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OPERATORS ON INTERVAL VALUED INTUITIONISTIC FUZZY SETS OF SECOND TYPE

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ABSTRACT

In this paper, we introduce some operators on Interval Valued Intuitionistic Fuzzy Sets of Second Type and establish some of their properties.

Key Words: *Fuzzy sets (FS), Intuitionistic Fuzzy Sets (IFS), Intuitionistic Fuzzy Sets of Second Type (IFSST), Interval Valued Fuzzy Sets (IVFS), Interval Valued Intuitionistic Fuzzy Sets (IVIFS), Interval Valued Intuitionistic Fuzzy Sets of Second Type (IVIFSST).*

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1. INTRODUCTION

An Intuitionistic Fuzzy Set (IFS) for a given underlying set X were introduced by K. T. Atanassov [2] which is the generalization of ordinary Fuzzy Sets (FS) and the theory of Interval Valued Intuitionistic Fuzzy Sets and established some operators and their properties.

The present authors further introduced the Interval Valued Intuitionistic Fuzzy Sets of Second Type and established its basic operations [5]. The rest of the paper is designed as follows: In Section 2, we give some basic definitions. In Section 3, we define some operators on Interval valued Intuitionistic Fuzzy Sets of second type and establish some of their properties. This paper is concluded in section 4.

2. PRELIMINARIES

In this section, we give some basic definitions.

Definition 2.1[6]: Let X be a non - empty set. A Fuzzy Set A in X is characterized by its membership function $\mu_A : X \to [0,1]$ and $\mu_A(x)$ is interpreted as the degree of membership of the element x in fuzzy set A, for each $x \in$ X. It is clear that A is completely determined by the set of tuples

$$A = \{< x, \mu_A(x) > \mid x \in X\}$$

Definition 2.2[2]: Let X be a non- empty set. An intuitionistic fuzzy set (IFS) A in X is defined as an object of the following form.

$$A = \{< x, \mu_A(x), \nu_A(x) > \mid x \in X\}$$

Where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x \in$ X, respectively, and for every $x \in$ X.

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

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Definition 2.3[2]: Let a set X be fixed. An intuitionistic fuzzy set of second type (IFSST) A in X is defined as an object of the following form.

$$A = \{< x, \mu_A(x), \nu_A(x) > \mid x \in X\}$$

Where the functions $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$.

$$0 \leq {\mu_A}^2(x) + {\nu_A}^2(x) \leq 1$$

Definition 2.4[2]: Let X be an universal set with cardinality n. Let [0, 1] be the set of all closed subintervals of the interval [0, 1] and elements of this set are denoted by uppercase letters. If M ∈ [0, 1] then it can be represented as $M = [M_L, M_U]$, where M_L and M_U are the lower and upper limits of M. For M ∈ [0, 1], $\overline{M} = 1 - M$ represents the interval $[1 - M_L, 1 - M_U]$ and $W_M = M_U - M_L$ is the width of M.

An interval-valued fuzzy set (IVFS) A in X is given by

$$A = \{< x, M_A(x) > \mid x \in X\}$$

where $M_A : X \to [0,1]$, $M_A(x)$ denote the degree of membership of the element x to the set A.

Definition 2.5[2]: An interval-valued intuitionistic fuzzy set (IVIFS) A in X is given by

$$A = \{< x, M_A(x), N_A(x) > \mid x \in X\}$$

where $M_A : X \to [0, 1]$, $N_A : X \to [0, 1]$. The intervals $M_A(x)$ and $N_A(x)$ denote the degree of membership and non - membership of the element x to the set A, where $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ and $N_A(x) = [N_{AL}(x), N_{AU}(x)]$ with the condition that

$$M_{AU}(x) + N_{AU}(x) \leq 1 \text{ for all } x \in X.$$

Definition 2.6[5]: An Interval-Valued Intuitionistic Fuzzy Sets of Second Type (IVIFSST) A in X is given by

$$A = \{< x, M_A(x), N_A(x) > \mid x \in X\}$$

Where $M_A: X \to [0, 1]$, $N_A: X \to [0, 1]$. The intervals $M_A(x)$ and $N_A(x)$ denote the degree of membership and the degree of non-membership of the element x to the set A, where $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ and $N_A(x) = [N_{AL}(x), N_{AU}(x)]$ with the condition that

$$M^2{}_{AU}(x) + N^2{}_{AU}(x) \leq 1 \text{ for all } x \in X.$$

Definition 2.7[5]: For every two IVIFSST A and B, we have the following relations and operations

