τ) is called RMG-space if every RMG-closed set is closed.

Lemma 2.10: [9] A space X is locally indiscrete if every open subset of X is closed.

3. CONTRA RMG-CONTINUOUS FUNCTIONS

In this section we introduce the notation of contra RMG-continuous, contra RMG-irresolute and almost contra RMG-continuous functions in topological space and study some of their properties.

Definition3.1: A function $f:(X,\tau)\to(Y,\sigma)$ is called Contra Regular Mildly Generalized Continuous (briefly contra RMG-continuous) if the inverse image of every open set in Y is RMG-closed set in X.

Example 3.2: Let X= {a, b, c, d}, Y={p, q} with the topology τ ={X, ∅, {a}, {b, c},{a, b, c}} and σ ={Y, ∅, {p}, {q}}. Let $f:(X,\tau)\to(Y,\sigma)$ be a map defined by f(a) =q, f(b) =p, f(c) =q, f(d) =q. Now $f^{-1}(\emptyset)=\emptyset$,$f^{-1}(Y)=X$, $f^{-1}(\{p\})=\{b\}$, $f^{-1}(\{q\})=\{a,c,d\}$ are RMG-closed sets in X. Thus f is contra RMG- continuous. Then inverse image of open set in Y is RMG-closed set in X.

Theorem 3.3: Every contra-continuous function is contra RMG-continuous.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be a contra-continuous map. Let V be an open set in Y. Since f is contra-continuous, $f^{-1}(V)$ is closed set in X. By the lemma 2.7(i), every closed set is RMG-closed set in X. $f^{-1}(V)$ is RMG-closed set in X. Therefore f is contra RMG-continuous.

The converse of the above Theorem need not be true as seen from the following example.

Example 3.4: Let X={a, b, c, d}, Y={a, b, c} with the topology τ ={X, ∅, {a}, {b, c},{a, b, c}} and σ ={Y, ∅, {a}}. Let $f:(X,\tau)\to(Y,\sigma)$ be defined by f(a) = d, f(b) = b, f(c) = a, f(d) = c. Then f is contra RMG- continuous but not contra-continuous, as inverse image of open set {a} in Y is {c} which is not closed set in X.

Corollary 3.5:

i) If a function $f:(X,\tau)\to(Y,\sigma)$ is contra pre-continuous, then it is contra RMG-continuous.
ii) If a function $f:(X,\tau)\to(Y,\sigma)$ is contra gα-continuous, then it is contra RMG-continuous.
iii) If a function $f:(X,\tau)\to(Y,\sigma)$ is contra w-continuous, then it is contra RMG-continuous.

Proof:

i) suppose f is contra pre-continuous function. Let V be an open set in Y. Since f is contra pre continuous. $f^{-1}(V)$ is pre-closed in X. Since every pre-closed set is RMG closed. By Lemma 2.7[ii], $f^{-1}(V)$ is RMG-closed in X. Hence f is contra RMG-continuous.
ii) Let V be open set of Y. Since f is contra gα-continuous, $f^{-1}(V)$ is a gα-closed in X. From lemma 2.7 [ii], $f^{-1}(V)$ RMG-closed. Therefore f is Contra RMG-continuous.
iii) Let V be open set of Y. Since f is contra w-continuous, $f^{-1}(V)$ is a w-closed in X. From lemma 2.7 [ii], $f^{-1}(V)$ RMG-closed. Therefore f is Contra RMG-continuous.

Remark 3.6: The converse of Corollary 3.5 need not be true as shown in the following example.

Let X= {a, b, c, d}with topology $\tau=\{X,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}$ and Y= {p, q} with topology $\sigma=\{Y,\emptyset,\{p\}\}$. Let function $f:(X,\tau)\to(Y,\sigma)$ be defined by $f(a)=q, f(b)=p, f(c)=q$ and $f(d)=q$. Now $f^{-1}(\emptyset)=\emptyset$,$f^{-1}(Y)=X$, $f^{-1}(\{p\})=\{b\}$ are RMG-closed sets in X. Hence, f is contra RMG-continuous function. However, since

i) {b} is not pre-closed set in X i.e. f is not contra pre-continuous on X.
ii) {b} is not gα-closed set in X i.e. f is not contra gα-contnuous on X.
iii) {b} is not w-closed set in X i.e. f is not contra w-continuous on X.

