International Journal of Mathematical Archive-8(8), 2017, 45-50
Available online through www.ijma.info ISSN 2229 – 5046

A STUDY ON SINGULAR VALUE DECOMPOSITION AND IT'S APPLICATIONS IN IMAGE PROCESSING

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(Received On: 14-07-17; Revised & Accepted On: 01-08-17)

ABSTRACT

This paper discusses about the application of Singular Value Decomposition (SVD) i.e. in Image processing. We use singular Value decomposition to decompose large data into smaller size. In this paper we discuss the mathematics behind SVD. By using Matlab we can find the singular value decomposition and the application of SVD to image compression without data loss. In Linear Algebra, SVD are the most useful tools to reduce any matrix into smaller matrices.

1. INTRODUCTION

Every $m \times n$matrix A can be representing an image. In the m x n matrix we have the elements mn. Each element represents a color block in the image. Now we have to reconstruct the image by using SVD. In the process for getting the original image, we need some iteration. The number of iterations depends upon the rank of "A". This process can be done easily by using singular value decomposition and the rank of A is number of nonzero singular values of A. If large data is given, we compress data without loss of data, and then we get the data by SVD.

2. MATHEMATICAL DISCUSSION ABOUT SVD

Let A be a $m \times n$ matrix with rank 'r', can be decomposed into the form $= UDV^T$; where D is a diagonal $m \times n$ matrix with real entries

$$\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \cdots \sigma_r \geq 0;$$
$$r \leq \min(m, n)$$

And U and V are orthogonal matrices such that U is $m \times m$ and V is $n \times n$ matrix. The diagonal matrix of D are $\sigma_1, \sigma_2, \sigma_3, \dots \sigma_r$ are called the singular value of A and the square root of positive eigen values of AA^T and A^TA .

The column of the matrix U are called the left singular vector of the matrix A and the column of the matrix V are called the right singular vectors of the matrix A. the matrix A can be approximated as $A_i = U_i \mathrm{D} \mathrm{V}_{\mathrm{i}}^{\mathrm{T}}$ where i corresponds to the first i rows and i column of each individual matrix A, U, V.

3. LINEAR TRANSFORMATION

Let $\bar{x}$ be vector, $A\bar{x}$ is known as performing a linear transforming of$\bar{x}$. This maps any vectors $\bar{x}$ onto the vector $A\bar{x}$ and it is denoted by $\bar{x} \to A\bar{x}$.

Invertible Square matrix

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Let $S = A^T A$ such that S is $n \times n$ invertible square matrix in the form $S = [\overline{S_1}\,\overline{S_2} \ldots \overline{S_\iota} \ldots \overline{S_n}]$

Where $\overline{S_\iota} = \begin{bmatrix} S_{i1} \\ S_{i2} \\ \ldots \\ S_{in} \end{bmatrix}$ $S = \begin{bmatrix} S_{11} & S_{12} & \ldots & S_{1n} \\ S_{21} & S_{22} & \ldots & S_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ S_{n1} & S_{n2} & \ldots & S_{nn} \end{bmatrix}$

Let $\|S\bar{x}\|$ is maximized for $\bar{x} = \overline{V_1}$, where $\overline{V_1}$ is the eigen vector of S corresponding to the strictly dominant eigenvalue. Since $\{\overline{V_1}\,\overline{V_2}\,\overline{V_3}\ \ldots \overline{V_n}\}$ is a orthogonal set of the unit eigenvector of S, then the matrix V is a $n \times n$ orthogonal matrix.

For any $m \times n$ matrix A there exists a factorization $A = U\mathrm{D}V^T$

If we know that matrix D, V and A, so a matrix U can be obtained in terms of the remaining three matrices

$$A = U\mathrm{D}V^T$$
$$AV = (U\mathrm{D})(V^T V)$$
$$AV = U\mathrm{D}I$$
$$AV = U\mathrm{D}$$
$$AV\mathrm{D}^{-1} = U$$

We can consider the matrix D as an $m \times n$ matrix in the form

$$D = \begin{bmatrix} \sigma_1 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & \ldots & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & \ldots & 0 & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots. & \ldots & \ldots & 0 & 0 \\ 0 & 0 & 0 & \ldots & \sigma_r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ 0 & \ldots & \ldots & \ldots. & \ldots. & 0 & 0 & 0 \end{bmatrix}$$

Definition: The rank of a matrix is the number of linearly independent vectors in the column space of the matrix.