1. $A \subset B$ iff $M_{AU}(x) \leq M_{BU}(x)$ & $M_{AL}(x) \leq M_{BL}(x)$&
   $N_{AU}(x) \geq N_{BU}(x)$&$N_{AL}(x) \geq N_{BL}(x)$
2. $A = B$ iff $A \subset B$ & $B \subset A$
3. $\bar{A} = \{< x, [N_{AL}(x), N_{AU}(x)], [M_{AL}(x), M_{AU}(x)] > \mid x \in X\}$
4. $A \cup B = \{< x, [\max(M_{AL}(x),\ M_{BL}(x)), \max(M_{AU}(x),\ M_{BU}(x))],$
   $[\min(N_{AL}(x),\ N_{BL}(x)), \min(N_{AU}(x),\ N_{BU}(x))] > \mid x \in X\}$
5. $A \cap B = \{< x, [\min(M_{AL}(x),\ M_{BL}(x)), \min(M_{AU}(x),\ M_{BU}(x))],$
   $[\max(N_{AL}(x),\ N_{BL}(x)), \max(N_{AU}(x),\ N_{BU}(x))] > \mid x \in X\}.$

3. SOME OPERATORS ON IVIFSST

In this section, we define the following modal type operators and establish some of their properties.

Definition 3.1: For every IVIFSST, we define the following

Necessity operator

$$\Box A = \left\{< x, [M_{AL}(x), M_{AU}(x)], \left[N_{AL}(x), \sqrt{1 - M^2{}_{AU}(x)}\right] > \mid x \in X\right\}$$

Possibility operator

$$\Diamond\, A = \left\{< x, \left[M_{AL}(x), \sqrt{1 - N^2{}_{AU}(x)}\right], [N_{AL}(x), N_{AU}(x)] > \mid x \in X\right\}$$

Definition 3.2: Given an IVIFSST A and for every α ∈ [0,1]. An operator $D_\alpha(A)$ is defined as follows

$$D_\alpha(A) = \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + (1-\alpha)^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right] > \mid x \in X\right\}$$

Definition 3.3: Given an IVIFSST A and for every $\alpha, \beta \in [0,1]$. An operator $F_{\alpha,\beta}(A)$ is defined as follows

$$F_{\alpha,\beta}(A) = \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right] > \mid x \in X\right\} \text{ for } \alpha + \beta \leq 1.$$

Proposition 3.1: *For every IVIFSST A, we have the following*

(i). $D_0(A) = \Box A$

(ii). $D_1(A) = \Diamond A$

Proof:

Let $A = \{< x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)] > | x \in X\}$

Then

$$\Box A = \left\{< x, [M_{AL}(x), M_{AU}(x)], \left[N_{AL}(x), \sqrt{1 - M^2{}_{AU}(x)}\right] > \mid x \in X\right\},$$

$$\Diamond A = \left\{< x, \left[M_{AL}(x), \sqrt{1 - N^2{}_{AU}(x)}\right], [N_{AL}(x), N_{AU}(x)] > \mid x \in X\right\}$$

and

$$D_\alpha(A) = \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + (1-\alpha)^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right] > \mid x \in X\right\} \quad (1)$$

(i). Put $\alpha = 0$ in (1), we have

$$\begin{aligned} D_0(A) &= \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + 0^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + (1-0)^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right] > \mid x \in X\right\} \\ &= \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + 1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)}\right] > \mid x \in X\right\} \\ &= \{< x, [M_{AL}(x), M_{AU}(x)], \left[N_{AL}(x), \sqrt{1 - M^2{}_{AU}(x)}\right] > \mid x \in X\right\} \\ &= \Box A \end{aligned}$$