Remark 3.7: The concept of RMG-continuity and contra RMG-continuity is independent as shown in the following example.

Example 3.8: Let X= {a, b, c, d} with topology $\tau=\{X,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}$ and Y={p, q} with topology $\sigma=\{Y,\emptyset,\{q\}\}$. Let function $f:(X,\tau)\to(Y,\sigma)$ be defined by $f(a)=q, f(b)=p, f(c)=p$ and $f(d)=q$. Clearly f is contra RMG-continuous but not RMG-continuous, since $f^{-1}(\{p\})=\{b,c\}$ is not RMG-closed sets in X where {p} is closed in Y.

Example 3.9: Let X = {a, b, c, d} with topology $\tau=\{X,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}$ and Y= {a, b, c} with topology $\sigma=\{Y,\emptyset,\{a,b\}\}$. Let function $f:(X,\tau)\to(Y,\sigma)$ be defined by $f(a)=b, f(b)=c, f(c)=a$ and $f(d)=a$. Clearly f is RMG-continuous but not contra RMG-continuous, since $f^{-1}(\{a,b\})=\{a,c\}$ is not RMG-closed sets in X where {a, b} is open in Y.

Corollary 3.10:

i) If a function $f:(X,\tau)\to(Y,\sigma)$ is contra RMG-continuous, then it is contra mildly g-continuous.
ii) If a function $f:(X,\tau)\to(Y,\sigma)$ is contra RMG-continuous, then it is contra wg-continuous.
iii) If a function $f:(X,\tau)\to(Y,\sigma)$ is contra RMG-continuous, then it is contra rwg-continuous.

Proof:

i) Let V be open set of Y. Since f is contra RMG-continuous, $f^{-1}(V)$ is a RMG-closed in X. From lemma 2.7 [iii], $f^{-1}(V)$ mildly g-closed. Therefore f is contra mildly g-continuous.

ii) Let V be open set of Y. Since f is contra RMG-continuous, $f^{-1}(V)$ is a RMG-closed in X. From lemma 2.7 [iii], $f^{-1}(V)$ wg-closed. Therefore f is contra wg-continuous.

iii) Let V be open set of Y. Since f is contra RMG-continuous, $f^{-1}(V)$ is a RMG-closed in X. From lemma 2.7 [iii], $f^{-1}(V)$ rwg-closed. Therefore f is contra rwg-continuous.

The converse of the above corollary need not be true as seen from the following example.

Example 3.11:

i) Let X=Y={a, b, c, d}with the topology τ ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and σ ={Y, ∅, {a}, {b, c}, {a, b, c}}. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by f(a) =c, f(b) =a, f(c) =d, f(d) =b. Then f is contra mildly-g-continuous but not contra RMG-continuous. Since $f^{-1}(\{a\}) = \{b\}$ is not RMG-closed in X, where {b} is open in Y.

ii) Let X=Y={a, b, c, d}with the topology τ ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and σ ={Y, ∅, {a}, {b, c}, {a, b, c}}. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by f(a) =d, f(b) =c, f(c) =a, f(d) =b. Then f is contra wg-continuous but not contra RMG-continuous. Since $f^{-1}(\{b, c\}) = \{b, d\}$ is not RMG-closed in X, where {b, c} is open in Y.

iii) Let X=Y={a, b, c, d}with the topology τ ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and σ ={Y, ∅, {a}, {b, c}, {a, b, c}}. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by f(a) =c, f(b) =a, f(c) =b, f(d) =d. Then f is contra rwg-continuous but not contra RMG-continuous. Since $f^{-1}(\{a\}) = \{b\}$ is not RMG-closed in X, where {a} is open in Y.

Remark 3.12: The concept of contra RMG-continuous is independent from the concept of contra semi-continuous, contra semi-pre-continuous, contra g-continuous, contra g*-continuous, contra πg-continuous, contra rg-continuous and contra rw-continuous are independent.