4. IMAGE PROCESSING BY USING SVD WITH THE HELP OF MAT LAB

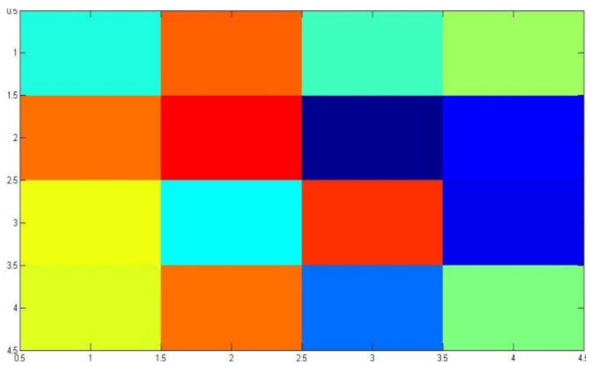
Example 1: To create a matrix A of order 4×4 with a rank of 4 and integer values ranging -60 to +60 using Matlab's SVD command find the matrices U, D and V corresponding to A. Using the Matlab command to discuss about the construction of image from matrix A and also discuss the reconstruction of the image of matrix A.

To create a matrix of random integersthe easiest way is to use the **randint** command. The command with this parameter reads

>> A = randint(4, 4, 60, 4);

$$A = \begin{bmatrix} 26 & 50 & 28 & 34 \\ 49 & 56 & 0 & 8 \\ 39 & 24 & 53 & 7 \\ 38 & 49 & 15 & 32 \end{bmatrix}$$

>>image(A);



Original image

Then, all that is required is ability to type the next line into the command

>> [U, D, V] = svd(A);

$$U = \begin{bmatrix} -0.5170 & -0.0452 & 0.6316 & 0.5760 \\ -0.5128 & 0.5343 & -0.6193 & 0.2607 \\ -0.4407 & -0.8178 & -0.3654 & -0.0591 \\ -0.5249 & 0.2091 & 0.2899 & -0.7725 \end{bmatrix}$$

$$V = \begin{bmatrix} -0.5674 & 0.0233 & -0.5897 & -0.5743 \\ -0.6810 & 0.4019 & 0.0800 & 0.6069 \\ -0.3426 & -0.9117 & 0.0916 & 0.2074 \\ -0.3115 & 0.0815 & 0.7984 & -0.5088 \end{bmatrix}$$

$$D = \begin{bmatrix} 133.4309 & 0 & 0 & 0 \\ 0 & 45.4848 & 0 & 0 \\ 0 & 0 & 29.1036 & 0 \\ 0 & 0 & 0 & 6.8052 \end{bmatrix}$$

If you wish it is possible to confirm the rank of the matrix A by typing the command:

```
>> Rank (A)
```

Seeing that all the singular values of A are non-zero then the rank of A is 4 confirmed.
With this mind entering a 4X4 matrix random integer should give a picture of 16 square blocks compressing one large block. The rank of A is 4 then we get the original image in 4 iterations

```
>>C=D;
for N=[1 2 3 4];
C(N+1:end,:)=0;
C(:,N+1:end)=0;
K=U*C*V';
figure;
image(K);
error=C-K;
end
```