Therefore, $D_0(A) = \Box A$

(ii). Again $\alpha = 1$ in (1), we have

$$\begin{aligned} D_1(A) &= \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + 1^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + (1-1)^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right] > \mid x \in X\right\} \\ &= \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + 1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x)}\right] > \mid x \in X\right\} \\ &= \left\{< x, \left[M_{AL}(x), \sqrt{1 - N^2{}_{AU}(x)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x)}\right] > \mid x \in X\right\} \\ &= \left\{< x, \left[M_{AL}(x), \sqrt{1 - N^2{}_{AU}(x)}\right], [N_{AL}(x), N_{AU}(x)] > \mid x \in X\} \\ &= \Diamond A \end{aligned}$$

Hence, $D_1(A) = \Diamond A$

Proposition 3.2: *For every IVIFSST A and for every $\alpha, \beta \in [0,1]$ such that $\alpha + \beta \leq 1$,*

(i). $F_{\alpha,\beta}(A)$ *is an IVIFSST* (iv). $\Diamond A = F_{1,0}(A)$

(ii). $D_\alpha(A) = F_{\alpha,\, 1-\alpha}(A)$ (v). $\overline{F_{\alpha,\beta}(\bar{A})} = F_{\beta,\alpha}(A)$

(iii). $\Box A = F_{0,1}(A)$

Proof:

(i). Let $A = \{< x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)] > | x \in X\}$

Then

$$\Box A = \left\{< x, [M_{AL}(x), M_{AU}(x)], \left[N_{AL}(x), \sqrt{1 - M^2{}_{AU}(x)}\right] > \mid x \in X\right\},$$

$$\Diamond A = \left\{< x, \left[M_{AL}(x), \sqrt{1 - N^2{}_{AU}(x)}\right], [N_{AL}(x), N_{AU}(x)] > \mid x \in X\right\}$$

Now,

$$F_{\alpha,\beta}(A) = \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \beta^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right] > \mid x \in \mathrm{X}\right\}$$

is an IVIFSST.

Since,

$$\left(\sqrt{M^2{}_{AU}(x) + \alpha^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right)^2 + \left(\sqrt{N^2{}_{AU}(x) + \beta^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right)^2$$

$$= M^2{}_{AU}(x) + N^2{}_{AU}(x) + \alpha^2(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)) + \beta^2(1 - M^2{}_{AU}(x) + N^2{}_{AU}(x))$$
$$\leq M^2{}_{AU}(x) + N^2{}_{AU}(x) + (\alpha^2 + \beta^2)\big(1 - (M^2{}_{AU}(x) + N^2{}_{AU}(x))\big)$$
$$\leq M^2{}_{AU}(x) + N^2{}_{AU}(x) + (\alpha^2 + \beta^2)(1 - 1)$$
$$\leq M^2{}_{AU}(x) + N^2{}_{AU}(x)$$
$$\leq 1.$$

(ii). $$D_\alpha(A) = \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + (1-\alpha)^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right] > \mid x \in \mathrm{X}\right\} \quad (2)$$

$$F_{\alpha,\beta}(A) = \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \beta^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right] > \mid x \in \mathrm{X}\right\}$$

Put β =1- α in $F_{\alpha,\beta}(A)$, we have

$$F_{\alpha,1-\alpha}(A) = \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + (1-\alpha)^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right] > \mid x \in \mathrm{X}\right\} \quad (3)$$

From (2) and (3), we have

$$D_\alpha(A) = F_{\alpha,1-\alpha}(A).$$

(iii). Now $$F_{\alpha,\beta}(A) = \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \beta^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right] > \mid x \in \mathrm{X}\right\}$$

Put $\alpha = 0$ and $\beta = 1$ in $F_{\alpha,\beta}(A)$, we have

$$F_{0,1}(A) = \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + 0^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + 1^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right] > \mid x \in \mathrm{X}\right\}$$
$$= \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + 1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)}\right] > \mid x \in \mathrm{X}\right\}$$
$$= \left\{< x, [M_{AL}(x), M_{AU}(x)], \left[N_{AL}(x), \sqrt{1 - M^2{}_{AU}(x)}\right] > \mid x \in \mathrm{X}\right\}$$
$$= \Box A$$