Example 3.13: Let X=Y={a, b, c, d} be with topologies τ ={X, ∅, {a}, {b, c}, {a, b, c}} and σ ={Y, ∅, {a}, {b}, {a, b}, {a, b, c}}. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by f(a) = d, f(b) = a, f(c) = c, f(d) =b. Clearly f is contra RMG-continuous, but $f^{-1}(\{a\})$ ={b} is not semi-closed, g^*-closed, g-closed, πg-closed, rw-closed, rg-closed in X. f is not contra semi-continuous, contra g^*-continuous, contra g-continuous, contra πg-continuous, contra rw-continuous and contra rg-continuous. .

Example 3.14: Let X={a, b, c, d} be with topology τ ={X, ∅, {a}, {b}, {a, b},{b, c}, {a, b, c}} and Y={a, b, c} be with topology σ ={Y, ∅, {a}}. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by f(a) = c, f(b) =a, f(c) = b, f(d) = a. Clearly f is contra semi-continuous, contra g^*-continuous, contra g-continuous, contra πg-continuous, contra rw-continuous and contra rg-continuous, but $f^{-1}(\{a\})$ ={b, d}is not a RMG-closed in X. Therefore f is not contra RMG continuous.

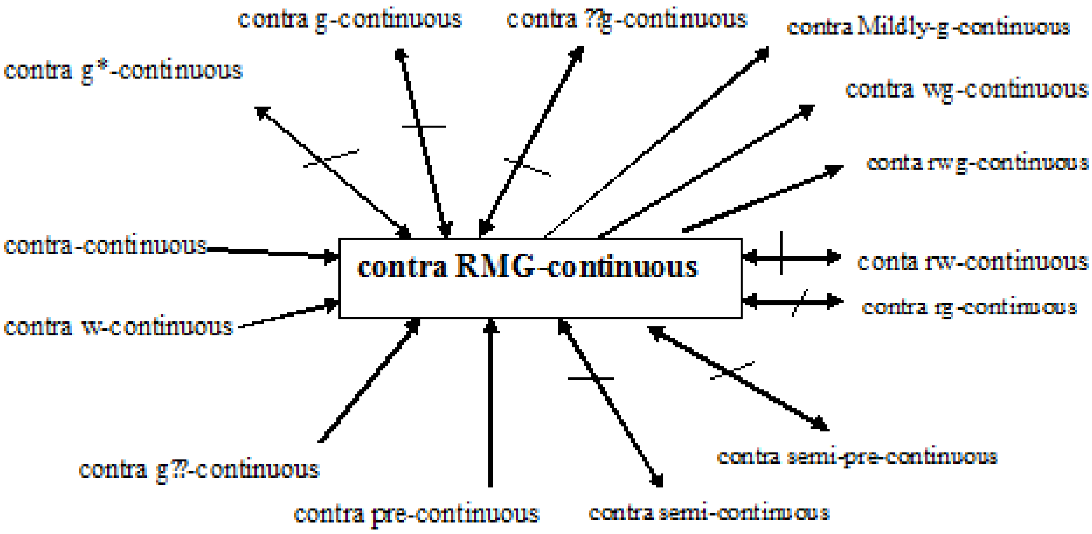
Example 3.15: Let X={a, b, c, d} be with topology τ ={X, ∅, {a}, {b}, {a, b}{b, c}, {a, b, c}} and Y={p, q, r} be with topology σ ={Y, ∅, {p}}. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by f(a) = q, f(b) = p, f(c) = q, f(d) = r. Clearly f is contra RMG-continuous, but $f^{-1}(\{p\})$ ={b}is not a semi-pre closed in X, but therefore f is not contra semi pre-continuous.

Example 3.16: Let X={a, b, c, d} be with topology τ ={X, ∅, {a}, {b}, {a, b},{b, c}, {a, b, c}} and Y={a, b, c} be with topology σ ={Y, ∅, {a}}. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by f(a) = a, f(b) = c, f(c) = b, f(d) = b. Clearly f is contra semi pre-continuous, but $f^{-1}(\{a\})$ ={a}is not a RMG-closed in X, but Therefore f is not contra RMG continuous.