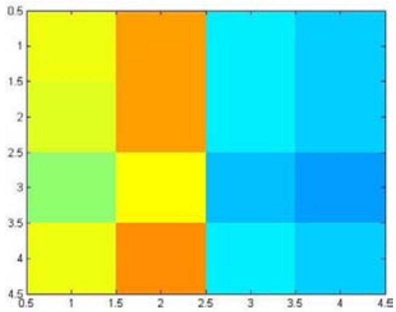


Figure-1: First Iteration

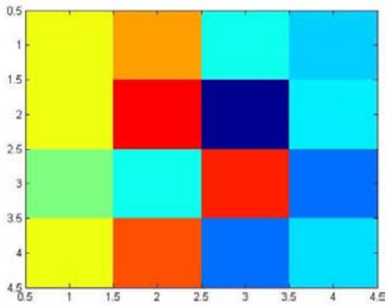


Figure-2: Second Iteration

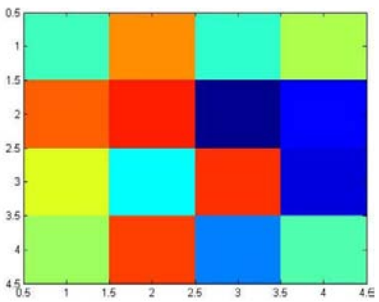


Figure-3: Third Iteration

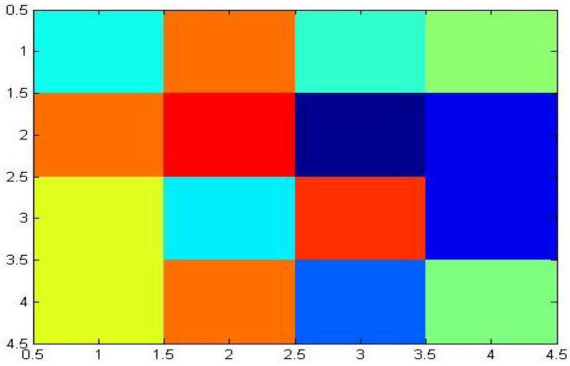


Figure-4: Fourth Iteration

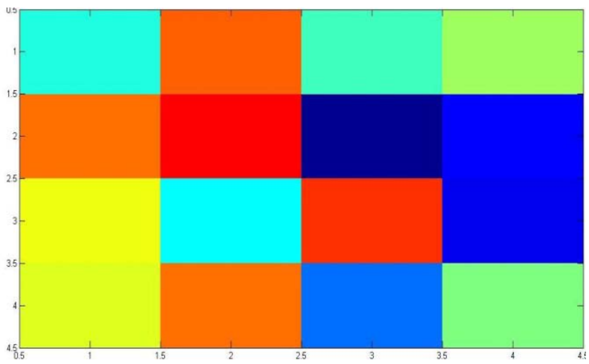


Figure-4: Original image

Example 2: using Mat lab construct A to be 20X15 matrix of random integer from -78 to +78 with rank 15. An exact representation of the original image should be obtained 15^{th} iteration

```
>>A=randint(20,15,78,15);
>>image (A);
>>[U,D,V]=svd(A);
>> SVD image(A,U,D,V):
```