Therefore, $\Box A = F_{0,1}(A)$

(iv). Now $$F_{\alpha,\beta}(A) = \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \beta^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right] > \mid x \in \mathrm{X}\right\}$$

Put $\alpha = 1$ and $\beta = 0$ in $F_{\alpha,\beta}(A)$ we have

$$F_{1,0}(A) = \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + 1^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + 0^2\big(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\big)}\right] > \mid x \in \mathrm{X}\right\}$$

$$= \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + 1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x)}\right] > \mid x \in \mathrm{X}\right\}$$
$$= \left\{< x, \left[M_{AL}(x),\ \sqrt{1 - N^2{}_{AU}(x)}\ \right],\ [N_{AL}(x), N_{AU}(x)] > \mid x \in \mathrm{X}\right\}$$
$$= \lozenge A$$

Therefore, $\lozenge A = F_{1,0}(A)$

(v). Let $A = \{< x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)] > | x \in \mathrm{X}\ \}$,

$\bar{A} = \{< x, [N_{AL}(x), N_{AU}(x)], [M_{AL}(x), M_{AU}(x)] > | x \in \mathrm{X}\ \}$

Then,

$$F_{\alpha,\beta}(\bar{A}\,) = \left\{< x, \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right],\right.$$
$$\left.\left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right] > \mid x \in \mathrm{X}\right\}$$
$$\overline{F_{\alpha,\beta}(\bar{A}\,)} = \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \beta^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right],\right.$$
$$\left.\left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right] \mid x \in \mathrm{X}\right\}$$
$$= F_{\beta,\alpha}\,(A)$$

Therefore, $\overline{F_{\alpha,\beta}(\bar{A}\,)} = F_{\beta,\alpha}\,(A)$

Proposition 3.3: *For every IVIFSST A, we have the following*

(i). $\lozenge\left(D_1(A)\right) = \lozenge A$

(ii). $\square\left(D_0(A)\right) = \square A$

(iii). $D_\alpha(\square A) = \square A$

(iv). $D_\alpha(\lozenge A) = \lozenge A$

Proof: Let $A = \{< x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)] > | x \in \mathrm{X}\ \}$,

Then

$$\square A = \left\{< x, [\mathrm{M}_{AL}(x), \mathrm{M}_{AU}(x)], \left[\mathrm{N}_{\mathrm{AL}}(x), \sqrt{1 - M^2{}_{AU}(x)}\right] > \mid x \in \mathrm{X}\right\}$$

and

$$\lozenge A = \left\{< x, \left[\mathrm{M}_{AL}(x), \sqrt{1 - N^2{}_{AU}(x)}\right], [\mathrm{N}_{\mathrm{AL}}(x), \mathrm{N}_{\mathrm{AU}}(x)] > \mid x \in \mathrm{X}\right\}$$

Now,

$$D_\alpha(A) = \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + \alpha^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right],\right.$$
$$\left.\left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + (1-\alpha)^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right] > \mid x \in \mathrm{X}\right\}$$

(i). Put $\alpha = 1$ in $D_\alpha(A)$, we have

$$D_1(A) = \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + (1)^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right],\right.$$
$$\left.\left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + (1-1)^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right] > \mid x \in \mathrm{X}\right\}$$
$$= \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + 1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)}\right],\right.$$
$$\left.\left[N_{AL}(x), \sqrt{N^2{}_{AU}(x)}\right] > \mid x \in \mathrm{X}\right\}$$
$$D_1(A) = \left\{< x, \left[M_{AL}(x), \sqrt{1 - N^2{}_{AU}(x)}\right], [N_{AL}(x), N_{AU}(x)] > \mid x \in \mathrm{X}\}\right.$$
$$\lozenge\,(D_1(A)) = \left\{< x, \left[M_{AL}(x), \sqrt{1 - N^2{}_{AU}(x)}\right],\ [N_{AL}(x), N_{AU}(x)] > \mid x \in \mathrm{X}\}\right.$$
$$= \lozenge A$$