Remark 3.17: From the above discussion and know results we have the following implications. In the following diagram, by

A $\longrightarrow$ B we mean A implies B but not conversely and

A $\longleftrightarrow$ B means A and B are independent of each other.



Example 3.19: Let X=Y=Z={a, b, c, d}. τ ={X, ∅, {a}, {b, c}, {a, b, c}}, σ ={Y, ∅, {a}, {b}, {a, b}, {b, c}, {a, b, c}} and η ={Z, ∅, {a}, {b}, {a, b}, {a, b, c}}. Let f: (X, τ)→(Y, σ) be defined by f(a) = c, f(b) = d, f(c) = b, f(d) =a and g: (Y, σ)→(Z, η) be defined by g(a) =d, f(b) =c, f(c) =a, f(d) =b. Then f and g are contra RMG-continuous, but their composition g∘f : (X, τ)→(Z, η) is not contra RMG-continuous, because F ={a} is open in (Z, η), but $(g\circ f)^{-1}$(F)= $(g\circ f)^{-1}$({a})= $f^{-1}(g^{-1}$({a})) = f^{-1}({c}) = {a} which is not RMG-closed in (X, τ **)**

Theorem 3.20: If f: (X, τ)→(Y, σ) is a contra RMG-continuous and g: (X, τ)→(Z, η) is continuous, then their composition g∘f : (X, τ)→(Z, η) is contra RMG-continuous.

Proof: Let V be any open set in (Z, η). Since g is continuous, $(g)^{-1}$(V) is open in (Y, σ). Then $f^{-1}\left(g^{-1}(V)\right)$ is closed in (X, τ). Since f is contra RMG-continuous, $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is a RMG-closed set in (X, τ). Hence g∘f is contra RMG-continuous.

Theorem 3.21: Let f: (X, τ)→(Y, σ) be map, then the following are equivalent.

i) f is contra RMG-continuous.
ii) The inverse image of each closed set F of Y is RMG-open in X.
iii) The inverse image of each open set U of Y is RMG-closed in X.

Proof: Suppose i) holds. Let F be an closed set in Y. Then Y-F is open set in Y. By (i) $f^{-1}(Y-F)$ =X-f^{-1}(F) is RMG closed in X. Therefore f^{-1}(U) is RMG-open in X. This proves (i)⟹(ii). The implications (ii)⟹(iii) and (iii)⟹(i) obviously.

Theorem 3.22: Let f: (X, τ)→(Y, σ) and g: (Y, τ)→(Z, η) be any two functions. Then

(i) g ∘ f: (X, τ)→(Z, η) is contra RMG-continuous if g is contra RMG-continuous and f is RMG-irresolute.
(ii) g ∘ f: (X, τ)→(Z, η) is contra RMG-continuous if g is continuous and f is contra pre-continuous.
(iii) g ∘ f: (X, τ)→(Z, η) is perfectly RMG-continuous if g is contra RMG-continuous and f is perfectly RMG-continuous.

Proof:

(i) Let U be a open set in (Z, η). Since g is contra RMG-continuous, then g^{-1}(U) is RMG-closed set in (Y, σ). Since f is RMG-irresolute, $f^{-1}(g^{-1}(U))$ is an RMG-closed set in (X, τ). Thus $(g\circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an RMG-closed set in (X, τ). Therefore g∘f is contra RMG-continuous.
(ii) Let U be a open set in (Z, η). Since g is continuous, g^{-1}(U) is open set in (Y, σ). Since f is contra pre-continuous then $f^{-1}(g^{-1}(U))$ is an pre-closed set in (X, τ). Hence by lemma 2.7[iii], every pre-closed set is RMG-closed, $(g\circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an RMG-closed set in (X, τ). Therefore g∘f is contra RMG-continuous.
(iii) Let U be a open set in (Z, η). By lemma 2.7(iii), every open set is RMG-open which implies U is RMG-open in (Z, η). Since g is contra RMG-continuous, then g^{-1}(U) is RMG-closed set in (Y, σ). Since f is perfectly RMG-continuous, $f^{-1}(g^{-1}(U))$ is both open and closed set in (X, τ). Thus $(g\circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is both open and closed set in (X, τ). Therefore gof is perfectly RMG-continuous.