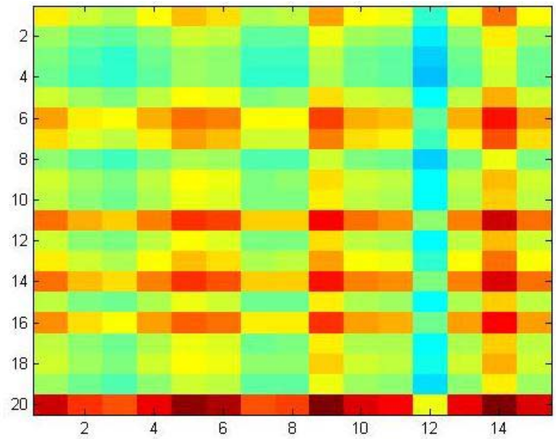


Figure-5: First Iteration

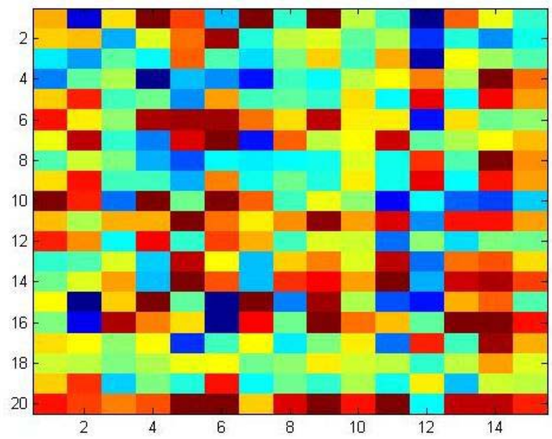


Figure-6: Fourth Iteration

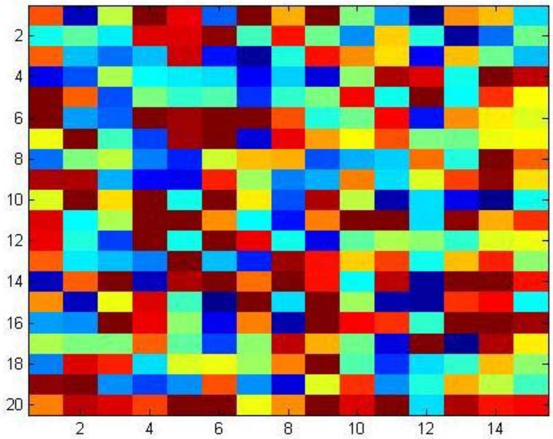


Figure-7: Eighth Iteration

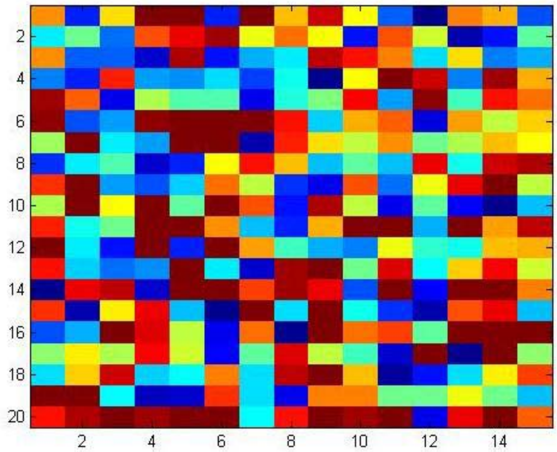


Figure-8: Twelve Iterations

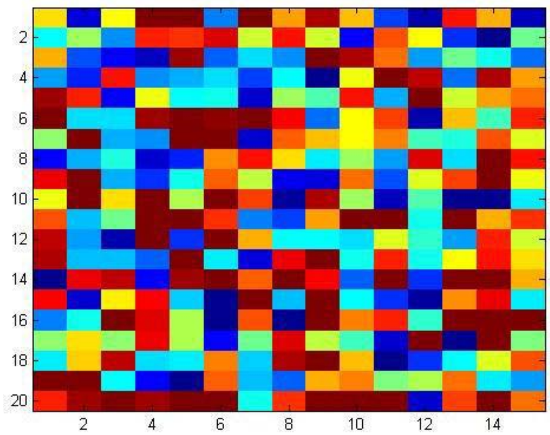


Figure-9: Fifteenth Iteration

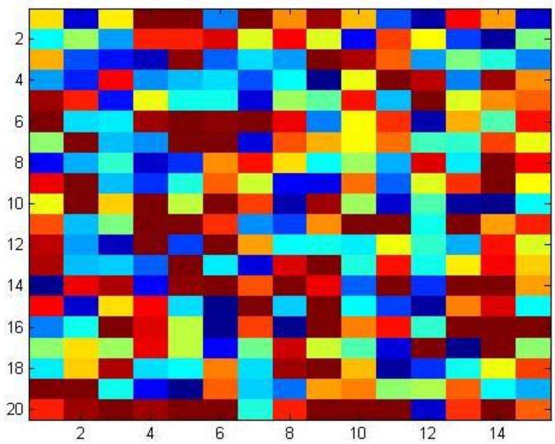


Figure-10: Original image

Notice that our good enough matrix A_{12} is a matrix of 144 entries is approximately equal to original image.

Example 3: The following image is 256X256 matrix or pixels

Figure-11: Original image

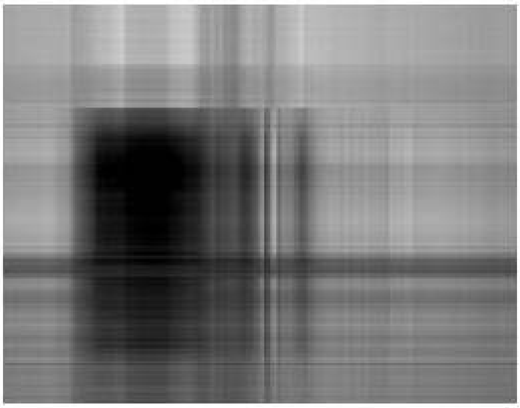


Figure-12: First Iteration

Figure-13: 10^{th} Iteration

Figure-14: 25^{th} Iteration

As you can see, after only ten *iterations y*ou can obtain the rough image.

Figure-15: 40^{th} Iteration

Figure-16: 65^{th} Iteration

By 25^{th} iteration the picture is clearly evident. By 65^{th} iterations we have essentially the original iterations. A 65X65 matrix with 4225 entries is significantly is reduced compare to a 256X256 matrix with 65536 entries.

5. CONCLUSION

It is known that Singular value decomposition is very much useful in linear algebra. This paper shows this the application of SVD in image processing. The paper demonstrates how an image represented by a matrix can be compressed into a smaller sized matrix without data loss. It is demonstrated that SVD is useful in image compression and hence in image processing .

6. REFERENCES

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Source of support: Nil, Conflict of interest: None Declared.