Therefore, $\lozenge\,(D_1(A)) = \lozenge A$

(ii). Put $\alpha = 0$ in $D_\alpha(A)$, we have

$$D_0(A) = \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x) + (0)^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right],\right.$$
$$\left.\left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + (1-0)^2\left(1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)\right)}\right] > \mid x \in \mathrm{X}\right\}$$
$$= \left\{< x, \left[M_{AL}(x), \sqrt{M^2{}_{AU}(x)}\right], \left[N_{AL}(x), \sqrt{N^2{}_{AU}(x) + 1 - M^2{}_{AU}(x) - N^2{}_{AU}(x)}\right] > \mid x \in \mathrm{X}\right\}$$
$$D_0(A) = \left.\{< x, [M_{AL}(x), M_{AU}(x)], \left[N_{AL}(x),\ \sqrt{1 - M^2{}_{AU}(x)}\right] > \mid x \in \mathrm{X}\right\}$$

Now,

$$\Box(D_0(A)) = \{< x, [M_{AL}(x), M_{AU}(x)],\ \left[N_{AL}(x), \sqrt{1 - M^2_{AU}(x)}\right] > \mid x \in \mathrm{X}\Big\}$$
$$= \Box A$$

Therefore, $\Box(D_0(A)) = \Box$ A

(iii). Let $A = \{< x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)] > | x \in \mathrm{X}\ \}$ and

$$\Box\ A = \{< x, [M_{AL}(x), M_{AU}(x)], \left[N_{AL}(x),\ \sqrt{1 - M^2_{AU}(x)}\right] > \mid x \in \mathrm{X}\Big\}$$

Now,

$$D_\alpha(\Box\ A) = \Big\{< x, \left[M_{AL}(x), \sqrt{M^2_{AU}(x) + \alpha^2(1 - M^2_{AU}(x) - (1 - M^2_{AU}(x)))}\right], \left[N_{AL}(x), \sqrt{1 - M^2_{AU}(x) + (1-\alpha)^2(1 - M^2_{AU}(x) - (1 - M^2_{AU}(x)))}\right] > \mid x \in \mathrm{X}\Big\}$$
$$= \Big\{< x, \left[M_{AL}(x), \sqrt{M^2_{AU}(x)}\right], \left[N_{AL}(x), \sqrt{1 - M^2_{AU}(x)}\right] > \mid x \in \mathrm{X}\Big\}$$
$$= \{< x, [M_{AL}(x), M_{AU}(x)], \left[N_{AL}(x), \sqrt{1 - M^2_{AU}(x)}\right] > \mid x \in \mathrm{X}\Big\}$$
$$= \Box A$$

Hence, $D_\alpha(\Box\ A) = \Box\ A$

(iv). Let $A = \{< x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)] > | x \in \mathrm{X}\ \}$ and

$$\Diamond\ A = \Big\{< x, \left[M_{AL}(x), \sqrt{1 - N^2_{AU}(x)}\right], [N_{AL}(x), N_{AU}(x)] > \mid x \in \mathrm{X}\}$$

Now,

$$D_\alpha(\Diamond\ A) = \left\{< x, \left[M_{AL}(x), \sqrt{1 - N^2_{AU}(x) + \alpha^2\left(1 - \left(1 - N^2_{AU}(x)\right) - N^2_{AU}(x)\right)}\right], \left[N_{AL}(x), \sqrt{N^2_{AU}(x) + (1-\alpha)^2\left(1 - \left(1 - N^2_{AU}(x)\right) - N^2_{AU}(x)\right)}\right] > \mid x \in \mathrm{X}\right\}$$
$$= \Big\{< x, \left[M_{AL}(x), \sqrt{1 - N^2_{AU}(x)}\right], \left[N_{AL}(x), \sqrt{N^2_{AU}(x)}\right] > \mid x \in \mathrm{X}\Big\}$$
$$= \Big\{< x, \left[M_{AL}(x), \sqrt{1 - N^2_{AU}(x)}\right], [N_{AL}(x), N_{AU}(x)] > \mid x \in \mathrm{X}\}$$
$$= \Diamond\ A$$

Hence, $D_\alpha(\Diamond\ A) = \Diamond\ A$.

4. CONCLUSION

We have defined the new modal operators like necessity, possibility, D_α and $F_{\alpha,\beta}$ on IVIFSST and also established some of their properties. It is still open to define some more operators on IVIFSST.

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