Theorem 3.23: Let f: (X, τ)→(Y, σ) be a function and X is a RMG-space. Then the following are equivalent.

i) f is contra-continuous.
ii) f is contra RMG-continuous.

Proof:
(i)⇒(ii): Let U be any open set in (Y, σ). Since f is contra continuous, $f^{-1}(U)$ is closed in (X, τ) and since every closed set is RMG-closed, $f^{-1}(U)$ is RMG-closed in (X, τ). Therefore f is contra RMG-continuous.

(ii)⇒(i): Let U be any open set in (Y, σ). Since f is contra RMG-continuous, $f^{-1}(U)$ is RMG-closed in (X, τ) and since X is a RMG-space, $f^{-1}(U)$ is closed in (X, τ). Therefore f is contra-continuous.

Definition 3.24: A map f: (X, τ)→(Y, σ) is called contra RMG-irresolute map if the inverse image of every RMG-open set in (Y, σ) is RMG-closed in (X, τ).

Definition 3.25: A map f: (X, τ)→(Y, σ) is called perfectly contra RMG-irresolute map if the inverse image of every RMG-open set in (Y, σ) is RMG-closed and RMG-open in (X, τ).

Theorem 3.26: A function f: (X, τ)→(Y, σ) is perfectly contra RMG-irresolute if and only if f is contra RMG-irresolute and RMG-irresolute.

Proof: It is directly follows from the definitions.

Theorem 3.27: Every contra RMG-irresolute map is contra RMG-continuous map.

Proof: Let U be a open set in Y. Since every open set is RMG-open which implies U is RMG-open in Y. Since f: (X, τ)→(Y, σ) is contra RMG-irresolute then $f^{-1}(\{U\})$ is RMG-closed in X. Therefore, f is contra RMG-continuous.

The converse of the above theorem need not be true.

Example 3.28: Let X={a, b, c, d} be with topology τ ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and Y={a, b, c, d} be with topology σ ={Y, ∅, {a},{b, c}, {a, b, c}}. Let f: (X, τ)→(Y, σ) be defined by f(a) =b, f(b) =a, f(c) =b, f(d) =c. Then f is contra RMG-continuous but not contra RMG-irresolute, as inverse image of RMG –open set {a} in Y is {b} which is not a RMG-closed in X.

Remark 3.29: The following examples show that the concept of RMG-irresolute and contra RMG-irresolute are independent of each other.

Example 3.30: Let X={a, b, c, d} be with topology τ ={X, ∅, {a}, {b, c}, {a, b, c}} and Y={a, b, c} be with topology σ ={Y, ∅, {a}}. Let f: (X, τ)→(Y, σ) be defined by f(a) =b, f(b) =c, f(c) =b, f(d) =a. Clearly f is contra RMG-irresolute but not RMG-irresolute. Since $f^{-1}(\{a\}) =\{d\}$is not a RMG-open in X, but Therefore f is not RMG irresolute.

Example 3.31: Let X={a, b, c, d} be with topology τ ={X, ∅, {a}, {b}, {a, b}} and Y={p, q} be with topology σ ={Y, ∅, {a}}. Let f: (X, τ)→(Y, σ) be defined by f(a) =q, f(b) =q, f(c) =p, f(d) =p. Clearly f is RMG-irresolute but not contra RMG-irresolute. Since $f^{-1}(\{q\}) =\{a, b\}$is not a RMG-closed in X, but Therefore f is not contra RMG irresolute.

Theorem 3.32: Every perfectly contra RMG-irresolute function is contra RMG-irresolute and RMG-irresolute.

Proof: The proof directly follows from the definitions.

Remark 3.33: The following two examples shows that a contra RMG-irresolute function may not be perfectly contra RMG-irresolute and RMG-irresolute may not be perfectly contra RMG-irresolute.

In example 3.30, f is contra RMG-irresolute but not perfectly contra RMG-irresolute.

In Example 3.31, f is RMG-irresolute but not perfectly contra RMG-irresolute.

Theorem 3.34: Let f: (X, τ)→(Y, σ) and g: (Y, τ)→(Z, η) be any two functions. Then

(i) g ∘ f: (X, τ)→(Z, η) is contra RMG-irresolute if g is contra RMG-irresolute and f is RMG-irresolute.
(ii) g ∘ f: (X, τ)→(Z, η) is contra RMG-continuous if g is RMG-continuous and f is contra RMG-irresolute.

Proof:

(i) Let U be a RMG-open set in (Z, η). Since g is contra RMG-irresolute, $g^{-1}(U)$ is RMG-closed set in (Y, σ). Since f is RMG-irresolute then $f^{-1}(g^{-1}(U))$ is an RMG-closed set in (X, τ). $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an RMG-closed set in (X, τ). Therefore gof is contra RMG-irresolute.

(ii) Let U be a open set in (Z, η). Since g is RMG-continuous, then $g^{-1}(U)$ is RMG-open set in (Y, σ). Since f is contra RMG-irresolute, $f^{-1}(g^{-1}(U))$ is an RMG-closed set in (X, τ). Thus $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an RMG-closed set in (X, τ). Therefore gof is contra RMG-irresolute

Definition 3.35: A map f: (X, τ)→(Y, σ) is called almost contra RMG-continuous map if the inverse image of every regular open set in (Y, σ) is RMG-closed in (X, τ).

Example 3.36: Let X={a, b, c, d} be with topology τ ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and Y={a, b, c} be with topology σ ={Y, ∅, {a}, {b}, {a, b}}. Let f: (X, τ)→(Y, σ) be defined by f(a) =c, f(b) =d, f(c) =a, f(d) =b. Clearly f is almost contra RMG-continuous. Since $f^{-1}(\{a\})=\{c\}$, $f^{-1}(\{b\})=\{d\}$ are RMG-closed in X, for every regular open sets {a}, {b} in Y.

Theorem 3.37: Every contra RMG-continuous map is almost contra RMG-continuous map.

Proof: Let U be a regular open set in Y. Since every regular open set is open which implies U is open in Y. Since f: (X, τ)→(Y, σ) is contra RMG-continuous then $f^{-1}(\{U\})$ is RMG-closed in X. Therefore, f is almost contra RMG-continuous.

The converse of the above theorem need not be true.

Example 3.38: Let X={a, b, c, d} be with topology τ ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and Y={a, b, c} be with topology σ ={Y, ∅, {a},{b, c}}. Let f: (X, τ)→(Y, σ) be defined by f(a) =b, f(b) =a, f(c) =a, f(d) =c. Clearly f is RMG-continuous. But $f^{-1}(\{a\})=\{b, c\}$ is not regular open in X. Therefore f is not almost contra RMG-continuous.

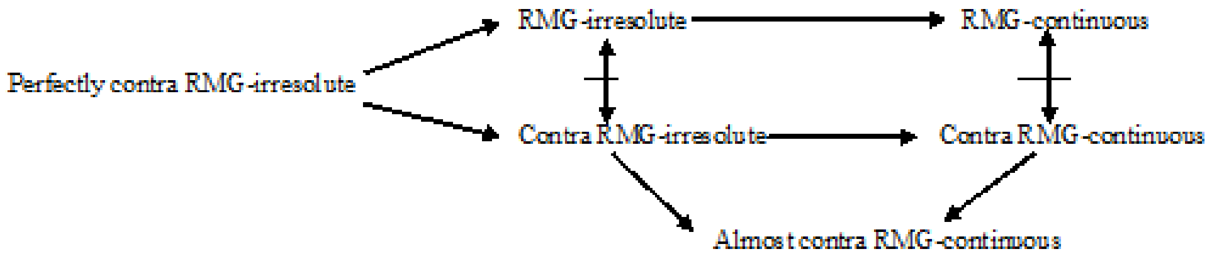
Theorem 3.39: Every contra RMG-irresolute map is almost contra RMG-continuous map but not conversely.

Proof: Let U be a regular open set in Y. Since every regular open set is RMG-open which implies U is RMG-open in Y. Since f: (X, τ)→(Y, σ) is contra RMG-irresolute then $f^{-1}(\{U\})$ is RMG-closed in X. Therefore, f is almost contra RMG-continuous.

The converse of the above theorem need not be true.

Example 3.40: Let X={a, b, c, d} be with topology τ ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and Y={a, b, c, d} be with topology σ ={Y, ∅, {a},{b, c}, {a, b, c}}. Let f: (X, τ)→(Y, σ) be defined by f(a) =a, f(b) =d, f(c) =b, f(d) =c. Then f is contra RMG-continuous but not contra RMG-irresolute, as inverse image of RMG-open set {a} in Y is {a} which is not a RMG-closed in X.

Remark 3.41: From the above discussions and known results we have the following implications.
A ⟶ B we mean A implies B but not conversely and
A ⟷ B means A and B are independent of each other.



Theorem 3.42: Let f: (X, τ)→(Y, σ) be map, then the following are equivalent.

i) f is almost contra RMG-continuous.
ii) $f^{-1}(F)$ is RMG-open in X for every regular closed F in Y.

Proof: (i)⇔(ii) Let F be any regular closed set of Y. Then (Y-F) is regular open and therefore $f^{-1}(Y-F)=X-f^{-1}(F) \in RMGC(X)$. Hence $f^{-1}(F)$ is RMG-open in X. The converse part is obvious.

Theorem 3.43: Let f: (X, τ)→(Y, σ) and g: (Y, τ)→(Z, η) be any two functions. Then

(i) g ∘ f: (X, τ)→(Z, η) is RMG-continuous and contra RMG-continuous if g is perfectly continuous and f is almost contra RMG-continuous.

(ii) g ∘ f: (X, τ)→(Z, η) is almost contra RMG-continuous if g is almost continuous and f is almost contra RMG-continuous

Proof:

(i) Let U be a open set in (Z, η). Since g is perfectly continuous, then $g^{-1}(U)$ is clopen in (Y, σ). Since f is almost contra RMG-continuous, $f^{-1}(g^{-1}(U))$ is RMG-open and RMG-closed set in (X, τ). Thus (g ∘ f)−1U=f−1(g−1U) is RMG-open and RMG-closed set in (X,τ). Therefore g∘f is RMG-continuous and contra RMG-continuous.

(ii) Let U be a regular open set in (Z, η). Since g is almost continuous, $g^{-1}(U)$ is open set in (Y, σ). Since f is contra RMG-continuous then $f^{-1}(g^{-1}(U))$ is an RMG-closed set in (X, τ). $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an RMG-closed set in (X, τ). Therefore gof is almost contra RMG-continuous.

Definition 3.44: A space is said to be locally RMG-indiscrete if every RMG-open set of X is closed in X.

Theorem 3.45: A contra RMG-continuous map f: (X, τ)→(Y, σ) is continuous when X is locally RMG-indiscrete.

Proof: Let O be a open set in Y. Since f is contra RMG-continuous then $f^{-1}(O)$ is RMG-closed in X. Since, X is locally RMG-indiscrete which implies $f^{-1}(O)$ is open in X. Therefore, f is continuous.

Theorem 3.46: Let f: (X, τ)→(Y, σ) is RMG-irresolute map with Y as locally RMG-indiscrete space and g: (Y, σ)→(Z, η) is contra RMG-continuous, then gof is RMG-continuous.

Proof: Let F be any closed set in Z. Since g is contra RMG-continuous, $g^{-1}(F)$ is RMG-open in Y. But Y is locally RMG-indiscrete, $g^{-1}(F)$ is closed in Y. Hence $g^{-1}(F)$ is RMG-closed in Y. Since, f is RMG-irresolute, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is RMG-closed in X. Therefore, g∘f is RMG-continuous.

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